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Optimal Provision of Public Goods

Suppose that consumers like to consume a public good x and a private good y. The good y represent all the other goods and referred to as the *numeraire* or simply as *money*. Suppose that there are two consumers 1 and 2 with utility functions $U^1(x,y)$ and $U^2(x,y)$. The price of public good x in units of y is p. The social planner needs to determine the optimal contribution by each consumer towards the public good. Let the contribution by consumer i towards the public good be x_i. The social planner then solves the following problem

$$\max_{x_1,x_2} U^1(x, y_1 - px_1) + U^2(x, y_2 - px_2)$$
$$s.t.$$
$$x_1 + x_2 = x$$

Notice that the amount of the public good consumed by all the consumers is the same, because the public good is *nonrival*. Substituting the constraint into the objective gives

$$\max_{x_1,x_2} U(x_1 + x_2, y_1 - px_1) + U(x_1 + x_2, y_2 - px_2)$$

First order conditions

$$U_1^1(x, y_1 - px_1) + U_1^2(x, y_2 - px_2) - pU_2^1(x, y_1 - px_1) = 0 \quad (1)$$
$$U_1^1(x, y_1 - px_1) + U_1^2(x, y_2 - px_2) - pU_2^2(x, y_2 - px_2) = 0 \quad (2)$$

Notice that if consumer 1 increases his contribution towards the public good by a bit, the cost to him is $-pU_2^1(x, y_1 - px_1)$ but the benefit is enjoyed by *both consumers* at the amount of $U_1^1(x, y_1 - px_1) + U_1^2(x, y_2 - px_2)$. This illustrates the *nonexcludability* nature of public goods - if it is supplied to one consumer, it is supplied to everybody else (e.g. national defense).

From equations (1) and (2) we see that the optimal provision of public good requires that the utility cost to each consumer is the same:

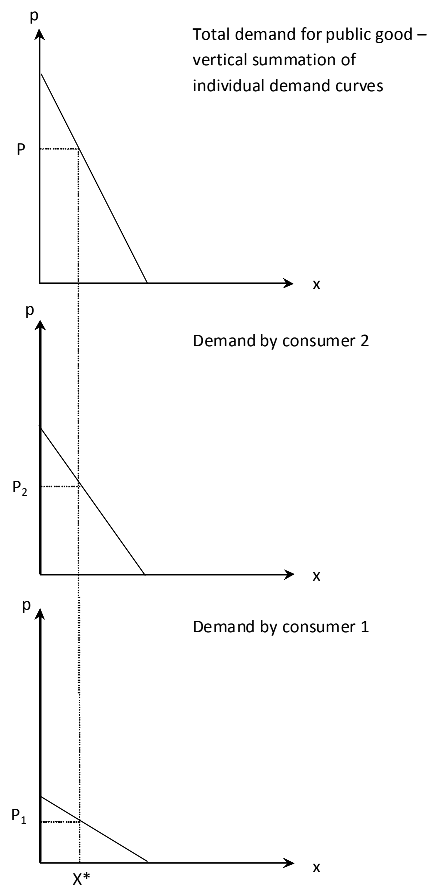
$$\begin{aligned} pU_2^1(x, y_1 - px_1) &= pU_2^2(x, y_2 - px_2) \\ \text{or } U_2^1(x, y_1 - px_1) &= U_2^2(x, y_2 - px_2) \end{aligned}$$

Dividing equation (1) by the equal cost gives

$$\underbrace{\frac{U_1^1(x, y_1 - px_1)}{U_2^1(x, y_1 - px_1)}}_{MRS_{x,y}^1} + \underbrace{\frac{U_1^2(x, y_2 - px_2)}{U_2^2(x, y_2 - px_2)}}_{MRS_{x,y}^2} = p \quad (3)$$

Equation (3) is the Samuelson's condition for optimal provision of public goods. The interpretation is that the cost of the public good p should be equal to the *sum of the benefits* to all consumers from the public good. Recall that $MRS_{x,y}$ is the marginal rate of substitution between x and y, i.e. how many units of y the consumer is willing to give up for 1 extra unit of x (the public good). In other words, the MRS represents the *marginal* benefit that each consumer derives from 1 extra unit of x, in terms of units of y. The optimal provision of public goods requires that the social cost of the public good be equal to the sum of the benefits to all consumers from the public good.

The next figure illustrates the Samuelson's condition for optimal provision of public goods.



The social cost of public goods is P. We add the total benefit from public good to both consumers, and the optimal quantity of public good (for each consumer and the economy as a whole) is X^*. The optimal contribution by each consumer is given by p_1 and p_2 respectively.

Numerical example: Suppose that in a two consumers economy, the demand for public good by each consumer is given by

$$\begin{aligned} D_1 &: \ p_1 = 100 - x \\ D_2 &: \ p_2 = 200 - 2x \end{aligned}$$

Determine the optimal quantity of public good in this economy when the social cost of a unit of public good is \$60.

Solution: The total marginal benefit is a vertical summation of the two marginal benefit curves:

$$p = \underbrace{100 - x}_{MRS^1_{x,y}} + \underbrace{200 - 2x}_{MRS^2_{x,y}} = 300 - 3x$$

The optimal amount of public good is given by

$$\begin{aligned} 60 &= 300 - 3x \\ x^* &= 80 \end{aligned}$$

The economy, and also each consumer, will consume 80 units at the optimum. Now plug this amount into each demand curve to determine how much will each consumer contribute:

$$\begin{aligned} p_1 &= 100 - 80 = \$20 \text{ per unit of public good} \\ p_2 &= 200 - 2 \cdot 80 = \$40 \text{ per unit of public good} \end{aligned}$$

Hence the total cost of the public good to the economy is $80 \cdot 60 = \$480$. Of this consumer 1 pays $80 \cdot 20 = \$160$ and consumer 2 pays $80 \cdot 40 = \$320$.