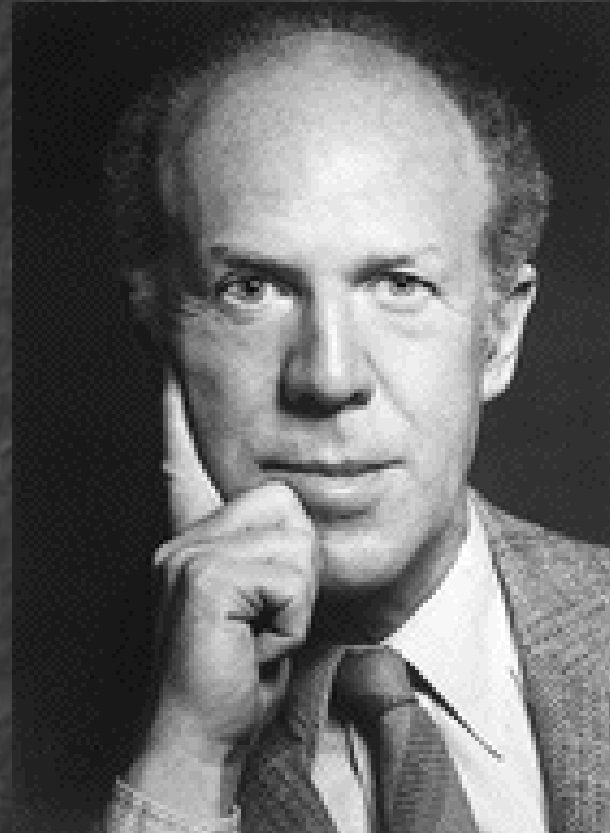


Gerard Debreu

Jared MacLane

Introduction

"Gerard Debreu has set the standard as well as posed most of the questions (and, in many cases, provided many of the solutions) that were to be addressed by mathematical economics."



Background – Influenced by – Contributions – Impact - Critique

- French born Economist and Mathematician.
- Joined the French army during WWII.
- 1948, Debreu came to the USA on a Rockefeller Fellowship.
- 1948-1960, Traveled the country working at the University of Chicago, Yale, and Stanford under the Cowles Foundation.
- 1960-61, Worked at Center for Advanced Study in the Behavioral Sciences at Stanford, focused on a complex proof of a general theorem on the existence of an economic equilibrium.
- 1963, began his career at the University of California, Berkeley.

Background – Influenced by – Contributions – Impact - Critique

- Emeritus professor of economics and mathematics at the University of California, Berkeley, lived until he was 83.
- Debreu won the Nobel Memorial Prize in Economic Sciences in 1983 for applying mathematical rigor to the fundamental theory of supply and demand in economics.



Economists

David Ricardo (1772–1823)



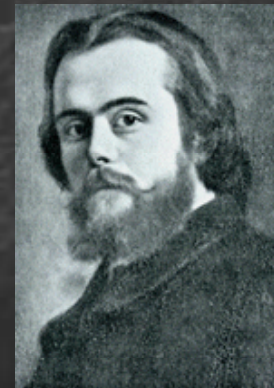
Wassily Leontief (1909–1999)



Vilfredo Pareto (1848-1923)

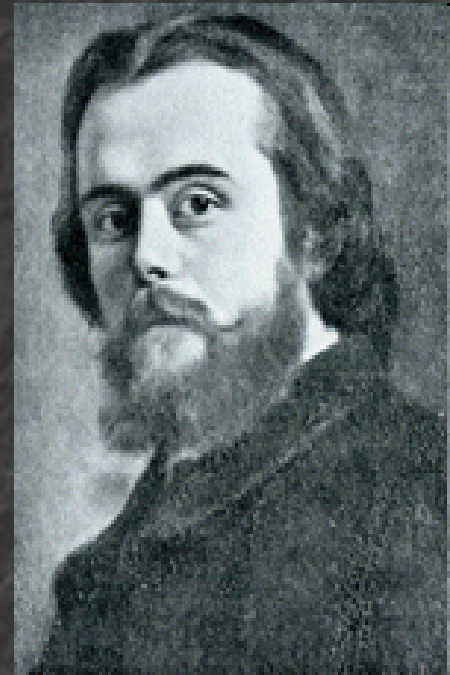


Leon Walrus (1834-1910)



Leon Walrus

- Regarded as the father of general equilibrium theory.
- Appointed to the Academy of Lausanne in 1870.
- Walrus wrote and published the Elements of Pure Economics (1874).



