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Population and Economic Growth

1 Introduction

The focus of our study in this course is GDP per capita, i.e.

$$\frac{GDP}{Population}$$

From the definition of GDP per capita it should be obvious that in order to study the evolution of GDP per capita, one has to study the evolution of population, and not only the GDP. In these notes we discuss the joint evolution of population and output.

1.1 Data

The standard of living did not increase much, and the kind of sustained growth that the world experienced in the last 200 years were unprecedented throughout the human history. Figure 1 shows the natural log of the GDP per capita in western Europe during the last two millennia. Recall that the slope of the ln is approximately equal to the growth rate of the original variable, so there was little or no growth in the standard of living in western Europe until 1800 AD. The average growth rate in GDP per capita during 0-1820 was 0.06% per

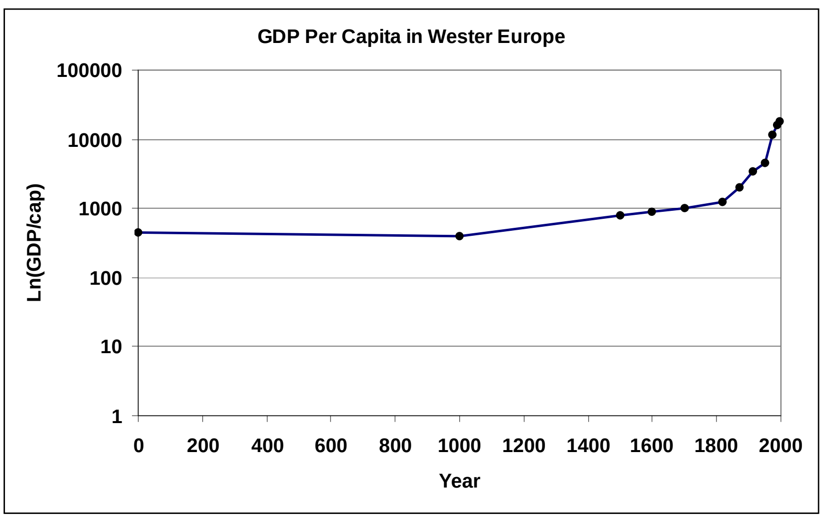

Figure 1: Source: Maddison, Angus. 2001. The World Economy: A Millennial Perspective. Paris: Development Center of the Organization for Economic Cooperation and Development.

year, while in the next 180 years (1820-1998) GDP per capita grew on average at 1.52% per year (27 times faster!). So in a sense, growth in standard of living is a new phenomenon.

It certainly didn't seem to Thomas Malthus that the standard of living grew at all. In his book "*An Essay on the Principle of Population*" in 1798 he predicted that even if resources grow, the population will grow even faster, so that eventually there will be no improvement in the standard of living (i.e., GDP per capita). Here are some quotes from his book.

1. "I think I may fairly make two postulata. First, that food is necessary to the existence of man. Secondly, that the passion between the sexes is necessary and will remain nearly in its present state."

2. "Assuming then my postulata as granted, I say, that the power of population is indefinitely greater than the power in the earth to produce subsistence for man. Population, when unchecked[1], increases in a geometrical ratio. Subsistence increases only in an arithmetical ratio. A slight acquaintance with numbers will shew the immensity of the first power in comparison of the second."

   Malthus brings anecdotal evidence from China to support his theory:

3. "When we are assured that China is the most fertile country in the world, that almost all the land is in tillage , and that a great part of it bears two crops every year, and further, that the people live very frugally, we may infer with certainty that the population must be immense".

Malthus' prediction that the humans are destined to live in poverty did not materialize and many countries today experience sustained growth in standard of living. Yet, there are many other countries that did not experience improvement in the standard of living. In the next section we write a model that will formalize Malthus' idea and will give the kind of pessimistic prediction that Malthus made. The key prediction of the model is that technological improvements lead to only temporary improvements in the standard of living, but eventually the "power of population" wins and the economy is back to the subsistence level of income per capita.

