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The Cutting Edge of Technology (Ch. 9)

In this chapter we focus on the technological leaders. An example of a technological leader is Great Britain during the years 1760 - 1870. We start by demonstrating how we can use simple growth accounting in order to measure productivity growth of countries during the times before good data was available. The resulting estimates of productivity are a very rough approximations, but they allow us to date the beginning of a regime change such as the industrial revolution. Next we attempt to answer the million dollar question "will the growth in productivity continue forever?". Finally, we will investigate the implications of differential technological progress - different sectors in the economy grow at different rates. Can we predict the impact of such biased technological progress for the macroeconomy as a whole?

1 Measuring Productivity Growth

1.1 Measuring productivity growth before the 18th century

In the old times land was important factor of production so that the land share in total income was about 60%. The labor share in income was about 30%. The rest (10%) is the capital share, which was mostly capital stock. If we had good data on the capital stock, we could measure the productivity growth as follows. Let the production function of the aggregate output be of the Cobb-Douglas form with inputs being capital (K), labor (L) and land (Λ):

$$Y = AK^{\phi}L^{\mu}\Lambda^{1-\phi-\mu}$$

where ϕ is the capital share, μ is the labor share and $1-\phi-\mu$ is the land share. The output per worker is then

$$y = \frac{Y}{L} = \frac{AK^{\phi}L^{\mu}\Lambda^{1-\phi-\mu}}{L} = Ak^{\phi}\left(\frac{\Lambda}{L}\right)^{1-\phi-\mu}$$

In terms of growth rates the above becomes

$$\hat{y} = \hat{A} + \phi\hat{k} + (1-\phi-\mu)\hat{\Lambda} - (1-\phi-\mu)\hat{L}$$

Now assuming that the land is fixed $\left(\hat{\Lambda} = 0\right)$ and since we don't have data on capital that goes back far enough, the approximate growth accounting formula becomes[1]

$$\hat{y} = \hat{A} - (1-\phi-\mu)\hat{L}$$

and the growth rate of productivity is

$$\hat{A} = \hat{y} + (1-\phi-\mu)\hat{L} \tag{1}$$

[1] See the appendix for derivation.

In order to use this data, one needs data on the growth rate of output per worker and the growth rate of the labor force. Alternatively, we can use the growth rate of GDP/cap and the growth rate of population in the absence of other data. In the textbook, the land share is denoted by β, so equation (1) becomes equation (9.3) in the textbook:

$$\hat{A} = \hat{y} + \beta \hat{L} \tag{2}$$

Example: Using the data in table 9.1, find the approximated growth rate of productivity in Europe during the years 500-1500 and 1500-1700. Assume that the land share is $\beta = 1/3$.

TABLE 9.1
Growth Accounting for Europe, A.D. 500–1700

Period	Annual Growth Rate of Income per Capita, $\hat{y}$	Annual Growth Rate of Population, $\hat{L}$	Annual Growth Rate of Productivity, $\hat{A}$
500–1500	0.0%	0.1%	0.033%
1500–1700	0.1%	0.2%	0.166% i

Solution: During 500-1500

$$\hat{A} = \hat{y} + \beta \hat{L} = 0 + \frac{1}{3} \cdot 0.1\% = 0.03\%$$

During 1500-1700

$$\hat{A} = \hat{y} + \beta \hat{L} = 0.1\% + \frac{1}{3} \cdot 0.2\% = 0.166\%$$

Which is 5 times bigger than in the previous period.

1.2 Measuring productivity growth nowadays

In modern economies the importance of land as a factor of production is almost insignificant. The economists typically model the aggregate production of modern economies as Cobb-Douglas, with only two inputs: labor and capital, where the capital also includes land. The capital share is around 1/3 and the labor share is 2/3. Thus, the aggregate production function is

