San Francisco State University
Department of Economics

Econ 312/Spring 2010
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# Econ 312: Problem Set #1

## (Solutions to Problem Set)

1.The table shows that $\Pr(X=0, Y=0)=0.045$, $\Pr(X=0, Y=1)=0.709$, $\Pr(X=1, Y=0)=0.005$, $\Pr(X=1, Y=1)=0.241$, $\Pr(X=0)=0.754$, $\Pr(X=1)=0.246$, $\Pr(Y=0)=0.050$, $\Pr(Y=1)=0.950$.

(a)

$$\begin{aligned} E(Y) = \mu_Y &= 0 \times \Pr(Y=0) + 1 \times \Pr(Y=1) \\ &= 0 \times 0.050 + 1 \times 0.950 = 0.950. \end{aligned}$$

(b)

$$\begin{aligned} \text{Unemployment Rate} &= \frac{\#(\text{unemployed})}{\#(\text{labor force})} \\ &= \Pr(Y=0) = 0.050 = 1 - 0.950 = 1 - E(Y). \end{aligned}$$

(c) Calculate the conditional probabilities first:

$$\Pr(Y=0|X=0) = \frac{\Pr(X=0, Y=0)}{\Pr(X=0)} = \frac{0.045}{0.754} = 0.0597,$$

$$\Pr(Y=1|X=0) = \frac{\Pr(X=0, Y=1)}{\Pr(X=0)} = \frac{0.709}{0.754} = 0.9403,$$

$$\Pr(Y=0|X=1) = \frac{\Pr(X=1, Y=0)}{\Pr(X=1)} = \frac{0.005}{0.246} = 0.0203,$$

$$\Pr(Y=1|X=1) = \frac{\Pr(X=1, Y=1)}{\Pr(X=1)} = \frac{0.241}{0.246} = 0.9797.$$

The conditional expectations are

$$\begin{aligned} E(Y|X=1) &= 0 \times \Pr(Y=0|X=1) + 1 \times \Pr(Y=1|X=1) \\ &= 0 \times 0.0203 + 1 \times 0.9797 = 0.9797, \end{aligned}$$

$$\begin{aligned} E(Y|X=0) &= 0 \times \Pr(Y=0|X=0) + 1 \times \Pr(Y=1|X=0) \\ &= 0 \times 0.0597 + 1 \times 0.9403 = 0.9403. \end{aligned}$$

(d) Use the solution to part (b),

Unemployment rate for college grads

$$= 1 - E(Y|X=1) = 1 - 0.9797 = 0.0203.$$

Unemployment rate for non-college grads

$$= 1 - E(Y|X=0) = 1 - 0.9403 = 0.0597.$$

(e) The probability that a randomly selected worker who is reported being unemployed is a college graduate is

$$\Pr(X=1|Y=0)=\frac{\Pr(X=1,Y=0)}{\Pr(Y=0)}=\frac{0.005}{0.050}=0.1.$$

The probability that this worker is a non-college graduate is

$$\Pr(X=0|Y=0)=1-\Pr(X=1|Y=0)=1-0.1=0.9.$$

(f) Educational achievement and employment status are not independent because they do not satisfy that, for all values of *x* and *y*,

$$\Pr(Y=y|X=x)=\Pr(Y=y).$$

For example,

$$\Pr(Y=0|X=0)=0.0597\neq\Pr(Y=0)=0.050.$$

2. Using obvious notation, $C=M+F$; thus $\mu_C=\mu_M+\mu_F$ and $\sigma_C^2=\sigma_M^2+\sigma_F^2+2\,\mathrm{cov}(M,F)$. This implies

(a) $\mu_C=40+45=\$85,000$ per year.

(b) $cor(M,F)=\frac{Cov(M,F)}{\sigma_M\sigma_F}$, so that $Cov(M,F)=\sigma_M\sigma_F cor(M,F)$. Thus $Cov(M,F)=12\times 18\times 0.80=172.80$, where the units are squared thousands of dollars per year.

(c) $\sigma_C^2=\sigma_M^2+\sigma_F^2+2\,\mathrm{cov}(M,F)$, so that $\sigma_C^2=12^2+18^2+2\times 172.80=813.60$, and $\sigma_C=\sqrt{813.60}=28.524$ thousand dollars per year.

(d) First you need to look up the current Euro/dollar exchange rate in the Wall Street Journal, the Federal Reserve web page, or other financial data outlet. Suppose that this exchange rate is *e* (say *e* = 0.80 euros per dollar); each 1$ is therefore with *e*E. The mean is therefore $e\mu_C$ (in units of thousands of euros per year), and the standard deviation is $e\sigma_C$ (in units of thousands of euros per year). The correlation is unit-free, and is unchanged.