2 Malthusian Model

2.1 Description of the model

Consumers: Like to consume food (Y_t). Each consumer supplies 1 unit of labor.

Producers: Produce food using land and labor. Output of food at time t is given by

$$Y_t = A_t \Lambda^{\theta} L_t^{1-\theta}, \quad 0 < \theta < 1$$

where A_t is productivity level at time t, Λ is (fixed) land, and L_t is the number of workers, which is also the size of the population.

Population: evolves according to

$$L_{t+1} = g\left(y_t\right) \cdot L_t$$

[1] Malthus mentioned two kinds of "checks" to population, positive and preventive. The positive check refers to death from diseases, wars, etc. The preventive check is voluntary restriction of fertility by delaying marriage, contraception, etc.

where $g(y_t)$ is the growth rate of population as a function of income per capita $y_t = Y_t/L_t$. It is assumed that there is some subsistence level of consumption per capita y^* such that $g(y_t) < 1$ when $y_t < y^*$, $g(y_t) > 1$ when $y_t > y^*$ and $g(y_t) = 1$ when $y_t = y^*$. In other words we assume that the shape of the function $g(\cdot)$ is as displayed in figure 2.This completes the description of the model. In the next section we derive the model's predictions and contrast them with the evidence.

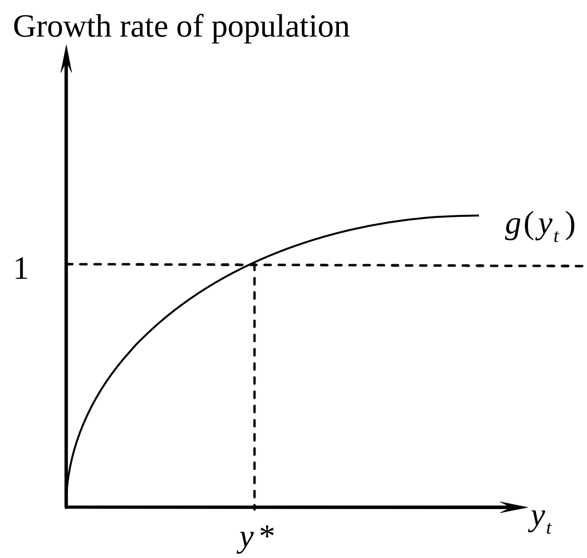

Figure 2: Relationship between the growth rate of population and income per capita.

2.2 Working with the model

The output per capita (or per worker) is

$$
\begin{aligned}
y_t &= \frac{Y_t}{L_t} = \frac{A_t \Lambda^\theta L_t^{1-\theta}}{L_t} \\
y_t &= A_t \left(\frac{\Lambda}{L_t}\right)^\theta \qquad (1)
\end{aligned}
$$

Notice that output per worker is increasing in the land per worker Λ/L_t. Higher population in this model reduces the land per worker, and therefore leads to lower output per capita.

Next we derive the law of motion of output per capita, that is, we want to see what happens to y_t over time.

$$\begin{aligned} y_{t+1} &= \frac{A_{t+1}\Lambda^\theta L_{t+1}^{1-\theta}}{L_{t+1}} = A_{t+1}\left(\frac{\Lambda}{L_{t+1}}\right)^\theta \\ y_{t+1} &= \left(A_{t+1}/A_t\right) A_t \left(\frac{\Lambda}{g\left(y_t\right) L_t}\right)^\theta \\ y_{t+1} &= \frac{A_{t+1}/A_t}{g\left(y_t\right)^\theta} y_t \end{aligned} \tag{2}$$

The key term in the Malthusian model is

$$\frac{A_{t+1}/A_t}{g\left(y_t\right)^\theta}$$

The numerator is the growth rate of technology and the denominator is the growth rate of output per capita (raised to the power of θ). This fraction describes the Malthusian race between the growth of resources and the growth of population, and he believed that "when unchecked", population will win the race (or at least will not loose it).

