

KENNETH J. ARROW



Early Life

- ▣ Born in New York, New York on August 23, 1921
- ▣ Graduated from Townsend Harris High School located in the Queens borough of New York City
- ▣ Townsend Harris High School is one of the top high schools in the country.
- ▣ Admission to the school when he was a student there was based on competitive examination.
- ▣ Received his bachelors degree at age 19 in social science as a math major at City College of New York.

Background

- ▣ Received his masters at Columbia in Mathematics in 1941.
- ▣ Served as a weather officer for the U.S. Army Air Corps during World War II from 1942-1946.
- ▣ From 1946-1949 he was a graduate student at Columbia and a research associate of the Cowles Commission for Research in Economics at the The University of Chicago.
- ▣ Received a Phd at Columbia in Economics in 1951.

Background

- ▣ Published the book “Social Choice and Individual Values” in 1951 which included his impossibility theorem.
- ▣ Published the book “Essays in the Theory of Risk- Bearing” which included his theory and measurement of risk aversion.
- ▣ Won the Nobel prize in 1972 along with John R. Hicks for his contribution to economic equilibrium theory and welfare theory.

Influences

- ▣ Influenced by Tjalling C. Koopmans and Jacob Marschak when he was a research associate of the Cowles Commission for Research at the University of Chicago and they were Staff.
- ▣ Arrow stated in his autobiography for the Nobel Prize “the brilliant intellectual atmosphere of the Cowles Commission, with eager young econometricians and mathematically-inclined economists under the guidance of Tjalling Koopmans and Jacob Marschak, was a basic formative influence for me,”

Background – **Influences** – Contributions – Impact – Questions

Influences

- ▣ Tjalling C. Koopmans was born August 28, 1910 in 's Graveland, the Netherlands. Father was a Principal for a protestant school.

▣



Influences

- ▣ Started his college education at University of Utrecht at age 17. Majored in Math for first three years.
- ▣ Switched his emphasis to theoretical physics. Three years later in 1933 he moved to Amsterdam to study mathematical economics and in 1936 received a degree from the University of Leiden.

Tjalling C. Koopmans

- ▣ Koopmans moved to the U.S. in 1940 worked for the British Merchant Shipping Mission in Washington D.C. as a statistician.
- ▣ Published a study on optimal (cost minimizing) routing at The Mission.
- ▣ In 1944 Koopmans was invited by Jacob Marschak to join the staff of the Cowles Commission at the University of Chicago where the focus was on constructing econometric models.

Tjalling C. Koopmans

- ▣ Econometrics is the combination of economic theory with statistics to analyze and test relationships.
- ▣ Succeeded Jacob Marschak as director of the commission until 1955 when the Cowles family moved their funding to Yale University.
- ▣ Went to Yale along with others from the commission and became part of the new Cowles foundation for research at Yale.

Tjalling C. Koopmans

- ▣ While at Yale he focused his research on optimum allocation over time.
- ▣ In 1975 Koopmans along with Leonid Vitaliyevich Kantorovich received the Nobel Prize in Economics for their contributions to the theory of optimum allocation of resources.

Background – **Influences** – Contributions – Impact – Questions

Jacob Marschak

- ▣ Born July 23 1898 in Kiev, Imperial Russia which is now the capital of Ukraine



Background - **Influences** - Contributions - Impact - Questions

Jacob Marshak

- ▣ Studied at the University of Berlin and Heidelberg and received his Ph.D from University of Heidelberg in 1922.
- ▣ Was a assistant professor at University of Heidelberg from 1930-1933
- ▣ Lecturer in Economics at All Souls College, Oxford University from 1933-1935.

Background – **Influences** – Contributions – Impact – Questions

Jacob Marschak

- ▣ Director of the Institute of Statistics at Oxford University from 1935-1939.
- ▣ Wrote the book “The New Middle Class” along with Emil Lederer which was published in 1937
- ▣ Moved to the U.S. in 1940 and taught Economics at The New School for Social Research in New York City.

