

Econ 311: Midterm II

Solutions

Please do not turn this page over until instructed to do so.

Instructions (Please Read Carefully Before Starting)

- This test has a total of **100 points**. Unless otherwise instructed, you have 1h 50m to solve it, that is, 110 minutes. There are 15 multiple choice questions (each is worth 3 points) and 4 written questions 20, 5, 15, and 15 points respectively).
- Show your work, unless you are explicitly told not to ! No credit will be given for correct answers if you do not justify your argument.
- Please be sure that your handwriting is **legible!**
- We will grade only what is written on your exam sheet. There should be plenty of space for all your answers. **Do not turn in anything aside from your exam sheet.**
- If time is running short, you should try to set up the problem without doing the final calculations.

Name : _____

Signature: _____

Multiple Choice (Just answer writing the letter corresponding to the statement you believe to be correct.)

Question	Answer
1	A
2	B
3	C
4	B
5	A
6	D
7	C
8	C
9	C
10	B
11	D
12	B
13	D
14	B
15	A
Score	

Part I. Multiple Choice (15 questions worth 3 points each).

1. Which of the following is not a true statement about the binomial probability distribution?
 - A) The random variable of interest is continuous.
 - B) Each outcome is independent of each other.
 - C) Each outcome can be classified as either success or failure.
 - D) The probability of success must be constant from trial to trial.

2. If the outcomes of a discrete random variable follow a Poisson distribution, then their
 - A) mean equals the standard deviation.
 - B) mean equals the variance.
 - C) median equals the variance.
 - D) median equals the standard deviation.

3. When sampling without replacement from a finite population such that the probability of a success, is no longer constant from trial to trial, the data must follow a
 - A) binomial distribution
 - B) Poisson distribution
 - C) hypergeometric distribution
 - D) None of the above

4. Which of the following statements regarding a binomial experiment is false, where n is the number of trials, and p is the probability of success in each trial?
 - A) The n trials are independent.
 - B) The standard deviation is $np(1 - p)$.
 - C) The mean is np .
 - D) There are only two possible outcomes.

5. Suppose that 19% of all sales are for amounts greater than \$1,000. In a random sample of 30 invoices, what is the probability that more than six of the invoices are for over \$1,000? Use the binomial approximation to the normal distribution, without the correction for continuity.
 - A) 0.4443
 - B) 0.9440
 - C) 0.5557
 - D) 0.0560

6. The area under the probability density function for the uniform distribution is between:
 - A) $-\infty$ and $+\infty$

- B) 0 and 1
- C) three standard deviations from the mean of the distribution
- D) the maximum and minimum values of the random variable

7. Which of the following is false regarding the normal distribution?

- A) The mean, median and mode are equal
- B) The frequency of values peaks at the mean, regardless of the value of the mean or variance
- C) 100% of the values fall between ± 3 standard deviations
- D) The shape of the distribution is symmetrical around the mean

8. Let X_1, X_2, \dots, X_k be a set of k continuous random variables. The function $F(x_1, x_2, \dots, x_k)$ which defines the probability that simultaneously $X_1 < x_1, X_2 < x_2$, and so on is called

- A) marginal distribution function
- B) marginal cumulative distribution function
- C) joint cumulative distribution function
- D) normal distribution function

9. Let the random variable Z follow a standard normal distribution. Find the value k , such that $P(-k < Z < k) = 0.78$.

- A) 1.78
- B) 1.37
- C) 1.23
- D) 0.78

10. Investment A has an expected return of 7.8% with a standard deviation of 2%. Investment B has an expected return of 7.2% with a standard deviation of 3.1%. Assume the returns on both of these stocks are normally distributed. Which stock is more likely to have a return greater than 10%?

- A) Stock A
- B) Stock B
- C) The probability is the same for both A and B.
- D) Unable to determine.

11. Let $a = -\mu/\sigma_x$ and $b = 1/\sigma_x$ in the linear function $Z = a + bX$. Which of the following statements is true?

- A) $\mu_x = \mu_x$
- B) $\sigma_z = 0$
- C) $\mu_z = 1$
- D) $\sigma_z = 1$

12. Suppose X and Y are two random variables with $E(X) = 1.50$, $E(Y) = 0.55$, $E(XY) = 0.80$, $Var(X) = 0.25$ and $Var(Y) = 0.2475$. What is the value of $Corr(X, Y)$?

- A) 6.533
- B) -0.1005
- C) 3.0151
- D) -0.6356

13. If Z is a standard normal random variable, then $P(-1.25 < Z < -0.75)$ is

- A) 0.6678.
- B) 0.1056.
- C) 0.2266.
- D) 0.1210.

14. The amount of time you have to wait at a particular stoplight is uniformly distributed between zero and two minutes. What is the probability that you have to wait between 15 and 45 seconds for the stoplight?

