Econ 312: Introduction to Econometrics

Review of Probability I

Sang-Yeob Lee

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Key Definitions

- A population is the collection of all items of interest or under investigation (N)
  We will think of populations as infinitely large (or “very big”)
- A sample is an observed subset of the population
- A parameter is a specific characteristic of a population
- A statistic is a specific characteristics of a sample

Random variables

- Numerical summary of a random outcome
- Discrete random variable: takes only discrete set of values
- Continuous random variable: takes on a continuum of possible values

Probability Distribution of a Discrete Random Variable

- Probability distribution: the list of all possible values of the variable and the probability that each value will occur.
- $\sum_x Pr(X = x) = 1$
- Example:
  - Let X be the number of times your computer crash while you are writing a term paper
  - the probability that $X = 0$ (no computer crash), denoted $Pr(X = 0)$
  - $Pr(X = 1)$ is the probability of a single computer crash; and so forth.

ctd.

- Cumulative probability distribution: the probability that the random variable is less than or equal to a particular value.
- $F(x_0) = Pr(X \leq x_0)$
- referred to as a cumulative distribution function, a c.d.f or a cumulative distribution.
- Example, $Pr(X \leq 1)$ is the probability of at most one crash.

Probability of Distribution of a Continuous Random Variable

- Because a continuous random variable can take on a continuum of possible values, the probability distribution use for discrete variables is not suitable for continuous variable.
- Note that $Pr(X = x) = 0$ when X is continuous.
- Instead, the probability is summarized by the probability density function (p.d.f).
- The area under the probability density function between any two points is that probability that the random variable falls between those two points.
- $Pr(a \leq X \leq b) = \int_b^a f_X(x)dx$

Expected Values

- expected value of a random variable X, $E(X)$
- $= \mu_x$
- =long-run average of X over repeated trials or occurrences.
- =mean of X
- $E(X) = \sum_i^k xPr(x_i)$ for a discrete random variable
- $E(X) = \int xf_X(x)dx$ for a continuous random variable

Example

Suppose you loan a friend \$100 at 10% interest. If the loan is repaid to you get \$110, but there is a risk of 1% that your friend will default and you will get nothing at all.

- Let X is the amount you are repaid.
- $P(X = 0) = 0.01$ and $P(X = 110) = 0.99$
- $E(X) = 0 \times 0.01 + 110 \times 0.99 = \$108.90$

Properties of Expected Values

1. For any constant c, $E(c) = c$
2. For any constants a and b, $E(aX + b) = aE(X) + b$
3. If $\{a_1, a_2, \ldots, a_n\}$ are constants and $\{X_1, X_2, \ldots, X_n\}$ are random variables, then $E(a_1X_1 + a_2X_2 + \cdots + a_nX_n) = a_1E(X_1) + a_2E(X_2) + \cdots + a_nE(X_n)$.

The Standard Deviation and Variance

measure the dispersion or the “spread” of a probability distribution.

- Variance:$\sigma_X^2 = E(X - \mu_x)^2$ (the expected value of the squared deviation of X form its mean)
- Standard deviation: $\sigma_X = \sqrt{variance}$
- $\sigma_X^2 = var(X) = E(X - \mu_x)^2 = \sum_{i=1}^{k}(x_i - \mu_x)^2 Pr(x_i)$ for a discrete random variable.

Properties of Variance and Standard Deviation

1. For any constant c, $Var(c) = 0$
2. For any constants a and b, $V(aX + b) = a^2 Var(X) + b$
3. For any constant a and b, $s.d(aX + b) = |a|\sigma_x$

Standardizing a Random Variable

Suppose that given a random variable X, we define a new random variable by subtracting off its mean μ_x and dividing by its standard deviation σ_x:

$Z = \frac{X - \mu_x}{\sigma_x}$

- Then, Z has expected value zero and variance one.

Skewness

measure of asymmetry of a distribution

- $\frac{E(X-\mu_x)^3}{\sigma^3}$
- skewness$=0$: distribution is symmetric
- skewness>0: distribution has long right tail
- skewness<0: distribution has long left tail

Kurtosis

measure of how much mass is its tails and, therefore, is a measure of how much of the variance X arise from extreme value.

- $\frac{E(X-\mu_x)^4}{\sigma^4}$
- kurtosis: measure of probability of large values
- kurtosis= 3: normal distribution
- kurtosis> 0: heavy tail

Exercises

1. Let X denote the number of "heads" that occurs when two coins are tossed.
    1. Derive the probability distribution of X.
    2. Derive the cumulative probability distribution of X.
    3. Derive the mean and variance of X.
2. The random variable X has a mean of 1 and a variance of 4. Let $Z = \frac{1}{2}(X - 1)$.Show that $\mu_z = 0$ and $\sigma_z^2 = 1$.