$$Y = AK^{\theta} L^{1-\theta}$$

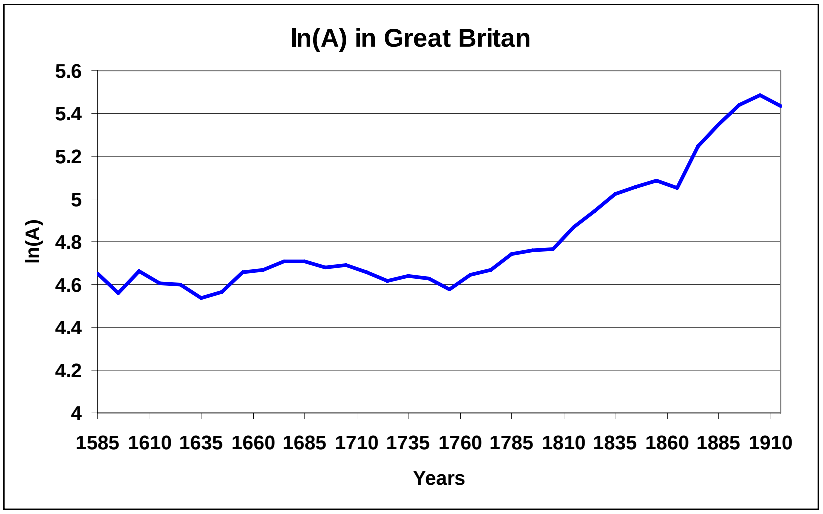
Typically, we do have data on Y (real GDP), K (real value of the capital stock) and L (number of workers or total working hours). We can calculate the time series of productivity using the above equation as follows:

$$A = \frac{Y}{K^{\theta} L^{1-\theta}}$$

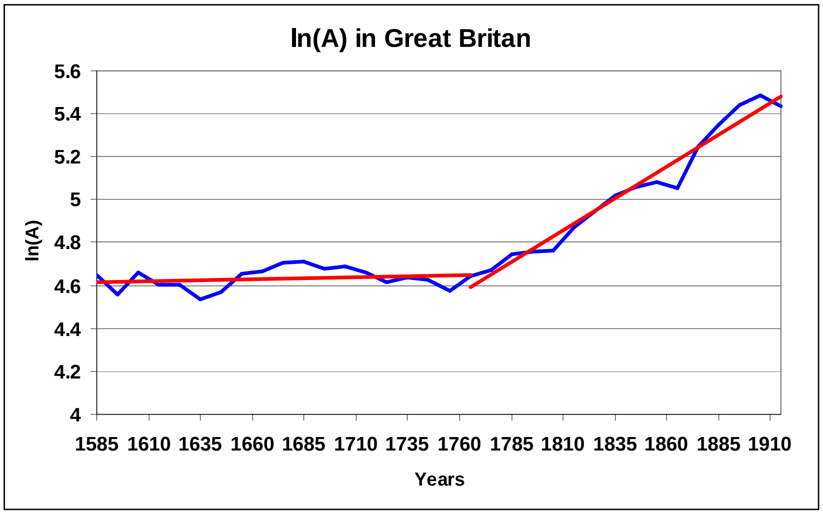
Once we obtained the time series of $\{A_t\}$, we can compute the growth rate of productivity.

1.3 When did the industrial revolution start?

Computing the time series of productivity allows us to estimate the date of regime changes, such as the onset of the industrial revolution. The next graph shows the ln of productivity in Great Britain.



When we look at the ln of TFP in Great Britain, we can clearly see that up to 1770 there is small or no productivity growth at all, while after 1770 there is faster growth. We can fit two linear trends for the two regimes, as the next graph shows.



This gives us an estimate of the starting date of the industrial revolution - around 1770 (where the two trends intersect).

2 Will the Growth in Productivity Continue Forever?

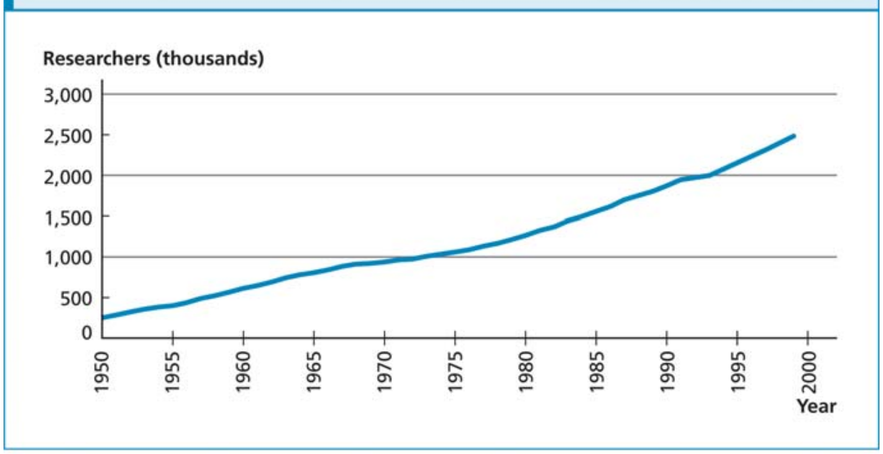
In chapter 8 we assumed the simplest technology production function that we could think of. In particular, we assumed that the growth rate of technology is proportional to the number of researchers in the economy:

