Chapter 4

Interest Rates and Rates of Return

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I. Debt Instruments: Bonds

- Discount (Zero-coupon)
- Coupon

} Payment type

- Treasurys
- Municipals
- Corporate

} Issuer

Quality/Risk

- Junk/High Yield vs. Investment Grade

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Discount Bonds

- Discount bond so-called because it sells at a “discount” i.e. at less than its face or payoff value
- Owner makes the difference between the sale price and the face value (aka par value)
- Example: Discount bond sells for $900 on 1 September 2000 pays $1000 on 1 September 2001

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Zero-Coupon Bonds

- Discount bonds are also called “zero-coupon” bonds because, well, they make no coupon payments

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Coupon Bonds

- similar to fixed-payment loan
- coupon = fixed payment or “interest payment”
- face or par = principal
- Example: Coupon Bond with a par value of $1,000 and a coupon of 6% per annum would pay $60 interest per year
- IMPORTANT: Coupon f(rate) + f(par value)

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Variable Coupon Bonds

- Most coupon bonds pay FIXED coupons
- i.e. the rates are fixed.
- Par value $1,000 w/ coupon rate of 10% = $100 per year REGARDLESS OF WHAT HAPPENS TO MARKET INTEREST RATES
- PRICE of the bond adjusts NOT the coupon rate/payment
- A FEW BONDS, HOWEVER, PAY VARIABLE COUPONS = f(market rates)

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Mechanics of Pricing

- Now that we know the contractual characteristics of various financial instruments, we need to know ...

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Calculating Return or Realized Yield

- Two concepts help us to calculate RETURN:
  - "yield" (or discount rate *i*)
  - "present value" or today's value of a payment to be received in the future when the interest rate is given

Various ways to
measure or estimate

- **YIELD TO MATURITY IS THE MOST ACCURATE MEASUREMENT OF YIELD or INTEREST RATE**

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Time Value of Money

- A dollar today is worth more than a dollar received at some future date.
- Money may be spent on consumption or saved by investing in real capital assets (e.g. machinery) or by buying financial assets (e.g. deposits or stock).

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Present Value

- The value today (at present) of a sum received at a future date discounted at the required rate of return.

$$PV = FV \frac{1}{(1+i)^n}$$

PV = FV/(1+*i*)^n

- Given the time value of money, one is indifferent between the present value today or the future value received in the future.

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Future Value or Compound Value

- The future value (FV) of a sum (PV) is

  FV = PV (1+i)^n.
- (1+i)^n is referred to as the ***Future Value Interest Factor***.
- Multiply by the dollar amount involved to calculate the FV of an investment.

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Valuing a Financial Asset

- There are two necessary ingredients for valuing financial assets.
  - Estimates of future cash flows.
    - The estimates include the timing and size of each cash flow.
  - An appropriate discount rate.
    - The discount rate must reflect the risk of the asset.
  - Or, if the price is known, the discount rate can be determined

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Pricing Zero Coupon Bonds

- Bonds that do not pay periodic interest payments are called zero-coupon bonds.
- Zero coupon bonds trade at a discount.
- The value of the "zero" bond is

$$PB = \frac{F_n}{(1+i)^n}$$

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Zero Coupon Example

$$PB = \frac{F_n}{(1+i)^n}$$

Face value = $1000; i = 5%; matures in 1 year

Price = $1000/(1+.05)^1$
Price = $958.32

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Basic Coupon Bond Pricing Formula

- The stream of coupon payments on a fixed rate bond is an annuity which allows the pricing of a bond with the following formula:

$$PB = \frac{C_1/m}{(1+i/m)^1} + \frac{C_2/m}{(1+i/m)^2} \ldots + \frac{C_{mn}/m + F_{mn}}{(1+i/m)^{mn}}$$

WHERE: PB = bond price; C = coupon payment; *i* = discount rate expressed as a decimal; n = number of periods to maturity; m = no. of times interest is compounded

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Example: Coupon Bond Pricing

$$PB = \frac{C_1/m}{(1+i/m)^1} + \frac{C_2/m}{(1+i/m)^2} + \frac{C_{mn}/m + F_{mn}}{(1+i/m)^{mn}}$$