2.2.1 No technological progress ($A_t = A \quad \forall t$)

The law of motion of output per capita then becomes

$$y_{t+1} = \frac{1}{g\left(y_t\right)^\theta} y_t$$

If for some reason the output per capita is above subsistence, $y_t > y^*$, then our assumptions about g imply that $g\left(y_t\right) > 1$ and output per capita will decline (i.e., $y_{t+1} < y_t$). If for some reason the output per capita is below subsistence, $y_t < y^*$, then our assumptions about g imply that $g\left(y_t\right) < 1$ and output per capita will increase (i.e., $y_{t+1} > y_t$). Thus, in the absence of technological progress, this economy (sadly) will converge to the subsistence level of output per capita, y^*. Also notice that in the long run the population is constant since $g\left(y^*\right) = 1$. We call a situation when all the endogenous variables are fixed, a **steady state**. Here the endogenous variables are the output per capita, the size of the population and the total output. But we are concerned only with the first two.

2.2.2 Finding the steady state (y^*, L^*)

First, to solve for the steady state output per capita from

$$g\left(y^*\right) = 1 \tag{3}$$

Then use equation (1) to find the steady state population

$$\begin{aligned} y^* &= A\left(\frac{\Lambda}{L^*}\right)^\theta \\ \left(\frac{y^*}{A}\right)^{1/\theta} &= \frac{\Lambda}{L^*} \\ L^* &= \Lambda / \left(\frac{y^*}{A}\right)^{1/\theta} = \Lambda\left(\frac{A}{y^*}\right)^{1/\theta} \end{aligned} \tag{4}$$

Example. Let $A = 2,\ \Lambda = 100,\ g(y_t) = 0.2y_t,\ \theta = 0.4$.

1. Find the steady state output per capita and the steady state size of population.

   Solution. The steady state output per capita:

$$\begin{aligned} g(y^*) &= 1 \\ 0.2y^* &= 1 \\ y^* &= 5 \end{aligned}$$

   The steady state population:

$$\begin{aligned} y^* &= A\left(\frac{\Lambda}{L^*}\right)^{\theta} \\ 5 &= 2\left(\frac{100}{L^*}\right)^{0.4} \\ \left(\frac{5}{2}\right)^{1/0.4} &= \frac{100}{L^*} \\ L^* &= \frac{100}{\left(\frac{5}{2}\right)^{2.5}} = 10.11929 \end{aligned}$$

2. Write the equation of the law of motion per capita and plot it, that is, plot y_{t+1} as a function of y_t.

   Solution. The law of motion is

$$\begin{aligned} y_{t+1} &= \frac{1}{(0.2y_t)^{\theta}}y_t = \frac{1}{0.2^{\theta}}y_t^{1-\theta} \\ y_{t+1} &= \frac{1}{0.2^{0.4}}y_t^{0.6} \end{aligned}$$

   Figure 3 shows the law of motion of output per worker.

Notice that this law of motion resembles the law of motion of capital per worker in the Solow model. Starting from any level of output per worker, there is convergence to the steady state. The steady state of output per worker is at the point of intersection between the 45^0 line and the law of motion of y_t.