Jacob Marcshak

- ▣ Became the Head of the Cowles Commission in 1943 when it moved to the University of Chicago.
- ▣ According to Koopmans with the Commission Marschak “created a rare kind of research environment, by shrewd selection of staff members and by a truly open style of work and discussion”

Background - **Influences** - Contributions - Impact - Critique

Jacob Marschak

- ▣ Rejoined the Cowles commission when it moved to Yale.
- ▣ Well known for contributions to Information Theory.

Background - Influences - **Contributions** - Impact - Critique

Contributions

1. Theory and Measurement of Risk Aversion
2. Arrow's Impossibility Theorem

Risk aversion

- ▣ Most of the important decisions we face involve making choices under uncertainty.
- ▣ Examples: getting married, buying a car, choosing a major, getting a mortgage,...
- ▣ The field of **finance** evolved from economics of uncertainty.

Risk aversion

- ▣ A lottery gives payments x_1, x_2, \dots, x_N with probabilities $\pi_1, \pi_2, \dots, \pi_N$.
- ▣ Example:

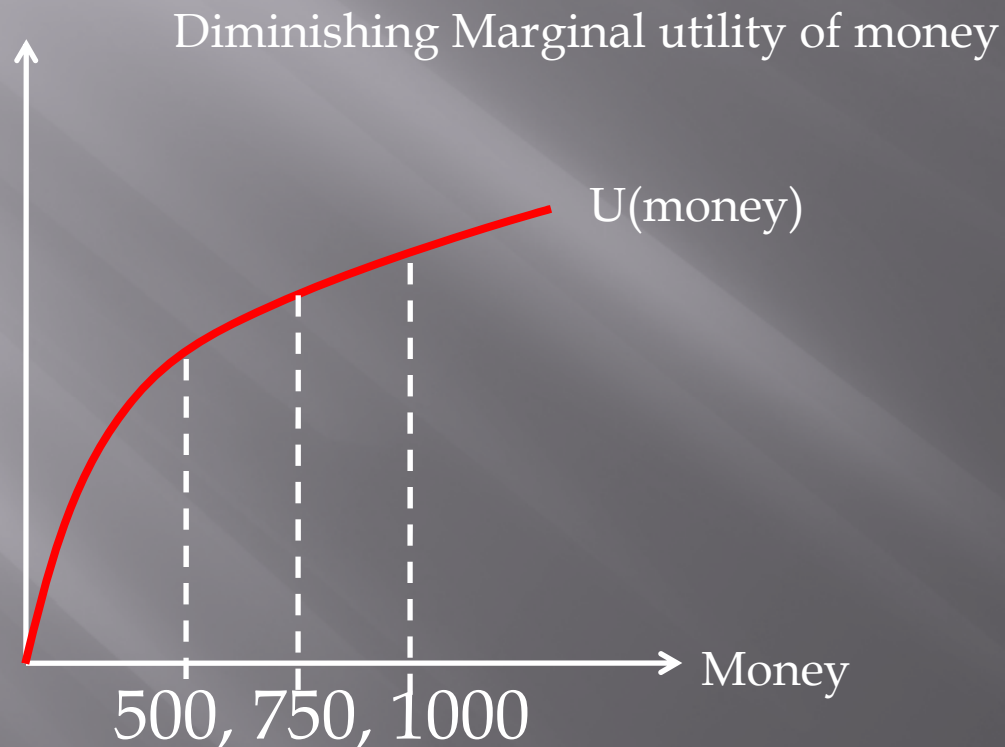
$$L = \begin{cases} \$1000 & \text{w.p. } 0.5 \\ \$500 & \text{w.p. } 0.5 \end{cases}$$

- ▣ This lottery pays \$1000 or \$500 with equal probabilities.
- ▣ **Expected value** of this lottery is \$750.