$$\hat{A} = \frac{\gamma_A L}{\mu} = \frac{L_A}{\mu}$$

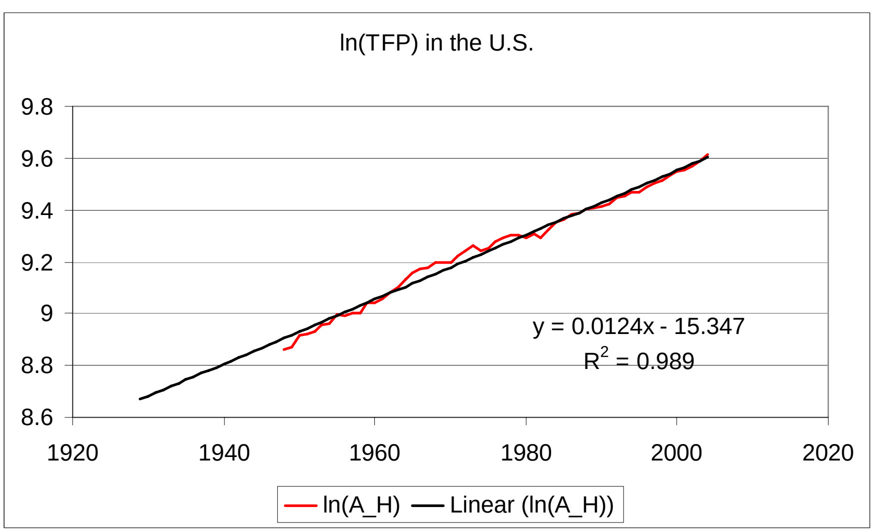
where $\hat{A}$ is the growth rate of productivity, L_A is the number of researchers, and μ is the cost of innovations in units of researchers. This production function is overly optimistic and not consistent with the data. The above production function predicts that if the number of researchers will not change, then the productivity will grow at some constant rate forever. If the number of researchers doubles, the productivity will grow twice as fast. According to this production function, the growth in productivity will continue forever. If the fraction of researchers in the labor force is fixed, and the labor force is growing, then the number of researchers will grow and the productivity will grow at an ever increasing rate.

The next two graphs show that despite the growth in the number of researchers in the most developed countries, the productivity is growing at constant rate, *not accelerating*.

FIGURE 9.4
R and D Researchers in the G-5 Countries, 1950–1999



Sources: Jones (2002); OECD Main Science and Technology Indicators database.



What can explain these observations and how can we modify the technology production function so that it will be consistent with the data? One effect that is not incorporated in the above production function is the idea that the more advanced the technology is, the more difficult it is to discover new ideas. This is sometimes called the "fishing out effect", because of the view that just like after fishing in the lake for a long time all the good fish is gone, all the simple discoveries were already made. Nowadays, when the technology is so advanced, in order to make new discoveries one needs to study a vast amount of material. Thus, greater level of technology today diminishes the growth of technology because of the fishing out effect.

Another effect that is neglected by the previous production function is the fact that doubling the number of researchers does not lead to doubling the number of discoveries, but typically less than doubling. There are many research teams that are working on the same areas and typically there are many repetitions of discoveries. This leads us to conclude that the appropriate technology production function should exhibit decreasing returns to scale in labor.

The following production function incorporates the two effects (1) fishing out effect, and (2) decreasing returns to scale:

$$\hat{A} = \frac{L_A^{\lambda} A^{-\phi}}{\mu} \tag{3}$$

where $0 < \lambda < 1$ and $0 < \phi < 1$. The fishing out effect is captured by the fact that on the right hand side there is $A^{-\phi}$ so the growth of technology is decreasing in A. The decreasing returns to scale property is captured by the fact that λ is less than 1. Suppose that $\lambda = 0.5$, this would mean that increasing the number of researcher by a factor of 4 will only double the output (productivity growth).

