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Walras Law

There are I goods indexed by $i \in \{1, 2, ..., I\}$ and J individuals indexed by $j \in \{1, 2, ..., J\}$. The prices of all goods are given by the price vector $p = (p_1, p_2, ..., p_I)$. Let $S_{ij}(p)$ be the quantity of good i that consumer j is planning to sell at the price vector p and let $D_{ij}(p)$ the quantity of good i that consumer j is planning to buy at price vector p.

Theorem 1 *(Walras Law). If each individual satisfies his budget constraint, so that the value of the goods sold equals the value of the goods bought, then the total value of all the sales by all individuals equals the total value of all the purchases by all individuals.*

Proof. For each consumer j and for any price vector p, the theorem assumes that the total sales equal total purchases.

$$p_1 S_{1j}(p) + p_2 S_{2j}(p) + ... + p_I S_{Ij}(p) = p_1 D_{1j}(p) + p_2 D_{2j}(p) + ... + p_I D_{Ij}(p)$$

or more compactly

$$\sum_{i=1}^{I} p_i S_{ij}(p) = \sum_{i=1}^{I} p_i D_{ij}(p)$$

Now summing over all J consumers and rearranging

$$\begin{aligned} \sum_{j=1}^{J}\sum_{i=1}^{I} p_i S_{ij}(p) &= \sum_{j=1}^{J}\sum_{i=1}^{I} p_i D_{ij}(p) \\ \sum_{i=1}^{I} p_i \sum_{j=1}^{J} S_{ij}(p) &= \sum_{i=1}^{I} p_i \sum_{j=1}^{J} D_{ij}(p) \\ \sum_{i=1}^{I} p_i S_i(p) &= \sum_{i=1}^{I} p_i D_i(p) \end{aligned}$$

In the last equation, $S_i(p)$ is the total quantity supplied of good i at the price p_i and $D_i(p)$ is the total quantity demanded of good i at the price p_i. This equation means that at any price vector $p = \{p_1, p_2, ..., p_I\}$, the value of total planned sales always equals the value of total planned purchases. ■

Corollary 2 *If all the markets, with the exception of market k, are in equilibrium, i.e. $S_i(p) = D_i(p) = Q_i^* \ \forall i \neq k$, and also $p_k \neq 0$, then we must have also $S_k(p) = D_k(p) = Q_k^*$. In other words, if all but one market are cleared, then the remaining market must also be cleared.*

Proof. The above condition, that the value of total planned sales always equals the value of total planned purchases, can be written as

$$\begin{aligned}\sum_{i=1}^{I} p_i S_i(p) &= \sum_{i=1}^{I} p_i D_i(p) \\ \sum_{i\neq k} p_i S_i(p) + p_k S_k(p) &= \sum_{i\neq k} p_i D_i(p) + p_k D_k(p)\end{aligned}$$

Since $S_i(p) = D_i(p) = Q_i^* \ \forall i \neq k$ we have

$$\begin{aligned}\sum_{i\neq k} p_i Q_i^*(p) + p_k S_k(p) &= \sum_{i\neq k} p_i Q_i^*(p) + p_k D_k(p) \\ p_k S_k(p) &= p_k D_k(p) \\ S_k(p) &= D_k(p)\end{aligned}$$

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Discussion

Walras law tells us something very intuitive, that if income equals spending for every individual, then the *sum* over all individuals' income will also equal the *sum* over all individuals' spending. This result is very general. It applies to virtually any economy, with any number of goods and consumers. The corollary says that if all but one market are in equilibrium, the remaining market must be in equilibrium. Thus, if one market is not in equilibrium, then some other markets must be in disequilibrium. The key assumption that Keynes made is that all the markets, *except for the labor market*, are cleared, and the labor market itself is not in equilibrium. This, Walras showed, is a mathematical impossibility.