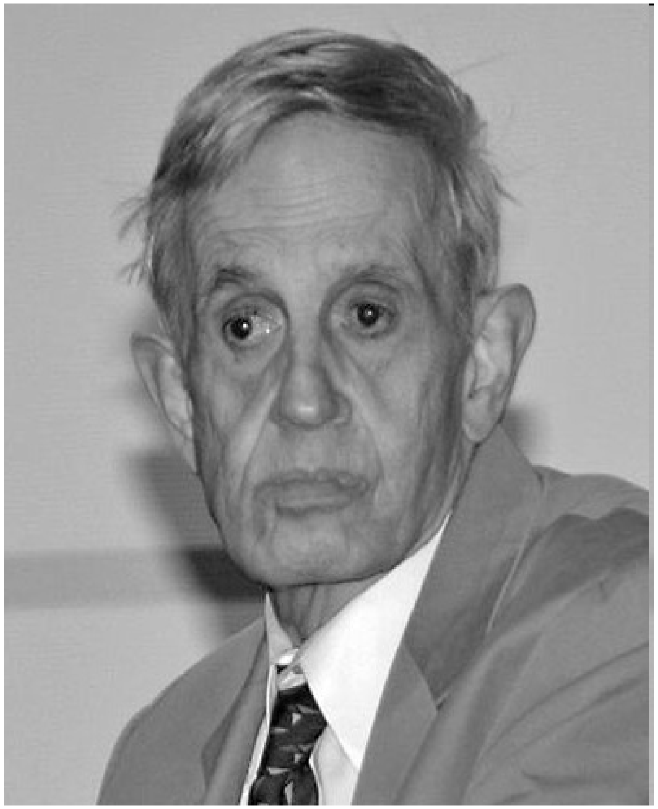
John F. Nash Jr.
(1928-)



Presented by: Chuangxing Wu

Background

- Nash was born in 1928 in Bluefield, West Virginia, USA, where he grew up.
- He was an undergraduate mathematics major at Carnegie Institute of Technology from 1945 to 1948.
- In 1948, he obtained both a B.S. and an M.S., and began graduate work in the Department of Mathematics at Princeton University. (One of his letters of recommendation, from a professor at Carnegie Institute of Technology, was a single sentence: “This man is a genius.”

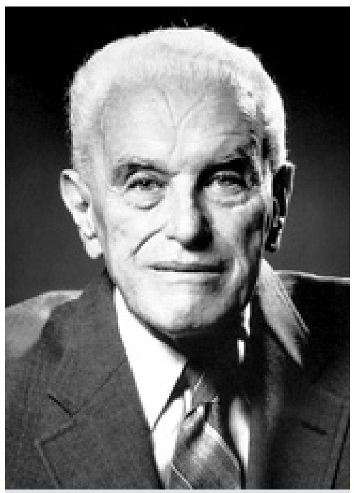
Background

- He completed his PhD the following year, graduating on his 22nd birthday.
- His thesis, introduces the equilibrium notion now known as “Nash equilibrium.”
- While in a graduate student at Princeton, Nash also wrote the seminal paper in bargaining theory.
- He went on to take an academic position in the Department of Mathematics at MIT, where he produced “a remarkable series of papers.”
- He shared the 1994 Nobel prize in economics with the game theorists John C. Harsanyi and Reinhard Selten.

Background

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 1994

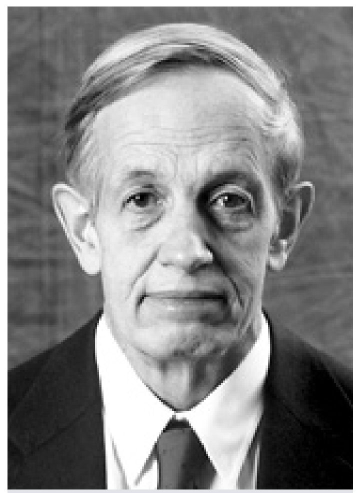
"for their pioneering analysis of equilibria in the theory of non-cooperative games"



John C. Harsanyi

1/3 of the prize

USA



John F. Nash Jr.

