

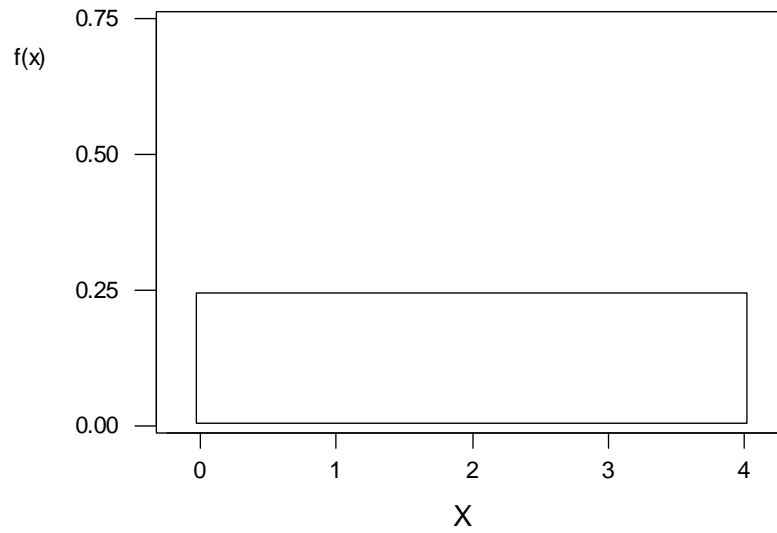
Econ 311: Problem Set #4

Suggested Solutions

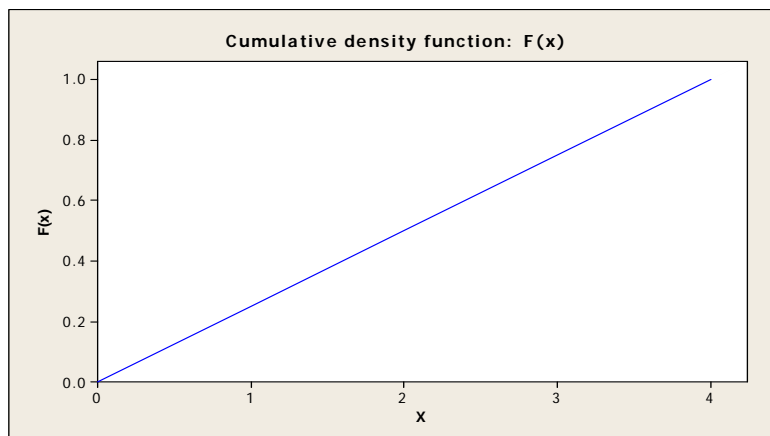
Question.1

a.

Probability density function: $f(x)$



b.



c. $P(x < 1) = .25$

d. $P(X < .5) + P(X > 3.5) = P(X < .5) + 1 - P(X < 3.5) = .25$

Question.2

$$\mu_Y = 6,000 + .08 \mu_X = 6,000 + 48,000 = \$54,000$$

$$\sigma_Y = |.08| \sigma_X = .08(180,000) = \$14,400$$

Question.3

X follows a normal distribution with $\mu = .2$ and $\sigma^2 = .0025$

- Find $P(X > .4)$. $P(Z > \frac{.4 - .2}{.05}) = P(Z > 4.00) = .5 - .5 = .0000$
- Find $P(.15 < X < .28)$. $P(\frac{.15 - .2}{.05} < Z < \frac{.28 - .2}{.05}) = P(-1.00 < Z < 1.60) = .3431 + .4452 = .7883$
- Find $P(X < .10)$. $P(Z < \frac{.10 - .20}{.05}) = P(Z < -2.00) = .5 - .4772 = .0228$
- Probability is .2 that X is greater than what number? $Z = .84$. $.84 = \frac{X - .20}{.05}$
 $X = .242$

Question.4

- Find $P(X < .15 \times 400)$. $E(X) = nP = 80$, $\text{Var}(X) = nP(1 - P) = 64$. So,
 $P(Z < (60 - 80)/8) = P(Z < -2.5) = 1 - F_Z(2.50) = .0062$
- Probability is .11 the percentage of successes is greater than? $Z = 1.23 = (X - 80)/8$. $X = 89.84$ and
Percent = $89.84/400 \times 100\% = 22.46\%$

Question.5

$$1 - e^{-5 \times 5} = 0.00$$

Question.6

Find the mean and variance of the random variable: $W = 5X - 4Y$ with correlation of $-.5$.

$$\mu_W = a\mu_x - b\mu_y = 5(1000) - 4(200) = 4200$$

$$\sigma_W^2 = a^2\sigma_x^2 + b^2\sigma_y^2 - 2ab\text{Corr}(X, Y)\sigma_x\sigma_y$$

$$= 5^2(500) + 4^2(400) - 2(5)(4)(-.5)(22.3607)(20) = 27,844.28$$