

Econ 311: Problem Set #2

Due: Monday, September 29, 2008

Q.1 A corporation takes delivery of some new machinery that must be installed and checked before it becomes available to use. The corporation is sure that it will take no more than 7 days for this installation and check to take place. Let A be the event “It will be more than 4 days before the machinery becomes available” and B be the event “It will be less than 6 days before the machinery becomes available.”

- Describe the event that is the complement of event A .
- Describe the event that is the intersection of event A and B .
- Describe the even that is the union of events A and B .
- Are events A and B mutually exclusive?
- Are events A and B collectively exhaustive?
- Show that $(A \cap B) \cup (\bar{A} \cap B) = B$.
- Show that $A \cup (\bar{A} \cap B) = A \cup B$.

Solution:

- The complement of event A is that it will take 4 days or less before the machinery become operational
- The intersection of A and B will be the event that it takes 5 days before the machinery become operational.
- The union of A and B is the event of 1 day, 2days, 3days, 4days, 5 days, 6 days or 7 days.
- A and B are not mutually exclusive because $P(A \cup B) \neq 0$.
- Yes, A and B are collectively exhaustive because they include all of the possible same points. f. $(A \cup B)$ is the event that it takes 5 days. $(\bar{A} \cup B)$ is the event that is takes 4 days, 3 days, 3 days, 1 day. The union of between these two events is that it takes less than 6 days (1 through 5) before the machinery is operational. This is the definition of event B , therefore $(A \cap B) \cup (\bar{A} \cap B) = \text{event } B$.
- $(\bar{A} \cup B)$ is the event that it takes 4 days, 3 days, 2 days, 1day. Since A is event 5days, 6 days, 7 days. Then $A \cup (\bar{A} \cup B)$ will the event of 1 through 7 days. This is the event of $A \cup B$. Therefore, $A \cup (\bar{A} \cap B)$ must equal $A \cup B$.

Q.2 A mutual fund company has six funds that invest in the U.S market and four that invest in international markets. A customer wants to invest in two U.S funds and two international funds.

- How many different sets of funds from this company could the investor choose?
- Unknown to this investor, one of the U.S funds and one of the international funds will seriously underperform next year. If the investor selects funds for purchases at random, what is the probability that at least one of the chosen funds will seriously underperform next year?

Solution: Let A -selecting two U.S funds and B -selecting two international funds. First notice that the selections are independent.

C_2^6 different sets of two US funds and C_2^4 different sets of two foreign funds. Since the selections are independent, there are $C_2^6 \times C_2^4$ different sets of funds from which to choose.

b. you need to find $P(\text{at least one fund under performs})$. Because $P(A) = 1 - P(\bar{A})$,
 $P(\text{at least one fund under performs}) = 1 - P(\text{no fund under performs})$. Also, it can be seen that
 $P(\text{no fund under performs}) = P(\text{no U.S fund under performs}) \times P(\text{no foreign fund under performs})$ since two events are independent.

$P(\text{no US fund under performs}) = \frac{C_2^5}{C_2^6} = 10/15$. $P(\text{no foreign fund under performs}) = \frac{C_2^3}{C_2^4} = 3/6$. So,
 $P(\text{no fund under performs}) = P(\text{no U.S fund under performs}) \times P(\text{no foreign fund under performs}) = 10/15 \times (3/6) = 2/3$.

Q.3 A stock market analyst claims expertise in picking stocks that will outperform the corresponding industry norm. This analyst is presented with a list of five high-technology stocks and a list of five air-line stocks, she is invited to nominate, in order, the three stocks that will do best on each of these two lists over the next year. The analyst claims that success in just one of these two tasks would be a substantial accomplishment. If, in fact, the choices are made randomly and independently, what is the probability of success in at least one of the two tasks merely by chance? Given this result, what do you think of the analyst's claim?

Solution: Let A -picking 3 high tech stocks in order and B - picking 3 airline stocks in order out of five. The number of ways of choosing three high tech stocks out of 5 is P_3^5 . So, $P(A) = 1/60$. Similarly, you can find $P(B) = 1/60$.

Notice that two events are independent. Then, $P(A \cup B) = 1/60 + 1/60 - 1/3600 = 0.331$. Therefore, the probability of getting either the high tech stocks or the airline stocks picked correctly is relatively small and would be an accomplishment.

Q.4 An analyst attempting to predict a corporation's earning next year believes that the corporation's business is quite sensitive to the level of interest rates. She believes that, if average rates in the next year are more than 1% higher than this year, the probability of significant earnings growth is 0.1. If average rates next year are more than 1% lower than this year, the probability of significant earnings growth is estimated to be 0.8. Finally, if average interest rates next year are within 1% of this year's rates, the probability for significant earnings growth is put at 0.5. The analyst estimates that the probability is 0.25 that rates next year will be more than 1% higher than this year and 0.15 that they will be more than 1% lower than this year.

- What is the estimated probability that both interest rates will be 1% higher and significant earnings growth will results?
- What is the probability that this corporation will experience significant earnings growth?
- If the corporation exhibits significant earnings growth, what is the probability that interest rates will have been more than 1% lower than in the current year?

Solution: Let H = Average rate in the next year are more than 1% higher than this year

L = Average rate in the next year are more than 1% lower than this year

S = Average rate in the next year are within 1% of this year's rates

G = Significant earnings growth.

Given that we know $P(G|H) = 0.1$, $P(G|L) = 0.8$, $P(G|S) = 0.5$, $P(H) = 0.25$, $P(L) = 0.15$, and $P(S) = 0.6$.

a. Find $P(G \cap H)$.

$$P(G \cap H) = P(G|H)P(H) = 0.1 \times 0.25 = 0.024.$$

b. Find $P(G)$.

$$P(G) = P(G \cap H) + P(G \cap L) + P(G \cap S) = P(G|H)P(H) + P(G|L)P(L) + P(G|S)P(S) = 0.445.$$

c. Find $P(L|G)$.

$$P(L|G) = \frac{P(G \cap L)}{P(G)} = \frac{P(G|L)P(L)}{P(G)} = 0.2697.$$

Q.5 A stock market analyst examined the prospects of the shares of a large number of cooperations. When the performance of these stocks was investigated one year later, it turned out that 25% performed much better than the market average, 25% much worse, and the remaining 50% about the same as the average. Forty percent of the stocks that turned out to do much better than the market were rated "good buys" by the analyst, as were 20% of those that did about as well as the market and 10% of those that did much worse. What is the probability that a stock rates a "good buy" by the analyst performed much better than the average?

Solution: Bayes' Theorem Let E_1 = Stock performs much better than the market average,

E_2 = Stock performs same as the market average

E_3 = Stock performs worse than the market average

A = Stock is rated a 'Buy'.

Given that $P(E_1) = 0.25$, $P(E_2) = 0.5$, $P(E_3) = 0.25$, $P(A|E_1) = 0.4$, $P(A|E_2) = 0.2$, $P(A|E_3) = 0.1$.

Now, we need to find $P(E_1|A)$.

By applying Bayes' Thm, we have

$$\begin{aligned} P(E_1|A) &= \frac{P(A|E_1)P(E_1)}{P(A|E_1)P(E_1) + P(A|E_2)P(E_2) + P(A|E_3)P(E_3)} \\ &= \frac{0.4 \times 0.25}{0.4 \times 0.25 + 0.2 \times 0.5 + 0.1 \times 0.5 + 0.1 \times 0.25} \\ &= 0.444 \end{aligned}$$