1/3 of the prize

USA

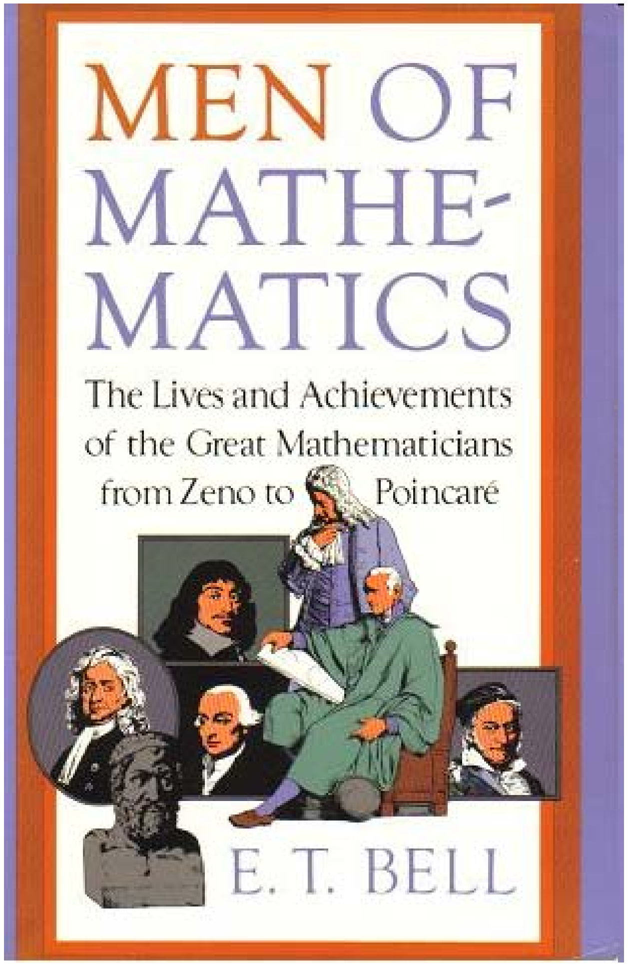
Reinhard Selten

1/3 of the prize

Federal Republic of Germany

Influenced by

- Men of Mathematics by mathematician E.T. Bell is a well-know book on the history of mathematics.
- To keep the interest of readers, the book typically focuses on unusual or dramatic aspects of the lives of mathematicians.
- It has inspired many young people, including the young John Nash, to become mathematicians.



Influenced by

- Antoine Augustin Cournot (1801-1877)
- Cournot competition model:
  - No product differentiation
  - No collusion
  - Firms have market power
  - Firms compete in quantities and choose quantities simultaneously
  - Profit maximization

Influenced by

- John Von Neumann and Oskar Morgenstern, who found the mathematical field of game theory

Pure Strategy

- **Pure Strategy** – a list of distinct actions a player can take each time she needs to make a decision.
- Example: prisoner's dilemma game,

Player 2

Player 1

	confess	silent
confess	-10,-10	0,-20
silent	-20,0	-1,-1

- Each player has 2 pure strategies: {**Confess**, **Remain silent**}.

Dominant strategy

- **Dominant strategy –** a strategy that gives the highest payoff, regardless of what the opponent plays.
- Problem – in real life it is almost impossible to find situations in which players have dominant strategies.
- Are there dominant strategies in the following games?
  - Chess
  - Sports games
  - Oligopoly (or economics in general)
  - Election campaign
  - International relations
  - Any other social situation?

Contribution– Nash equilibrium

- Q: How do we predict the outcomes of games, if in most situations players don't have dominant strategies?
- A: Nash developed the concept of Nash equilibrium, which does not require existence of dominant strategies.

Nash equilibrium

- **Best response –** Given a strategy for the opponent, the best response is the strategy that gives the highest payoff.
- **Nash equilibrium -** is a combination of strategies, such that each player's strategy is a best response to other players' strategies.
- Informally, no player wants to deviate from his strategy, once he found out the strategies of other players.

Example: Oligopoly and Cournot-Nash Equilibrium

- Market demand: P = 100 – Q
- Two firms: 1,2, producing outputs Q_1 and Q_2 with unit costs $10 per unit.
- Each firm chooses how much to produce, for any given output of the opponent.
- Pure strategies: $Q_i \in [0, Q_{\max}]$

Example: Oligopoly and Cournot-Nash Equilibrium

- Profit maximization:

$$\max_{Q_1} \pi_1 = (100 - Q_1 - Q_2)Q_1 - 10Q_1$$

$$\max_{Q_2} \pi_2 = (100 - Q_1 - Q_2)Q_2 - 10Q_2$$

Example: Oligopoly and Cournot-Nash Equilibrium

- Best response function of firm 1:

$$\frac{\partial}{Q_1}\pi_1 = 90 - 2Q_1 - Q_2 = 0$$

$$Q_1 = \frac{90 - Q_2}{2}$$

- If firm 2 produces {0, 10, 30, 60, 90}, what is the optimal output for firm 1? Are there any **dominant strategies?**
- Similarly, best response function of firm 2:

$$Q_2 = \frac{90 - Q_1}{2}$$

Example: Oligopoly and Cournot-Nash Equilibrium

- Nash equilibrium: $\{Q_1^*, Q_2^*\}$
- Such that these quantities are both best responses to each other.
- Solve the two best response equations:

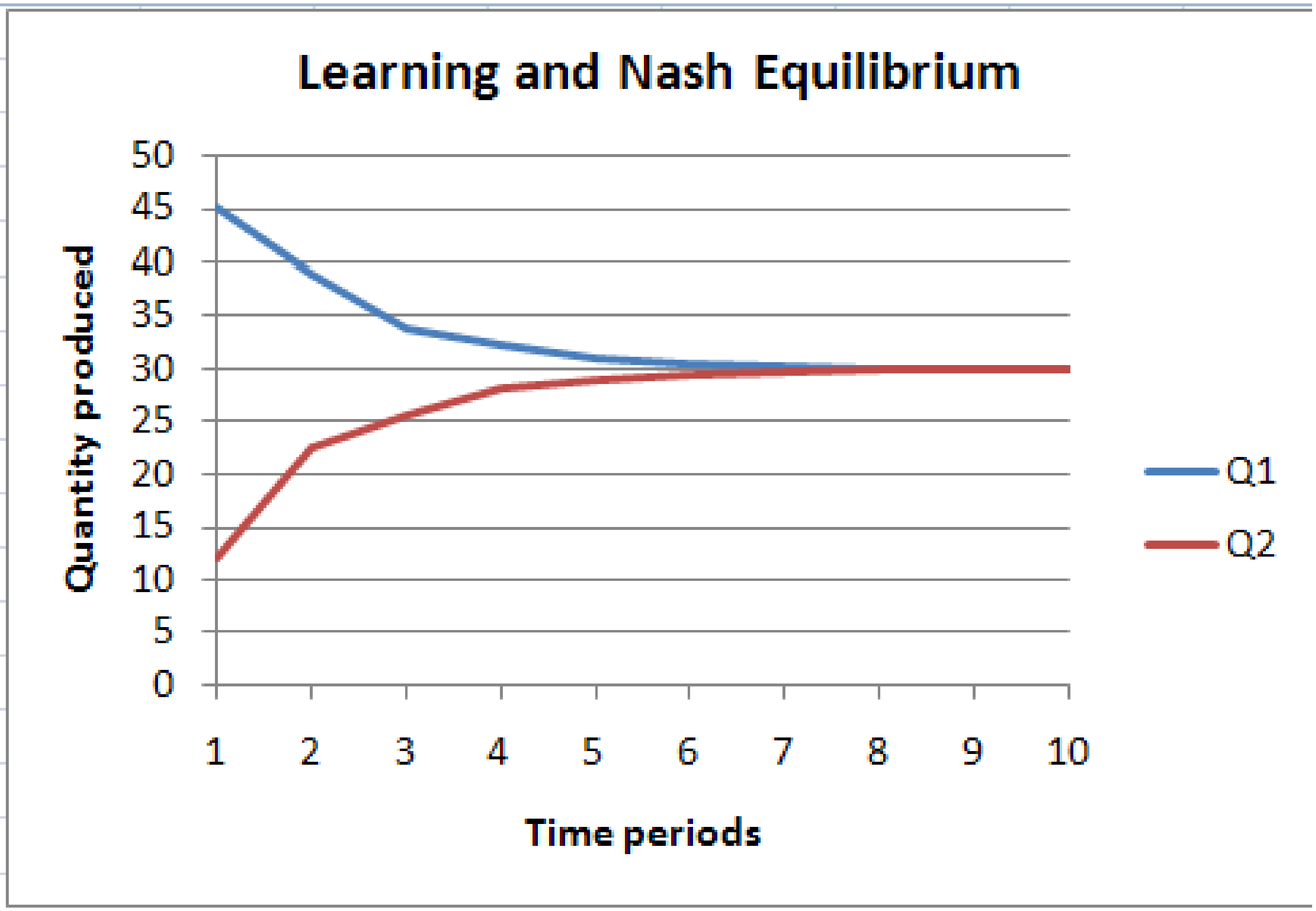
$$Q_1 = \frac{90 - Q_2}{2}$$

$$Q_2 = \frac{90 - Q_1}{2}$$

- The solution is: NE = {30, 30}

Convergence to Nash Equilibrium

Time	Q1	Q2
1	45	12
2	39	22.5
3	33.75	25.5
4	32.25	28.125
5	30.9375	28.875
6	30.5625	29.53125
7	30.23438	29.71875
8	30.14063	29.88281
9	30.05859	29.92969
10	30.03516	29.9707



Nash Equilibrium in Mixed Strategies

- Not every game has Nash equilibrium in **pure strategies.**
- Example: Rock, Scissors, Paper.