2.2.3 Once and for all technological improvement

Suppose that $A_0 = A, A_1 = A, ..., A_\tau = A, A_{\tau+1} = A', A_{\tau+2} = A', ...$, where $A' > A$. Thus, nothing happens before time τ, so the economy is at steady state y^*, L^*. At time $\tau + 1$ there is technological improvement and $y_{\tau+1}$ will increase, as can be seen from the law of motion of output per worker

$$y_{\tau+1} = \frac{A_{\tau+1}/A_\tau}{g(y^*)^{\theta}}y^*$$

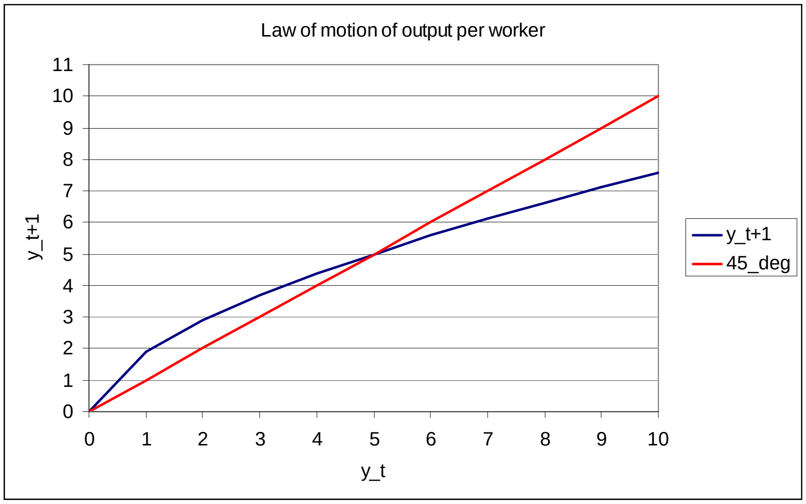

Figure 3: Law of motion of output per capita (or per worker).

Notice that $A_{\tau+1}/A_{\tau} > 1$. In all the following periods we are back to no technological improvements, $A_{t+1}/A_t = 1$ for all $t \geq \tau + 1$. This means that there will be convergence to the original steady state output per worker y^* and a new steady state level of population L^{**}. The new steady state population is found from (4):

$$L^{**} = \Lambda \left(\frac{A'}{y^*} \right)^{1/\theta}$$

Thus, following a technological progress, the economy will experience a temporary improvement in the standard of living (consumption per worker will go up), but eventually we are back to the subsistence. Figures 4-6 show the time paths of output per worker, population growth rate, and the size of population as a result of a once-and-for-all increase in A.

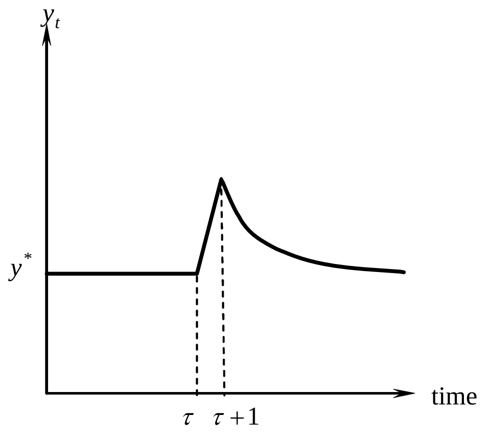

Figure 4: Time path of y_t.

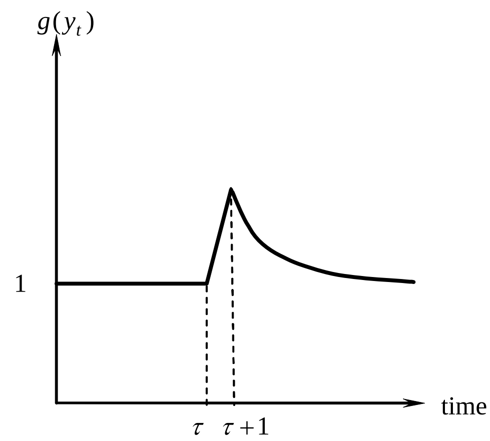

Figure 5: Time path of $g\left(y_t\right)$.

