Econ 311: Problem Set #5

Due: Monday, December 8, 2008

Q.1 What is a simple random sample?

A simple random sample is selected such that every object has an equal probability of being selected and the objects are selected independently-the selection does not change the probability of probability of selecting any other objects.

Q.2 What is the Central Limit Theorem?

Let $X_1, X_2, \cdots, X_n$ be a set of n independent random variables having identical distribution with mean μ and variance σ^2, and with X as the sum and $\bar{X}$ as the mean of these random variables. As n become larger, the central limit theorem states that the distribution of

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

approaches the standard normal distribution.

Q.3 Given a population with mean $\mu = 400$ and variance $\sigma^2 = 1,600$, the central limit theorem applies when the sample size $n \geq 25$. A random sample of size $m = 35$ is obtained.

a What are the mean and variance of the sampling distribution for the sample means?
$\mu_{\bar{x}} = \mu = 400, \sigma^2_{\bar{x}} = \sigma^2/n = 1600/35 = 45.7143, \sigma_{\bar{x}} = \sqrt{45.7143}$

b What is the probability that $\bar{x} > 412$?
$P(\bar{x} > 412) = P(z > \frac{412-400}{\sqrt{45.7143}}) = 1 - F(1.77) = 0.0384$

c What is the probability that $393 \leq \bar{x} \leq 407$?
$P(393 \leq \bar{x} \leq 407) = P(\frac{393-400}{\sqrt{45.7143}} \leq z \leq \frac{407-400}{\sqrt{45.7143}}) = P(-1.04 \leq z \leq 1.04) = 0.7016$

d What is the probability that $\bar{x} < 389$?
$P(\bar{x} < 389) = \frac{389-400}{\sqrt{45.7143}} = P(z < -1.63) = 1 - F(1.63) = 1 - 0.9484 = 0.0516$

Q.4 According to the Internal Revenue Services, 75% of all tax returns lead to a refund. A random sample of 100 tax returns is taken.

a What is the mean of the distribution of the sample proportion of returns leading to refunds?
$E(\hat{p}) = 0.75$

b What is the variance of the sample proportion?
$\sigma^2_{\hat{p}} = \frac{0.75(1-0.75)}{100} = 0.001875$

c What is the standard error of the sample proportion?
$\sigma_{\hat{p}} = 0.0433$

d What is the probability that the sample proportion exceeds 0.8?
$P(Z > \frac{0.8-0.75}{0.0433} = P(z > 1.15) = 0.1251$

Q.5 Monthly rates of return on the shares of a particular common stock are independent of one another and normally distributed with a standard deviation of 1.7. A sample of 12 months is taken.

a Find the probability that the sample variance is less than 2.5.
$P(\frac{(n-1)s^2}{\sigma^2} < \frac{11(2.5)}{1.7^2}) = P(\chi^2_{11} < 9.519) =$ between 0.1 and 0.9

b Find the probability that the sample variance is more than 1.0. $P(\frac{(n-1)s^2}{\sigma^2} > \frac{11(1)}{1.7^2}) = P(\chi^2_{11} > 3.81) =$ between 0.975 and 0.99