Player 2

Player 1

	Rock	Scissors	Paper
Rock	0,0	1,-1	-1,1
Scissors	-1,1	0,0	1,-1
Paper	1,-1	-1,1	0,0

Nash Equilibrium in Mixed Strategies

- **Mixed strategy –**A **mixed strategy** is an assignment of a probability to each pure strategy.
- Nash proved that **every game**, with any number of players and any number of pure strategies, has at least one **mixed strategies NE.**

Nash Equilibrium in Mixed Strategies

- There's no NE in pure strategies, as rock>scissors, scissors>paper, paper>rock
- If Player A: (1/2,1/4,1/4)……
- Player B: play more paper!
- Therefore, the mixed strategy (1/3,1/3,1/3) for both players, is the only NE in mixed strategies in this game.

Impact

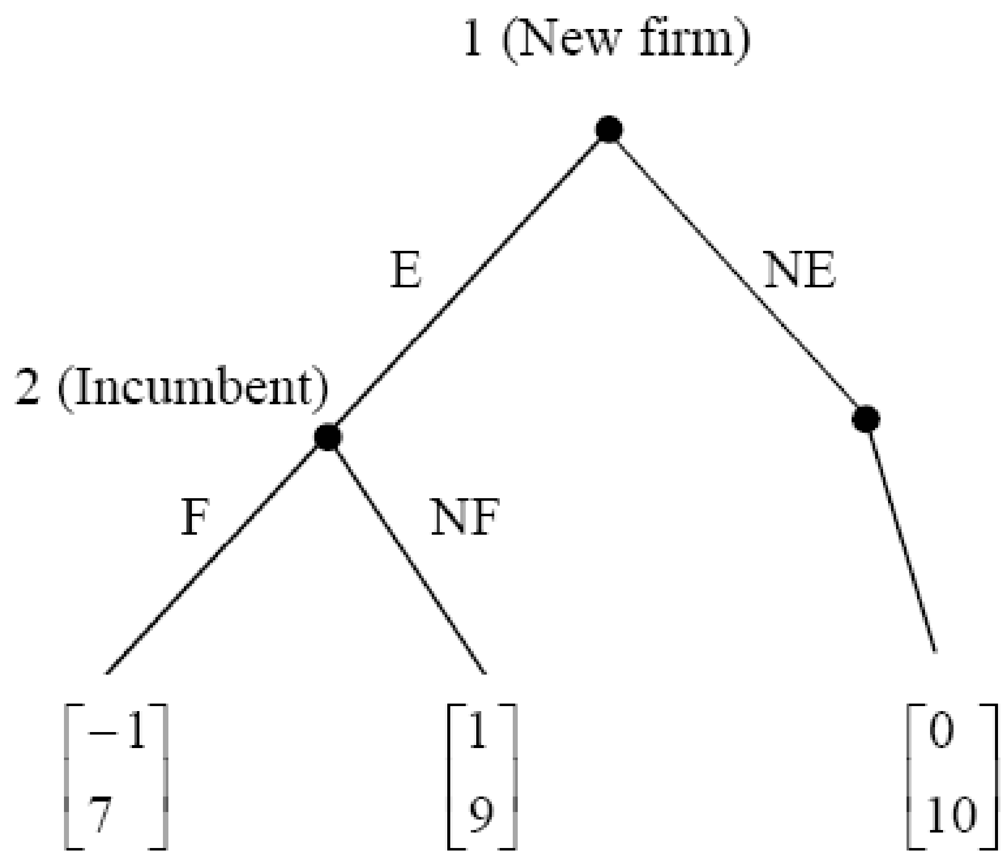
- The Nash equilibrium is the single most applied solution concept that in game theory.
- Applications in economics include
  - Oligopoly
  - Entry and exit
  - Market equilibrium
  - Location
  - Bargaining
  - Insurance
  - Advertising, etc.
- It can also be applied in areas of politics, biology, psychology, military strategies, personal relationship,…

Critique

- Sometimes there are too many Nash equilibria.
- Some Nash equilibria can be eliminated, as "unreasonable".
- Economists developed **refinements** to Nash equilibrium, to eliminate some unreasonable ones.