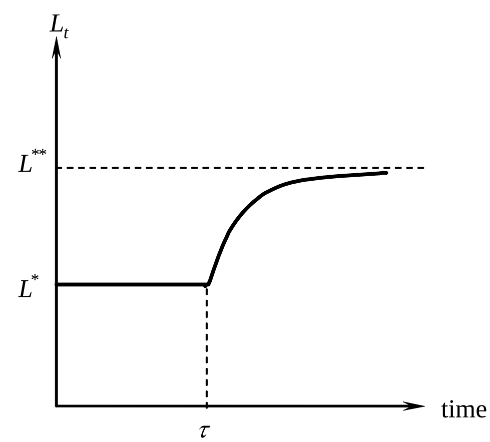

Figure 6: Time path of L_t.

This experiment implies that a country that has frequent technological advances, will experience faster growth of population and therefore will have very big population. The Malthusian mechanism suggests that since China was a technological leader for many centuries, its population must be very large.

The above result is in stark contrast to the prediction of the Solow model. Recall that a once-and-for-all technological improvement in the Solow model leads to a permanent increase in output per capita.

2.2.4 Changes in $g(\cdot)$

The Malthusian model assumes that there is a stationary relationship between the growth rate of population and output per capita. However, as Malthus himself noticed in the U.K., some people changed their fertility behavior and started restricting fertility (preventive check). In the next section we will see how the population depends on fertility and mortality.

But at this point, lets perform an experiment of changing the function $g(\cdot)$ such that for any given output per worker the associated population growth is smaller.

Figure 7 shows a shift to the right of the relationship between population growth rate and output per capita. Notice that the new steady state has greater output per worker than at the old steady state: $y^{**} > y^{*}$. Thus, according to the Malthusian model, the economy can escape poverty (subsistence) if the population dynamics changes in such a way that population grows slower. It seems that the one child law enacted in China in 1979 was motivated by this model.

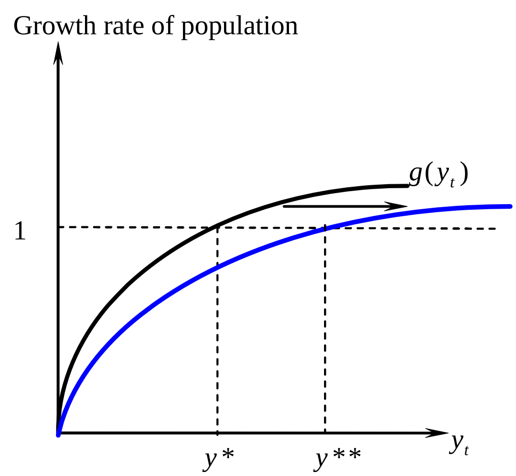

Figure 7: Change in the relationship between population growth rate and output per capita.

2.3 Summary: Malthus vs. Solow

Malthus vs. Solow

Malthus

- **Positive** relationship between population growth rate and income per capita
- Once-and-for-all increase in productivity leads to **temporary** increase in standard of living

Solow

- **Negative** relationship between population growth rate and income per capita
- Once-and-for-all increase in productivity leads to **permanent** increase in standard of living

3 Evolution of Population

The evolution of population depends on mortality and fertility[2]. In the next section we discuss some measures of mortality and fertility, and explain how they effect the evolution of population.

3.1 Demographic terminology

One of the most widespread measures of mortality in a country is life expectancy. Formally,

- μ_i - age specific mortality rate, i.e. probability of dying between age i and $i+1$. For example, $\mu_0 = 0.2$, means that there is 20% chance that a new born will not survive to the his first birthday, and it is called the Infant Mortality Rate (IMR). Another example, μ_5 is a probability that a five year old will not survive to his sixth birthday. A crude way of estimating μ_i is collecting a sample of people of age i and counting the fraction of them that did not survive until the age $i+1$. For example, if out of 1000 newborns, 50 did not survive to their first birthday, then we say that $\mu_0 = 50/1000 = 5\%$, that is $IMR = 5\%$.

- $P_i = 1 - \mu_i$ - age specific survival rate, i.e. the probability of surviving from age i to age $i+1$.