Bond = 4 years to maturity, 10 percent coupon, paid semiannually, current market interest percent, $1,000 face or par value

Last coupon plus Principal

$$PB = \frac{(100/2)}{1050} + \frac{50}{} + 50 \ldots +$$

Twice a year
Four years

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Yield to Maturity: Discount Bonds

- The interest rate that equates the present value of payments received from a credit market instrument with its value today.
- There is no reinvestment of coupon payments with zeros and thus, no reinvestment risk. The yield to maturity, i, is the actual yield received if held to maturity.

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Calculating Yield to Maturity: Discount Bonds

- EQUATION: $i = (F - P_d) / P_d$
  - where F = face value of the discount bond
  - P_d = current price of the discount bond
- EXAMPLE: What is the yield to maturity for a discount bond with a $1,000 face value, a market price of $950, and a maturity of one year?
- $i = (1000-950)/950 = 50/950 = .0526 = 5.26\%$
- EXAMPLE: What is the yield to maturity for a discount bond with a $100 face value, a market price of $50, and a maturity of one year?
- $i = (100-50)/50 = 50/50 = 1 = 100\%$

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Discount Bond Equation for Varying Maturities

- what if the maturity is 11 months? 6 days? Etc.
- For terms other than one year, solve for *i* as above, then use that term in the equation: $[(1+i)^n - 1]$ where n = total number time units in a year/number of time units until maturity
- EXAMPLE: What is the yield to maturity for a discount bond with a \$100 face value, a market price of \$50, and a maturity of eleven months? $[(1+1)^{12/11} - 1] = [2.13 - 1] = 1.13$ or 113%
- a maturity of 16 days? $[(1+1)^{365/16} - 1]$

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Yield to Maturity: Coupon Bonds

- Can be a MAJOR PAIN
- Example What is the yield of a bond if:
  - 30 years to maturity
  - 7.765 percent coupon paid monthly
  - par value = \$1,000
  - current market price = \$1,067.22
- Solution:

Calculators and Bond Tables

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Sample Bond Table

10.00% Bond Values per \$100 of Face Value

	Years to Maturity									
Yield (%)	1	2	3	4	5	6	7	8	9	10
10.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00	100.00
10.25	99.77	99.56	99.37	99.20	99.04	98.90	98.77	98.66	98.55	98.46
10.50	99.54	99.12	98.74	98.40	98.09	97.82	97.56	97.34	97.13	96.95
10.75	99.31	98.68	98.12	97.61	97.16	96.75	96.38	96.04	95.74	95.47
11.00	99.08	98.25	97.50	96.83	96.23	95.69	95.21	94.77	94.38	94.02
11.25	98.85	97.82	96.89	96.06	95.32	94.65	94.05	93.52	93.04	92.61
11.50	98.62	97.39	96.28	95.30	94.41	93.63	92.92	92.29	91.72	91.22
11.75	98.39	96.96	95.68	94.54	93.52	92.61	91.80	91.08	90.44	89.86
12.00	98.17	96.53	95.08	93.79	92.64	91.62	90.71	89.89	89.17	88.53
12.25	97.94	96.11	94.49	93.05	91.77	90.63	89.62	88.73	87.93	87.23
12.50	97.72	95.69	93.90	92.31	90.91	89.66	88.56	87.58	86.72	85.95
12.75	97.49	95.28	93.32	91.59	90.06	88.71	87.51	86.46	85.52	84.70

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Examples:

- What is the yield to maturity of a bond with 10 years to maturity, a \$100 face value, a 10% coupon, that sells for \$95.47?
- What is the yield to maturity of a bond with 1 year to maturity, a \$1000 face value, a 10% coupon, that sells for \$974.90?
- What is the price of a \$1000 par value bond with 5 years to maturity if the discount rate is 12 percent?