Risk aversion

- ▣ A **risk averse** individual prefers the mean (expected value) of the lottery over the lottery itself.
- ▣ Q: Why are most people risk averse?
- ▣ A: **Diminishing marginal utility of money.**

Risk aversion



- ▣ This is why the chances of losing certain amount does not compensate for the chances of winning the same amount.

Risk aversion

- ▣ Not all the people dislike risk at the same extent. Some are more risk averse, others less risk averse.
- ▣ Arrow, together with John W. Pratt, designed a way of measuring risk aversion.
- ▣ The idea is to measure how concave the utility of money is (how curved it is).

The Theory and Measurement of Risk Aversion

- ▣ Y =wealth
- ▣ $U(Y)$ =total utility of wealth Y
- ▣ Assume utility of wealth is twice differentiable
- ▣ $U'(Y)$ =marginal utility of wealth
- ▣ $U''(Y)$ =rate of change of marginal utility with respect to wealth.
- ▣ Assume wealth is desirable for a risk averter so $U'(Y) > 0$

The Theory and Measurement of Risk Aversion

- ▣ Absolute risk aversion = $-U''(Y)/U'(Y)$
 - Shows the unwillingness to risk absolute magnitudes of money.
- ▣ Relative risk aversion = $-YU''(Y)/U'(Y)$
 - Shows the unwillingness to risk certain proportions of wealth.
- ▣ Both measures are positive when there is risk aversion, that is, when $U''(Y) < 0$

The Theory and Measurement of Risk Aversion

▣ Example

$$U(Y) = \ln(Y)$$

$$U'(Y) = \frac{1}{Y}$$

$$U''(Y) = -\frac{1}{Y^2}$$

$$\text{Absolute risk aversion : } \alpha = \rho = -\frac{U''(Y)}{U'(Y)} = -\frac{-1/Y^2}{1/Y} = Y$$

$$\text{Relative Risk aversion : } \rho = -\frac{U''(Y)}{U'(Y)} Y = -\frac{-1/Y^2}{1/Y} \cdot Y = 1$$

Background - Influences - **Contributions** - Impact - Critique

Arrow's Impossibility Theorem

- ▣ Economists represent individual preferences with utility functions.
- ▣ Question: can we aggregate all the individual preferences and construct a **social welfare function**?
- ▣ It would be great if we could, because then we would be able to rank alternatives, such as presidential candidates, laws, military actions, and all the choices that we make as a society.

Arrow's Impossibility Theorem

- ▣ Arrow showed that it is impossible to construct a **social welfare function (utility function for the entire nation)** that will fulfill certain requirements.
- ▣ In order to represent individual preferences with a utility function, we require **transitivity**.
- ▣ Transitive - a relation in which one element in relation to a second element and the second in relation to a third element implies the first element is in relation to the third element, as the relation "less than or equal to." If $a > b$ and $b > c$, then $a > c$.
- ▣ In the next example, we show that even if individual preferences are transitive, it is possible that the **aggregation** of these preferences is not transitive.

Background - Influences - **Contributions** - Impact - Critique

Arrow's Impossibility Theorem

2008 Presidential Election For A Ranked-Voting System

Voter A: Obama > McCain > Nader

Voter B: McCain > Nader > Obama

Voter C: Nader > Obama > McCain

- ▣ There is no transitivity at the aggregate level. This group prefers Obama to McCain, McCain to Nader, and Nader to Obama.

Significance

- ▣ Work on uncertainty which includes his theory and measurement of risk aversion even now is still a standard source for economists with their work.
- ▣ Arrow's Impossibility Theorem created a research industry in Social Welfare Theory.

Questions

- ▣ Give example of choice under uncertainty that you have made in your life.
- ▣ Explain how the decision to act against global warming might be related to degree of risk aversion.
- ▣ Explain how the decision to develop alternative energy sources might be related to degree of risk aversion.
- ▣ What is the importance of Arrow's Impossibility Theorem?