- π_i - probability of being alive during age i.

$$\pi_i = P_0 \cdot P_1 \cdot \ldots \cdot P_i = \prod_{t=0}^{i} P_t$$

- LE - life expectancy (in some year), is the age at which a new born is expected to die given that year's age specific mortality rates. Life expectancy is one of the measures of mortality in the economy, since it is computed using all the age specific mortality rates. One way of calculating life expectancy is

$$LE = \sum_{i=0}^{\infty} \pi_i$$

  The proof of this formula is in the appendix. For example, see the Excel for HW3.

- F_i - age specific fertility rate. The crude way of estimating for example F_{25} is to collect a sample of 1000 women of age 25 and count the number of babies that they had during that year. Suppose that they had 200 babies. Then $F_{25} = 200/1000 = 0.2$. We then say that the age specific fertility rate of women at age 25 is 0.2.

[2] Leaving migration aside for now.

- TFR - Total Fertility Rate. This is the total number of children that a woman is expected to give birth to if she went through her reproductive years with the age specific fertility rates of that year. The formula for calculating TFR is

  $$TFR = \sum_{i=0}^{\infty} F_i$$

  For example, suppose that the reproductive years of a woman are $20-39$ (20 years). Suppose that during the first 10 years (age $20-29$) the age specific fertility rate is 0.2 and during the second 10 years the age specific fertility rate is 0.15. Using our notation, this can be written as

  $$\begin{aligned} F_i &= 0.2, \text{ when } i = 20, ..., 29 \\ F_i &= 0.15, \text{ when } i = 30, ..., 39 \end{aligned}$$

  With these age specific fertility rates, we get

  $$TFR = 10 \cdot 0.2 + 10 \cdot 0.15 = 3.5$$

- NRR - Net Reproduction Rate, is the number of daughters that a woman is is expected to have during her lifetime if she went through her life with the current year's age specific fertility and mortality rates. NRR is a combined measure of fertility and mortality that is used to represent the extent to which the daughter generation replaces the mothers generation by the time the daughters reach the reproductive ages. To calculate NRR we can use the formula

  $$NRR = \frac{1}{2} \sum_{i=0}^{\infty} \pi_i F_i$$

  Notice that this formula is similar to the TFR, except that now we take into account the possibility that a woman will not survive through all her reproductive years, so we multiply the age specific fertility rate F_i by the probability that the woman will live through this age π_i. Also, since we count only daughters, we multiplying the result by $1/2$. Notice that NRR is increasing in fertility rates and decreasing in mortality rates. With the previous example, suppose that mortality occurs only in the first year of life, and $\mu_0 = 0.3$. This means that there is 0.7 chance that the woman will live through her reproductive years. Thus,

  $$\begin{aligned} NRR &= \frac{1}{2} \sum_{i=0}^{\infty} \pi_i F_i = \frac{1}{2} \left(\sum_{i=20}^{29} 0.7 \cdot 0.2 + \sum_{i=30}^{39} 0.7 \cdot 0.15 \right) \\ &= \frac{1}{2} (10 \cdot 0.7 \cdot 0.2 + 10 \cdot 0.7 \cdot 0.15) = 1.225 \end{aligned}$$

  $NRR = 1$ means that if the current age specific fertility and mortality rates will remain unchanged, then the generation of daughters will be of the same size as the generation of mothers.