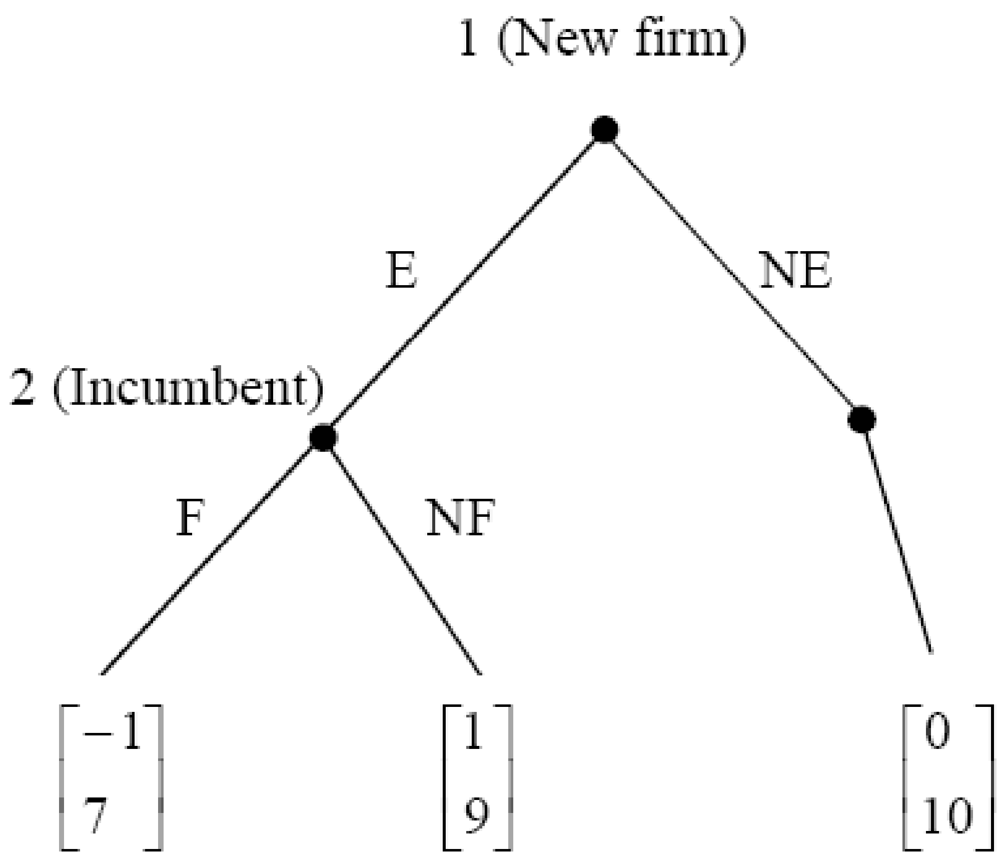
Example

- This is **sequential moves** game (new firm moves first, and the existing firm moves second).
- New firm's strategies: {Enter, Not Enter}.
- Existing firm strategies: {Fight if Enter, Not Fight if enter}



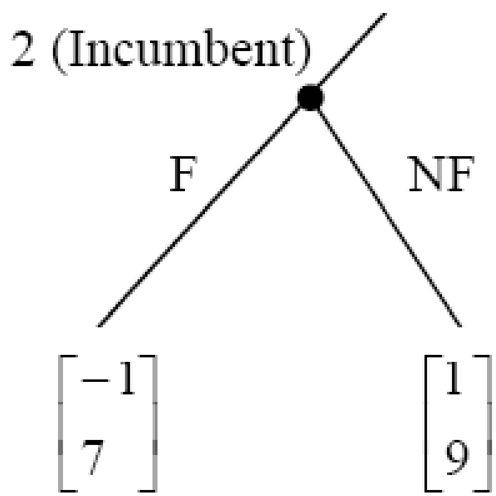
Example

- How many NE in this game?
- Two. (NE, F if E),(E, NF if E)
- Based on our knowledge of game theory and using the above model, what do we predict about entry of potential competitors?
- Either the competitors will enter or they won't.
- Take a closer look, which strategy will the potential competitor be more likely to choose?
- Enter, because {F if E} is **incredible threat**.



Critique

- Selten introduced the concept of **Subgame Perfect Nash Equilibrium (SPNE).**
- This is a refinement of the original Nash equilibrium.
- A set of strategies is a SPNE if it induces a Nash equilibrium in every subgame of the original game.
- In our example, fight-if-enter is not a best response of the incumbent to the entry. Thus, it is not NE of the subgame:



Questions

- How many strategies does each firm have in our duopoly example?

$$\max_{Q_1} \pi_1 = (100 - Q_1 - Q_2)Q_1 - 10Q_1$$

$$\max_{Q_2} \pi_2 = (100 - Q_1 - Q_2)Q_2 - 10Q_2$$

- Does any of the above firms have a **dominant** strategy?
- How do we predict outcomes of games that don't have dominant strategies.