Vilfredo Pareto

- Italian economist, leader of the Lausanne School who succeeded Walrus.
- His "tastes-and-obstacles" approach to general equilibrium theory were resurrected during the great 'Paretian Revival' of the 1930s and have guided much of economics since.



David Ricardo

- Ricardo dominated economic thinking throughout the 19th C.
- In 1815 he introduced the law of diminishing returns.
- For Ricardo, the appropriate theory was the "labor-embodied" theory of value or LTV.
- The relative "natural" prices of commodities are determined by the relative hours of labor expended in their production.



Saunders MacLane

- MacLane was Chairman of the Mathematics Department at Chicago from 1952-1958.
- “He was one of the most important figures in the University of Chicago Mathematics Department, or indeed in American mathematics,” David Eisenbud PhD Student.
- MacLane’s Department influenced and aided Debreu during his time in Chicago.



Wassily Leontief

- Input- Output Analysis: Every output in an economy can be analyzed in terms of final consumption and all the inputs necessary for its production.
- Used matrix algebra and subsequently computer technology to trace the relationship between industries.
- Debreu attended several weeks at the Salzburg Seminar in American Studies where Leontief was a member of the faculty. (1948)



Kenneth Arrow

- "It was a wonderful experience, he was just so brilliant to work with," Arrow said.
- "One of us would say a single word, and the other would just understand immediately. I learned quite a bit from him."



John von Neumann

- 1944 book, *Theory of Games and Economic Behavior* invented the entire field of Game Theory .
- This book introduced the axiomatization of utility theory.
- A theory pursued by Kenneth Arrow and Gerard Debreu.



Major Contributions

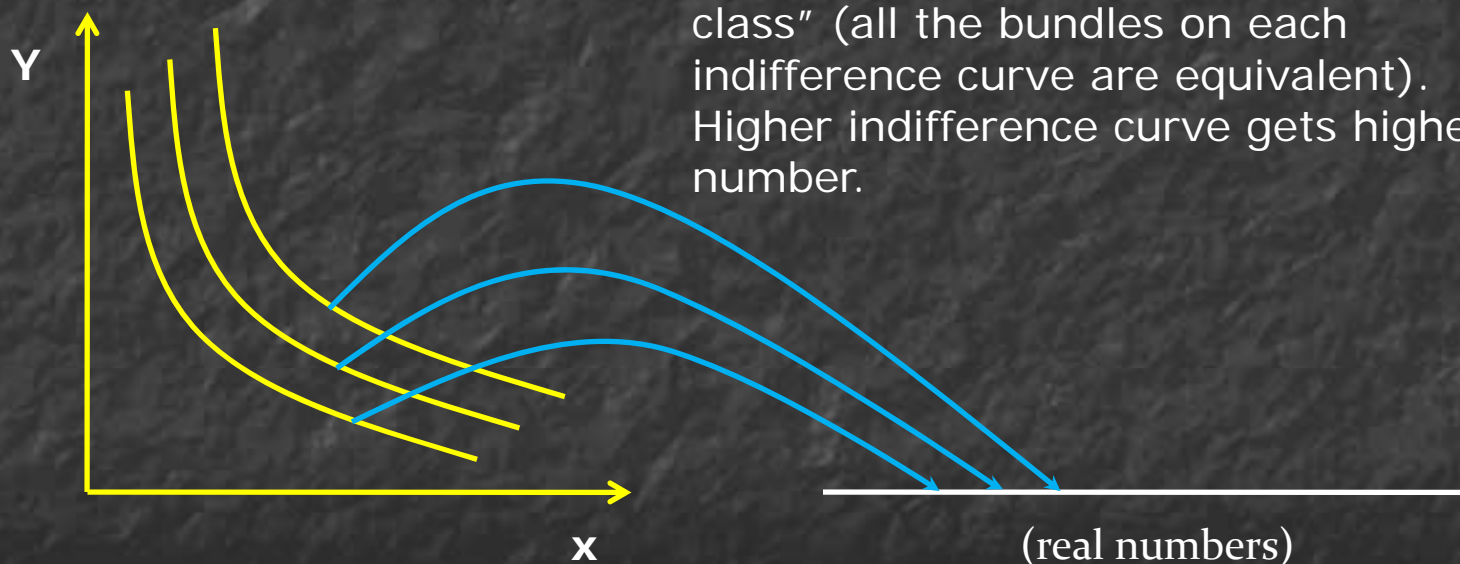
1. Rigorously derived the **Existence of Utility Functions**.¹¹¹ (1951,1954)
2. The **Theory of Value** “an analysis organized around the concept of a price system, or of a value function defined in the commodity space.” (1959)
3. Rigorous proofs of **welfare theorems**.
4. Provided one of the first simple proofs of existence of a **Competitive Equilibrium** using mathematics. (1954)