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MAJOR POINTS:

- As bond prices INCREASE, yields DECREASE
- And as bond prices DECREASE, yields INCREASE
- OR, as yields INCREASE, bond prices DECREASE
- And as yields DECREASE, bond prices INCREASE

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Major Point Reiterated

TABLE 1 Yields to Maturity on a 10 percent Coupon Rate Bond Maturing in Ten Years (Face Value = \$1000)

Price of Bond (\$)	Yield to Maturity (%)
1200	7.13
1100	8.48
1000	10.00
900	11.75
800	13.81

Estimating Yield to Maturity for Coupon Bonds: Current Yield

- since calculating the yield to maturity for coupon bonds (with maturity dates) is so difficult, it is acceptable under certain circumstances to use CURRENT YIELD instead.
- EQUATION: current yield = $i_c = c/P_b$, where:
  - c = yearly coupon payment
  - P_b = current price of the coupon bond

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Current Yield Examples:

- EXAMPLE: What is the current yield of a coupon bond with a yearly coupon payment of $100 and a current price of $1000?
- $i_c = 100/1000 = .1 = 10\%$
- EXAMPLE 2: What is the current yield of a coupon bond with a face value of $1000, a coupon rate of 5%, that currently sells for $900?
- $i_c = 50/900 = .055 = 5.5\%$
- Note that the coupon payment = face value of the coupon bond * coupon rate

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Acceptable Use of Current Yield:

- The current yield formula may be used instead of the much more difficult yield to maturity formula IFF (if and only if):
- #1 the coupon bond's term to maturity is > 20 years
- #2 the coupon bond's market price is close to its par or face price

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Potential Sources of a Bond's Return

- Periodic coupon interest payments
- Income from reinvestment of the periodic interest payments (interest-on-interest)
- Any capital gain or capital loss when the bond matures or is sold

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Interest Rate Risk

- Reinvestment risk– the risk that a decline in interest rates will lead to a decline in income from a bond portfolio.
- Price risk– the risk of a decline in a bon's price due to an increase in interest rates.
- Price risk and reinvestment risk offset one another, depending upon maturity and coupon rates.

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Total Rate of Return

- Also called realized yield.
- The total rate of return is the sum of current yield and actual capital gain or loss.
- Rate of return can differ from yield to maturity.
- The formula for total rate of return is: $R = C/P_t + (P_{t+1} - P_t)/P_t$.

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Realized Yields

- The realized yield is the ex-post, actual rate of return, given the cash flows actually received and their timing. Realized yields may differ from the promised yield to maturity due to:
  - A change in the amount and timing of the promised cash flows.
  - A change in market interest rates since the purchase of the bond, thus affecting the reinvestment rate of the coupons.
  - The bond may be sold before maturity at a market price varying from par price /purchase price.

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Returns Example:

- EXAMPLE #1: We purchase a bond for $1,000, that pays a yearly coupon of $100, and sell it in one year for $1,100.
- RET = (100 + 1100 - 1000)/1000 = (100 + 100)/1000 = 20%
- Notice that 10% came from the coupon payment and 10% from an increase in the market price of the bond.
- EXAMPLE #2: We purchase a bond for $1,000, that pays of yearly coupon of $100, and sell it in one year for $900.
- RET = (100 + 900-1000)/1000 = 0%

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Negative Returns:

- Returns, therefore, can become negative. In other words, bond investors can LOSE money, up to 100%.
- EXAMPLE #3: We purchase a bond for $1,000, that does not pay a coupon, and sell it in one year for $1.
- RET = (0 + 1-1000)/1000 = -99.99%
- Increases in interest rates are the problem here. As interest rates increase, the price of bonds decreases. Those who bought in when prices were high lose out.

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Real Returns

- We must keep 2 other things in mind before we can decide which investments to undertake:
- 1. Tax implications
  - income and capital gains from municipal bonds are exempt from federal and domestic state taxes
  - this, and other tax issues, should be considered before investing
- 2. Real vs. Nominal Interest rates
  - Fisher equation:
  - $i_r = i - \pi^e$, or real i = nominal i - expected inflation

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Think about…

- If current price = face value, then yield to maturity = current yield = coupon rate.
- If current price < face value, then yield to maturity > current yield > coupon rate.
- If current price > face value, then yield to maturity < current yield < coupon rate.

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