Suppose that the productivity growth is constant. We can derive the relationship between the growth rate of researchers and the growth rate of productivity that is implied by this production function. Then dividing equation (3) for time $t+1$ by the same equation at time t gives

$$\begin{aligned} 1 &= \frac{L_{A,t+1}^{\lambda} A_{t+1}^{-\phi}}{L_{A,t}^{\lambda} A_t^{-\phi}} = \left(\frac{L_{A,t+1}}{L_{A,t}}\right)^{\lambda} \left(\frac{A_{t+1}}{A_t}\right)^{-\phi} \\ 1 &= \left(1 + \hat{L}_A\right)^{\lambda} \left(1 + \hat{A}\right)^{-\phi} \end{aligned}$$

Taking natural log from both sides gives

$$0 = \lambda \ln\left(1 + \hat{L}_A\right) - \phi \ln\left(1 + \hat{A}\right)$$

which is approximately

$$\begin{aligned} 0 &= \lambda \hat{L}_A - \phi \hat{A} \\ \hat{A} &= \frac{\lambda}{\phi} \hat{L}_A \end{aligned}$$

Thus, the new production function delivers the prediction that if the growth rate of researchers is fixed, then the growth rate of productivity will be fixed. In the data for the U.S.

the growth rate in the number of researchers is about 5% per year, and at the same time the growth rate in TFP is about 1%. Thus we can estimate of $\lambda/\phi = 1/5$.

The technology production function in equation (3) gives us a different answer to the question "will the growth in productivity continue forever". This production function predicts that as long as the number of researchers continues to grow, the productivity will continue to grow. How can the number of researchers grow? First, the fraction of the labor force that is engaged in research can grow, but only up to certain limit. Somebody has to produce output, so we cannot have the entire labor force engaged in R&D. Second, the labor force itself can grow simply because the population grows. The growth of population is not limitless however, and we observe that in many developed countries (e.g. Japan, Italy) the population is shrinking. Based on this technology production function we must conclude that the growth of productivity must slow down and eventually stop. Only time will tell if this prediction is true or false.

3 Differential Technological Progress

So far we treated technological progress as affecting the entire economy. In the real world however some sectors in the economy experience fast technological improvement while others do not. For example, telecommunication technology is growing very fast while the technology of haircut industry and education did not advance as fast. In this section we examine the macroeconomic implications of differential technological progress.

3.1 Example: perfect complement goods

Suppose that there are two sectors that produce goods 1 and 2 according to

$$\begin{aligned} Y_1 &= A_1 L_1 \\ Y_2 &= A_2 L_2 \end{aligned}$$

where Y_i is the output of sector i, A_i is the productivity of sector i and L_i is the labor input in sector i, and $L_1 + L_2 = L$, which is the total labor in the economy. For simplicity we assume that the only input in production is labor. Suppose that the consumers want to consume equal amounts of each good[2] (e.g. one slice of bread with one slice of cheese). The productivity of sector 1 is growing at 2% per year while the productivity in sector 2 is growing at only 1% per year. How are the resources allocated in this economy, in other words what fraction of the labor force will be employed in each sector? What is the growth rate of productivity that will be measured in the entire economy?

Solution. First, allocation of labor. Since people like to consume equal amounts of the two goods, equilibrium requires that the two goods are also produced in equal amounts.

[2] The preferences of consumers are described by $U(Y_1, Y_2) = \min\{Y_1, Y_2\}$.

Thus, equating $Y_1 = Y_2$ and solving for L_1 gives

$$\begin{aligned} A_1 L_1 &= A_2 (L - L_1) \\ A_1 L_1 &= A_2 L - A_2 L_1 \\ L_1 (A_1 + A_2) &= A_2 L \\ \frac{L_1}{L} &= \frac{A_2}{A_1 + A_2} \\ \frac{L_1}{L} &= \frac{1}{A_1/A_2 + 1} \end{aligned}$$

Thus, if A_1 is growing faster than A_2 we have $A_1/A_2 \to \infty$, and $L_1/L \to 0$. Similarly

$$\frac{L_2}{L} = \frac{A_1}{A_1 + A_2} = \frac{1}{1 + A_2/A_1} \to 1$$

Interestingly, the sector with the faster growing technology repels workers instead of attracting them. In the long run, all the workers in this economy will be employed in the less advanced industry. This seems counterintuitive, but with the requirement that both goods must be produced in the same amount and because sector 1's technology is growing faster, there demand for workers in that sector is decreasing over time.

What about aggregate productivity and aggregate output? Since the output in each sector grows at the same rate, the aggregate output will also grow at that rate. We compute the growth rate of the output in sector 1, but we would get the same results if we computed the growth rate of output in sector 2.

