Econ 311: Problem Set #4

Due: Monday, November 3, 2008

Q.1 (Uniform Distribution) A repair team is responsible for a stretch of oil pipeline 4 miles long.The distance (in mile) at which any fracture occurs can be represented by a uniformly distributed random variable, with probability density function

$$f(x) = 0.25$$

Find the cumulative function and the probability that any given fracture occurs between 1 mile and 2 miles along the stretch of pipeline.

Answer: The cumulative distribution function is $F(x_0) = \int_0^{x_0} 0.25dx = 0.25x_0$ for $0 < x_0 < 4$. The probability that a fracture occurs between 1 mile and 2 miles along the pipe is

$$P(1 < X < 2) = F(2) - F(1) = 0.25 \times 2 - 0.25 \times 1 = 0.25$$

This is the area under the probability density function from $x = 1$ to $x = 2$.

Q.2 The profit for a production process are equal to $2,000 minus 2 times the number of units produced. The mean and variance for the number of units produced are 500 and 900 respectively. Find the mean and variance of the profit.

Answer: $W = a + bX$. If the available funds=2000-2X, when X = number of units produced, find the mean and variance of the profit if the mean and variance for the number of units produced are 500 and 900 respectively. $\mu_w = a + b\mu_x = 2000 - 2 \times 500 = 1000$. $\sigma_w^2 = b^2\sigma_x^2 = (-2)^2 \times 900 = 3600$.

Q.3 Let the random variable X follow a normal distribution with $\mu = 50$ and $\sigma^2 = 64$.

a Find the probability that X is greater than 60. $P(Z > \frac{60-50}{8}) = P(Z > 1.25) = 1 - F(1.25) = 0.1056$

b Find the probability that X is greater than 35 and less than 62. $P(\frac{35-50}{8} < Z < \frac{62-50}{8}) = F(1.5) - (1 - F(1.875)) = 0.9028$

c Find the probability that X is less than 55. $P(Z < \frac{55-50}{8}) = F(0.625) = 0.734$

d The probability is 0.2 that X is greater than what number? $Z = 0.84$, so $X = 50 + 8 \times 0.84 = 56.72$

e The probability is 0.05 that X is in the symmetric interval about the mean between which two numbers? $Z = +/-0.06$. Thus, $X = 50 + (/-)0.06 \times 8$. $X = 49.52$ and 50.48.

Q.4 (Normal Probability) A client has an investment portfolio whose mean value is equal to $500, with a standard deviation of $15. She has asked you to determine the probability that the value of her portfolio is between $485 and $530.

Answer:$P(485 \leq X \leq 530) = P(-1 \leq Z \leq 2) = F(2) - (1 - F(1)) = 0.9722 - (1 - 0.8413) = 0.8185$.

Q.5(Exponential Probabilities) An industrial plaint in Britain with 1000 employees has a mean number of lost time accidents per week equal to $\lambda = 0.8$, and the number of accidents follows Possion distribution. What is the probability that the time between accidents is less than 2 weeks?

Answer: $P(T < 2) = F(2) = 1 - e^{-0.8\times 2} = 0.7981.$