1. Existence of Utility Function (Debreu 1954)

- In economics, consumer's preferences are represented with preference relation \succsim . That is $A \succsim B$ reads "bundle A is at least as good as bundle B".
- It would be convenient if we could represent preferences with utility functions.
- We could then solve utility maximization problem. (we know from calculus how to maximize functions).
- Question:
 - Under what conditions (on \succsim), can \succsim be represented with a utility function?

1. Existence of Utility Function (Debreu 1954)

- Representing preferences with utility functions means that bundle A is at least as good as bundle B if and only if $U(A) \geq U(B)$.
 - The idea is to assign a real number to each indifference curve or “equivalence class” (all the bundles on each indifference curve are equivalent). Higher indifference curve gets higher number.



1. Existence of Utility Function (Debreu 1954)

- **Transitivity** is defined as follows:

$$A \succ B \text{ and } B \succ C \implies A \succ C$$

2. Theory of Value

- (1) “The explanation of the prices of commodities resulting from the interaction of the agents of a private ownership economy through markets.”
- (2) “The explanation of the role of prices in an optimal state of an economy.”

Theory of Value Explained

- Establishes the existence of equilibrium using an approach to showing there exists a price system for which the aggregate excess demand correspondence vanishes by proving a type of fixed point theorem based on the Kakutani fixed point theorem.
- He introduces the notion of a contingent commodity, “which is a promise to deliver a good should a state of nature realize.”

1st Fundamental Theorem

The first Fundamental theorem states that agents trade competitively, each acting in his own self interest (maximizes his utility), then the resulting allocation is efficient. This theorem is a formalization of Adam Smith's invisible hand argument and it expresses our confidence in the market economy.

2nd Fundamental Theorem

The Second Theorem states that every efficient allocation can be supported by some set of prices. All that is required to reach a particular outcome is a redistribution of initial endowments of the agents after which the market can be left alone to do its work.

This suggests that the issues of efficiency and equity can be separated and need not involve a trade off. However, the conditions for the Second Theorem are stronger than those for the First, as now we need consumers' preferences to be convex

