Econ 311: Problem Set #5

Due: Monday, December 14, 2009

Q.1 What is a simple random sample?

A simple random sample is selected such that every object has an equal probability of being selected and the objects are selected independently-the selection does not change the probability of probability of selecting any other objects.

Q.2 What is the Central Limit Theorem?

Let $X_1, X_2, \cdots, X_n$ be a set of n independent random variables having identical distribution with mean μ and variance σ^2, and with X as the sum and $\bar{X}$ as the mean of these random variables. As n become larger, the central limit theorem states that the distribution of

$$Z = \frac{\bar{X} - \mu_{\bar{X}}}{\sigma_{\bar{X}}}$$

approaches the standard normal distribution.

Q.3 Given a population with mean $\mu = 200$ and variance $\sigma^2 = 625$, the central limit theorem applies when the sample size $n \geq 25$. A random sample of size $m = 25$ is obtained.

a What are the mean and variance of the sampling distribution for the sample means?

b What is the probability that $\bar{x} > 209$?

c What is the probability that $198 \leq \bar{x} \leq 211$?

d What is the probability that $\bar{x} < 202$?

Q.4 An administrator for a large group of hospitals believes that of all patients 30% will generate bills that become at least 2 month overdue. A random sample of 200 patients is taken.

a What is the standard error of the sample proportion that will generate that become at least 2 month overdue?

b What is the probability that the sample proportion is less than 0.25.

c What is the probability that the sample proportion is more that 0.33.

d What is the probability that the sample proportion is between 0.27 and 0.33?

Q.5 Monthly rates of return on the shares of a particular common stock are independent of one another and normally distributed with a standard deviation of 1.6. A sample of 12 months is taken.

a Find the probability that the sample variance is less than 2.5. $P(\frac{(n-1)s^2}{\sigma^2} < \frac{11(2.5)}{1.6^2}) = P(\chi^2_{11} < 10.742) =$ between 0.1 and 0.9

b Find the probability that the sample variance is more than 1.0. $P(\frac{(n-1)s^2}{\sigma^2} > \frac{11(1)}{1.6^2}) = P(\chi^2_{11} > 4.297) =$ between 0.975 and 0.950