- A) 0.15
- B) 0.25
- C) 0.35
- D) 0.45

15. Checkout times in a Wal-Mart store follow an exponential distribution with a mean time of 8 minutes. What is the probability that the checkout time for the next customer will be 6 minutes or less?

- A) 0.528
- B) 0.472
- C) 0.972
- D) 0.328

Part II. Written Questions.

Question 1. (20pts) The San Francisco Giant are to play a series of six games in New York against the New York Yankees. For any one game it is estimated that the probability of a Giant win is 0.6. The outcomes of the six games are independent of one another.

a. (4pts) What is the probability that the Giant will win all six games?

$$P(x = 6) = C_9^6 0.6^6 * (1 - 0.4)^0 = 0.6^6 = 0.0467$$

b. (4pts) What is the probability that the Giant will win a majority of the six games?

Binomial with $n = 6$ and $p = 0.6$. So, $P(x) = C_x^n 0.6^n (1 - 0.4)^{n-x}$
 $P(x \geq 4) = P(x = 4) + P(x = 5) + P(x = 6) = 0.544$

c. (4pts) If the Giant win the first two games, what is the probability that they will a majority of the six games?

Binomial with $n = 4$ and $p = 0.6$. So, $P(x) = C_x^n 0.6^n (1 - 0.4)^{n-x}$
 $P(x \geq 2) = P(x = 2) + P(x = 3) + P(x = 4) = 0.821$

d.(4pts) Before the series begins, what is the expected number of Giant wins in these six games?

$$E(X) = np = 6 \times 0.6 = 3.6$$

e.(4pts) If the Giant win the first three games, what is the expected number of Giant's wins in the six game series?

$$E(X) = 3 + np = 3 + 3 \times 0.6 = 4.8$$

Question.2(5pts) A package of six light bulbs contains 2 defective bulbs. If three bulbs are selected for use, find the probability none are defective.

Hypergeometric with $N=6$, $S=2$, $n=3$.

$$P(x = 0) = \frac{C_0^6 \times C_3^2}{C_3^6} = 0.2$$

Question.3(15pts) An investment portfolio contains stocks of a large number of corporations. Over the last year the rates of return on these corporate stocks followed a normal distribution with mean 12.2% and standard deviation 7.2%.

a.(5pts) For what proportion of these corporations was the rate of return higher than 20%?

$$P(X > 20) = P\left(Z > \frac{20-12.2}{7.2}\right) = 1 - F(1.083) = 1 - 0.8599 = 0.1401$$

b.(5pts) For what proportion of these corporations was the rate of return negative?

$$P(X < 0) = P\left(Z < \frac{0-12.2}{7.2}\right) = 1 - F(1.694) = 1 - 0.9546 = 0.0454$$

c.(5pts) For what proportion of these corporations was the rate of return between 5% and 15%?

$$P(5 < X < 15) = P\left(\frac{5-12.2}{7.2} < Z < \frac{15-12.2}{7.2}\right) = F(0.39) - [1 - F(1)] = 0.6480 - 1 + 0.8413 = 0.4893$$

Question.4(15pts) An investor has \$100 to invest and two investment opportunities, each requiring a minimum of \$50. The profit per \$10 from the first can be represented by a random variable X, having the following probabilities function:

$$P(X = -5) = 0.4 \text{ and } P(X = 20) = 0.6$$

The profit per \$10 from the second is given by the random variable Y, whose probability function is

$$P(Y = 0) = 0.6 \text{ and } P(Y = 25) = 0.4$$

Random variables X and Y are independent. The investor has the following possible strategies:

1. \$100 in the first investment.
2. \$100 in the second investment.
3. \$50 in each investment.

a.(12 pts) Find the mean and variance of the profit from each strategy.

$$E(X) = \sum_x xP(x) = -0.5 \times 0.4 + 20 \times 0.6 = 10$$

$$E(Y) = \sum_y yP(y) = 0 \times 0.6 + 25 \times 0.4 = 10$$

$$Var(X) = \sum_x (x - \mu_x)^2 P(x) = (-0.5 - 10)^2 \times 0.4 + (20 - 10)^2 \times 0.6 = 150$$

$$Var(Y) = \sum_y (y - \mu_y)^2 P(y) = (0 - 10)^2 \times 0.6 + (25 - 10)^2 \times 0.4 = 150$$

1. $W_1 = 10X$: $E(W_1) = 10E(X) = 100$ and $Var(W_1) = 10^2 Var(X) = 15000$
2. $W_2 = 10Y$: $E(W_2) = 10E(Y) = 100$ and $Var(W_2) = 10^2 Var(Y) = 15000$
3. $W_3 = 5X + 5Y$: $E(W_3) = 10E(X) = 100$ and $Var(W_3) = 5^2 Var(X) + 5^2 Var(Y) = 7500$

b.(3 pts) Which should he choose?

Strategy 3 should be chosen since the strategy has the same expected profits as other strategies with a lower variance (risk).