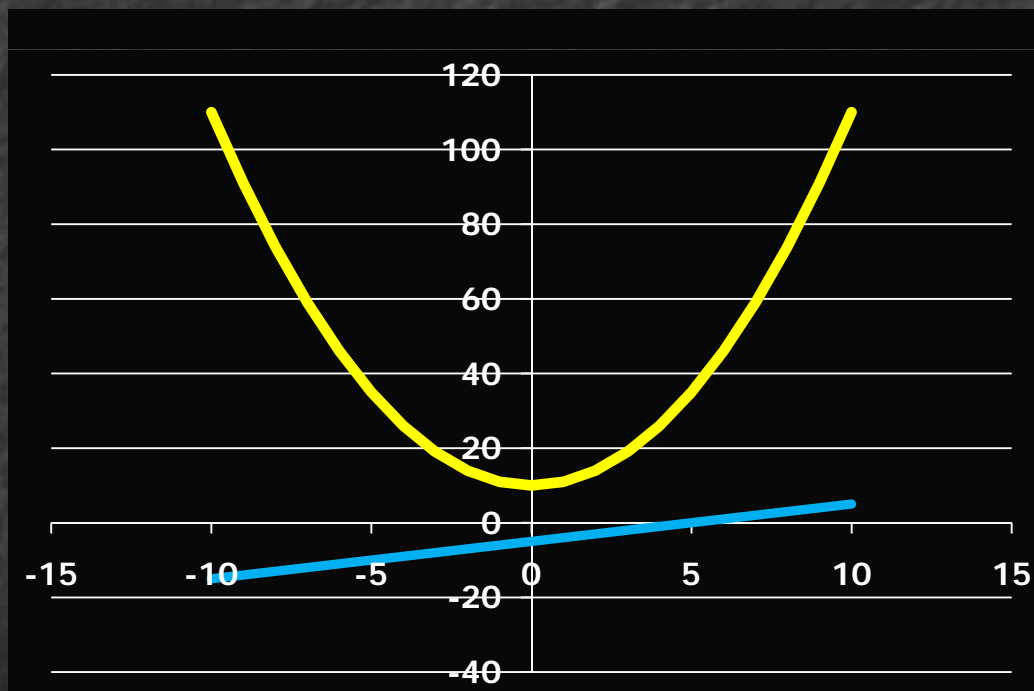
(convexity roughly corresponds to the idea of diminishing marginal utility, or to preferences where "averages are better than extrema").

4. Existence of Equilibrium

- Competitive **general** equilibrium is a situation where all the markets are in equilibrium (supply = demand in all markets in the economy).
- Walras argument: Suppose that we have N markets, i.e. N supply curves + N demand curves = $2N$ equations.
- Equilibrium in each market is a pair of unknowns (p, q) , price and quantity. Thus, in N markets we have in total $2N$ unknowns.
- Conclusion: $2N$ equations & $2N$ unknowns imply that a solution must exist, right?
- Debreu: **"WRONG!"**

3. Existence of Competitive Equilibrium (Arrow Debreu 1954)

- **Example:** the following system does not have a solution.



$$y = 10 + x^2$$
$$y = x - 5$$

Gerard Debreu (1921-2004)

Existence of Equilibrium

- Existence of **general** competitive equilibrium turns out to be non-trivial.
- We use the competitive equilibrium as the model's prediction about the economy,
- We have the 1st theorem that says that competitive equilibrium is **efficient**,
- But what if in the real world, there does not exist such thing as competitive equilibrium??? ☹
- Arrow and Debreu established the most general conditions under which **general** competitive equilibrium exists. These turn out to be not so unrealistic assumptions.

Existence of Equilibrium

First, that a competitive equilibrium exists if each person in the economy possesses some quantity of every good available for sale in that market. (1954)

Second, that exploitable labor resources exist which are capable of being used in the production of desired goods/services.

The overall impact was huge. Every graduate course in macro and micro-economics studies analysis developed by Debreu.

His contributions touch finance, involving asset pricing (Arrow-Debreu Securities), macro-economics and financial economics.

- Debreu introduced new tools of economic analysis – Real Analysis, Topology.
- The conditions of the "invisible hand" in the marketplace were clarified.
- The Theory of Value remains the definitive statement of Neo-Walrasian theory in its purest, axiomatic form.

- It is hard to criticize a mathematical economist like Debreu.
- Once you prove a mathematical theorem correctly, it is forever going to be true.
- Debreu did not do empirical work to test whether the assumptions of his theorems are supported by the evidence.
- In his defense, it is the job of applied economists to do this work. There is a specialization within each science.

Questions & Discussion

1. Why is it useful to represent preferences with utility functions?
2. Show that if preferences are not transitive, they cannot be represented by a utility function.
3. Give example of a system of two equations with two unknowns, that do not have a solution.
4. Is it possible that two **linear equations** with two unknowns, do not have a solution?