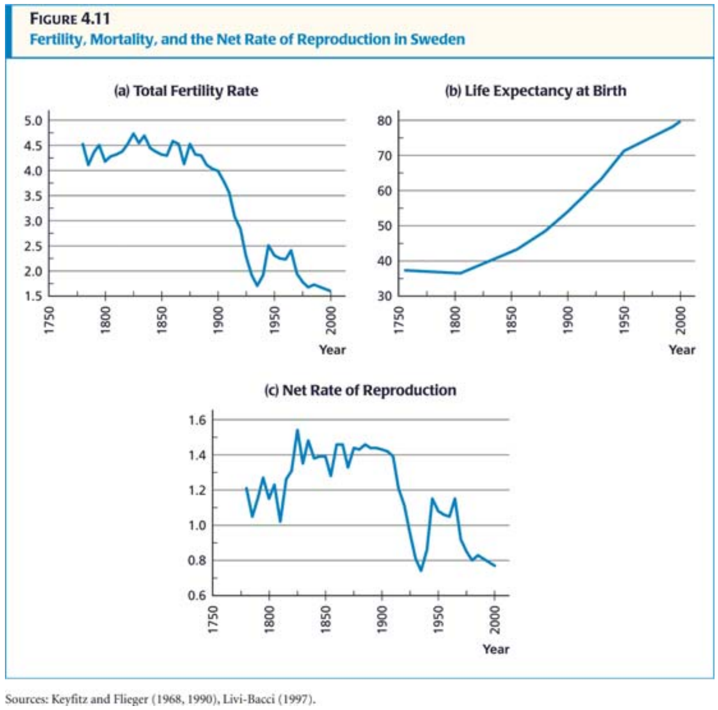

Figure 8: Demographic transition in Sweden.

- Demographic transition - the transition from high mortality and high fertility rates, to low mortality and low fertility rates. All the developed countries experienced the demographic transition. Figure 8 shows time paths of fertility and mortality in Sweden.

Recall that life expectancy is a measure of mortality. Panel (b) shows an increase in the life expectancy in Sweden as a result of declining mortality rates. Panel (a) shows the time series of TFR, which is declining from the end of the 19th century (with the exception of a baby boom in 1950's-1960's). Panel (c) shows the time series of NRR, which is a combined measure of fertility and mortality. A decline in mortality rates tends to increase NRR because more women are surviving through their reproductive years, but decline in TFR decreases NRR. Notice that In the first half of the 19th century, Sweden experienced an increase in NRR. This was a result of declining mortality, as you can see in panel (a) that fertility did not increase during those years. Later, in the first half of the 20th century, there was a decline in NRR, despite the continued decline in mortality, because the fertility decline was very large and managed to offset the effect of decline in mortality on NRR.

3.2 Predicting future population

Using the demographic terminology that was provided in the previous section, we can predict the future population. Figure 9 demonstrates how we can use age specific mortality and fertility rates in order to predict future population. The left side shows the size of the population in each age group in the current year, while the right side shows the same for the next year. Suppose that currently there are $20,000$ people of age 0. If the infant mortality