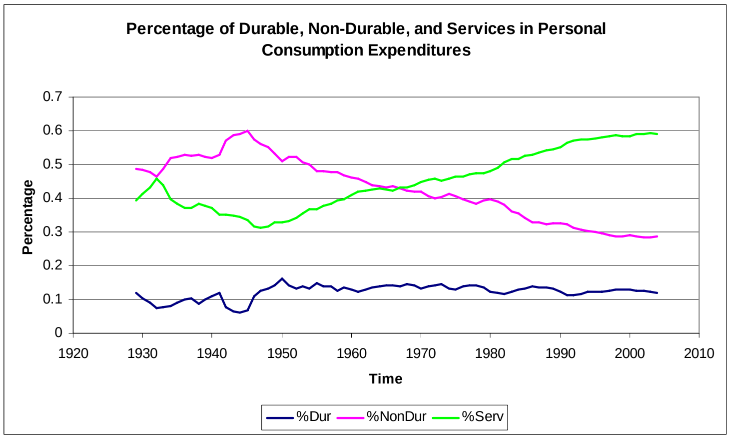
$$\begin{aligned} \frac{Y_{1t+1}}{Y_{1t}} &= \frac{A_{1t+1} L_{1t+1}}{A_{1t} L_{1t}} = \frac{A_{1t+1}}{A_{1t}} \left(\frac{A_{2t+1}}{A_{1t+1} + A_{2t+1}} L \right) / \left(\frac{A_{2t}}{A_{1t} + A_{2t}} L \right) \\ \frac{Y_{1t+1}}{Y_{1t}} &= \frac{A_{1t+1}}{A_{1t}} \frac{A_{2t+1}}{A_{2t}} \frac{A_{1t} + A_{2t}}{A_{1t+1} + A_{2t+1}} \\ \frac{Y_{1t+1}}{Y_{1t}} &= \frac{A_{2t+1}}{A_{2t}} \frac{1 + A_{2t}/A_{1t}}{1 + A_{2t+1}/A_{1t+1}} \end{aligned}$$

Now, since A_1 is growing faster than A_2 then $A_{2t}/A_{1t} \to 0$ as $t \to \infty$. Thus,

$$\frac{Y_{1t+1}}{Y_{1t}} \to \frac{A_{2t+1}}{A_{2t}} \text{ as } t \to \infty$$

Thus, in the long run, the total output and the productivity of this economy will grow at the rate of $\hat{A}_2$ - the slower sector. Although sector 1 is experiencing faster growth in productivity, the resources of the economy are reallocated to the sector with slower growing productivity. The growth of total output slows down because of that reallocation.

The above example is extreme, but we can learn something from it. The U.S. economy is experiencing reallocation of resources towards services, as the next figure shows



Although the productivity in the durable and nondurable sectors probably grows faster than the productivity in the services, resources are reallocated towards the production of services. This reallocation of resources towards the slower growing sector, is reflected in a slower overall growth in productivity.

4 Appendix: derivation of equation (1)

The output per worker at time t is

$$y_t = A_t k_t^{\phi} \left(\frac{\Lambda}{L_t}\right)^{1-\phi-\mu}$$

Notice that since the land is fixed, we do not index it by t. Dividing the above equation for $t+1$ by the one for time t:

$$\begin{aligned} \frac{y_{t+1}}{y_t} &= \frac{A_{t+1} k_{t+1}^{\phi} \left(\frac{\Lambda}{L_{t+1}}\right)^{1-\phi-\mu}}{A_t k_t^{\phi} \left(\frac{\Lambda}{L_t}\right)^{1-\phi-\mu}} \\ \frac{y_{t+1}}{y_t} &= \left(\frac{A_{t+1}}{A_t}\right) \left(\frac{k_{t+1}}{k_t}\right)^{\phi} \left(\frac{L_{t+1}}{L_t}\right)^{-(1-\phi-\mu)} \\ 1+\hat{y} &= \left(1+\hat{A}\right) \left(1+\hat{k}\right)^{\phi} \left(1+\hat{L}\right)^{-(1-\phi-\mu)} \end{aligned}$$

Taking ln of both sides

$$\ln(1+\hat{y}) = \ln\left(1+\hat{A}\right) + \phi \ln\left(1+\hat{k}\right) - (1-\phi-\mu)\ln\left(1+\hat{L}\right)$$

which for small growth rates, is approximately

$$\hat{y} = \hat{A} + \phi\hat{k} - (1-\phi-\mu)\,\hat{L}$$

In the absence of data for k we have

$$\hat{y} = \hat{A} - (1-\phi-\mu)\,\hat{L}$$