Econ 311: Problem Set #3

Suggested Answers

Q.1 The number of computers sold per day at Dan's Computer works is defined by the following probability distribution:

X	0	1	2	3	4	5	6
P(x)	0.05	0.10	0.20	0.20	0.20	0.15	0.10

a $P(3 \leq x < 6)$? $P(3 \leq x < 6) = P(3) + P(4) + P(5) = 0.20 + 0.20 + 0.15 = 0.55$

b $P(x > 3)$? $P(x > 3) = P(4) + P(5) + P(6) = 0.20 + 0.15 + 0.10 = 0.45$

c $P(x \leq 4)$? $P(x \leq 4) = P(1) + P(2) + P(3) + P(4) = 1 - (P(5) + P(6)) = 1 - 0.15 - 0.10 = 0.75$

d $P(2 < x \leq 5)$? $P(2 < x \leq 5) = P(3) + P(4) + P(5) = 0.55$

Q.2 A corporation produces packages of paper clips. The number of clips packages varies, as indicated in the accompanying table.

Number of clips	47	48	49	50	51	52	53
Proportion of packages	0.04	0.13	0.21	0.29	0.20	0.10	0.03
Cumulative prob.	0.04	0.17	0.38	0.67	0.87	0.97	1.00

a Draw the probability function.

b Calculate and draw the cumulative probability function.

c What is the probability that a randomly chosen package will contain between 49 and 51 clips (inclusive)?

$P(49 \geq X \leq 51) = P(49) + P(50) + P(51) = 0.70.$

d Two packages are chosen at random. What is the probability that at least one of them contains at least 50 clips?

$1 - P(X < 50) \times P(X < 50) = 0.8556$ (or $P(X \geq 50) + P(X \geq 50) - P(X \geq 50)P(X \geq 50)$)

e Find the mean and standard deviation of the number of paper clips per package.

$E(X) = \sum_x xP(x) = 47(0.04)+48(0.13)+49(0.21)+50(0.29)+51(0.20)+52(0.10)+53(0.03) = 49.9$

$\sigma_x = \sqrt{(47-49.9)^2(0.04)+(48-49.9)^2(0.13)+(49-49.9)^2(0.21)+(50-49.9)^2(0.29)+(51-49.9)^2(0.20)+(52-49.9)^2(0.10)+(53-49.9)^2(0.03)} = 1.3694$

f The cost (in cents) of producing a package of clips is $16+2X$, where X is the number of clips in the package. The revenue from selling the package, however many clips it contains is \$1.50. If profit is defined as the difference between revenue and cost, find the mean and standard deviation of profit per package.

Profit=1.5 - (0.16 + .02X), So $E(Profit) = 1.5 - 0.16 - 0.02E(X) = \$0.342$ and $\sigma_{\text{profit}} = 0.02\sigma_x = \$0.0279$.

Q.3 A production manager knows that 5% of components produced by a particular manufacturing process have some defect. six of these components, whose characteristics can be assumed to be independent of each other, are examined.

Solution: Let X be the number of components have a defect. Then, X follows a binomial distribution with $n = 6$ and $P = 0.05$.

a What is the probability that none of these components has a defect?

$P(X=0) = C_0^6 0.05^0(1-0.05)^6 = 0.7351.$

b What is the probability that one of these components has a defect?

$P(X=1) = C_1^6 0.05^1(1-0.05)^5 = 0.2321.$

c What is the probability that at least two of these components have a defect?

$P(X \geq 2) = 1 - P(X<2) = 1 - (P(X=0)+P(X=1)) = 1 - 0.7351 - 0.2321 = 0.0328.$

Q.4 A family of mutual funds maintains a services that allows clients to switch money among accounts through a telephone call. It was estimated that 3.2% of callers either get a busy signal or are kept on hold so long that they may hang up. Fund management assesses any failure of this sort as a \$10 goodwill loss. Suppose that 2,000 calls are attempted over a particular period.

a Find the mean and standard deviation of the number of callers who will either get a busy signal or may hang up after being kept on hold.

Let X be the number of callers either get a busy signal or are kept on hold so long. Then, X follows a binomial distribution with $n = 2000$ and $P = 0.032$. It is a binomial distribution with $n = 2,000$ and $P = 0.032$. So, $E(X) = nP = 2,000 \times 0.032 = 64$ and $\sigma_x = \sqrt{nP(1-P)} = \sqrt{2,000 \times 0.032(1-0.032)} = 7.871$.

b Find the mean and standard deviation of the total goodwill loss to mutual fund company from these 2,000.

Let Z be the total goodwill loss. Then, $Z = 10X$. So, $E(Z) = 10E(X) = \$640$ and $\sigma_z = 10\sigma_x = \$78.71$.

Q.5 A bond analyst was given a list of 12 corporate bonds. From that list she selected 3 whose ratings she felt were in danger of being downgraded in the next year. In actuality, a total of 4 of the 12 bonds on the list had their ratings downgrade in the next year. Suppose that the analyst had simply chosen 3 bonds randomly from this list. What is the probability that at least 2 of the chosen bonds would be among those whose ratings were to be downgrading in the next year?

Solution: If "downgrade" is identified as "success", the lists contains $N = 12$ corporate bonds and $S = 4$ of the 12 bonds that are successes. A sample of 3 bonds are selected. Then number of success,X, in the sample has a hypergeometric distribution with the probability function.
So, we need to find $P(X \geq 2) = P(2) + P(3)$ (or $1 - P(X < 2) = 1 - P(0) - P(1)$).
The probability of no success in the sample is

$$P(0) = \frac{C_0^4 C_3^8}{C_3^{12}} = 0.2545$$

The probability of 1 success in the sample is

$$P(1) = \frac{C_1^4 C_2^8}{C_3^{12}} = 0.5091$$

Therefore, we find that the probability that at least 2 of the chosen bonds would be among those whose ratings were to be downgrading in the next is $P(X \geq 2)$=0.2364.

Q.6 Customers arrive at a busy checkout counter at an average rate of three per minute. If the distribution of arrivals is Poisson, find the probability that in any given minute there will be two or fewer arrivals.

Solution: Since the mean number of arrivals in a minute is 3, then $\lambda = 3$. So, we need to find that $P(X \leq 2) = P(0) + P(1) + (2)$.

$$P(X = 0) = \frac{e^{-3}3^0}{0!} = 0.0498$$

$$P(X = 1) = \frac{e^{-3}3^1}{1!} = 0.1494$$

$$P(X = 2) = \frac{e^{-3}3^2}{2!} = 0.2240$$

Thus, the probability of two or fewer arrivals in a minute is

$$P(X \leq 2) = 0.0498 + 0.1494 + 0.2240 = 0.4232$$

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