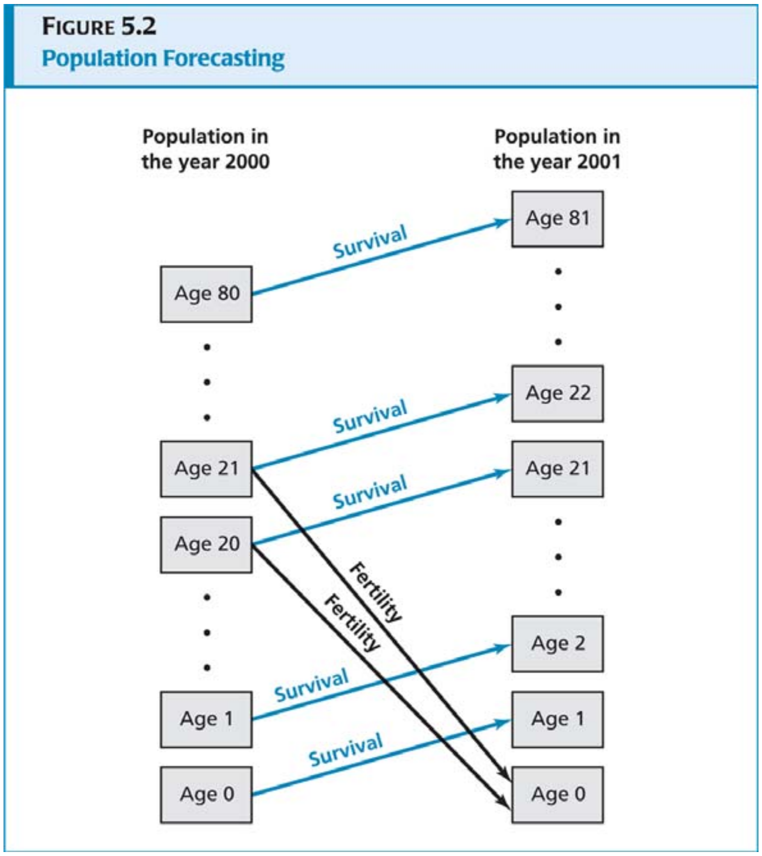

Figure 9: Predicting futurre population

rate is $\mu_0 = 0.2$, then the number of people of age 1 next year will be $20,000 \cdot 0.8 = 16,000$. Using the age specific fertility rates allows us to predict the number of babies (age 0) in the next year. Notice that in picture there are arrows that point from people of ages above 20 to people of age 0 in the next year. Thus, the number of babies in the next year is determined by the fertility of the people of child bearing age. Notice that there is no arrows from current age 80 to future babies. The Excel file posted on the web demonstrates in detail how to predict future population using current age specific mortality and fertility rates.

Example. The following table shows data for a country of Fantasia. Fantasians live for a maximum of five years. Also, all the people are women, there are no men.

Age	POP_{2000}	F_i	P_i
0	100	0	0.9
1	110	0.7	1
2	120	0.5	1
3	90	0	0.5
4	80	0	0

Calculate the population of Fantasia in 2001.

Solution.

Age	POP_{2000}	F_i	P_i	POP_{2001}
0	100	0	0.9	$0.7 \cdot 110 + 0.5 \cdot 120 = 137$
1	110	0.7	1	$0.9 \cdot 100 = 90$
2	120	0.5	1	$1 \cdot 110 = 110$
3	90	0	0.5	$1 \cdot 120 = 120$
4	80	0	0	$0.5 \cdot 90 = 45$
Total	**500**			**502**

3.3 Aging population

In the above example the fraction of older people in population declined. In many real world examples[3] the consequence of the demographic transition is that the fraction of working age population in the total population becomes smaller. This happens because fertility declines and people live longer. One direct effect of this demographic change on growth is the decline in the number of workers per population. Recall the formula that relates the output per worker $\left(y^L\right)$ and the output per capita $\left(y^N\right)$:

$$y^N = \alpha y^L$$

where α is the fraction of workers in population. A decline in this fraction alone, even when output per worker is unchanged, will lead to a decline in the standard of living measured by output per capita.

[3] See homework questions about Australia.

4 Appendix: Deriving Life Expectancy Formula

The straight forward way of calculating the expected length of life is

$$\begin{aligned}
LE &= 0 \cdot \Pr(\text{dying at age } 0) + 1 \cdot \Pr(\text{dying at age } 1) + 2 \cdot \Pr(\text{dying at age } 2) + ... \\
&= 1 \cdot P_0(1 - P_1) + 2 \cdot P_0 P_1 (1 - P_2) + 3 \cdot P_0 P_1 P_2 (1 - P_3) + ... \\
&= P_0 - P_0 P_1 + 2P_0 P_1 - 2P_0 P_1 P_2 + 3P_0 P_1 P_2 - 3P_0 P_1 P_2 P_3 + ... \\
&= P_0 + P_0 P_1 + P_0 P_1 P_2 + P_0 P_1 P_2 P_3 + ... \\
&= \pi_0 + \pi_1 + \pi_2 + \pi_3 + ... \\
&= \sum_{i=0}^{\infty} \pi_i
\end{aligned}$$