San Francisco State University Michael Bar

# Human Capital

So far we have treated the labor input in the production as being homogeneous (the same) across countries. In particular, we modeled the production of aggregate output in an economy as

$$Y = AK^{\theta}L^{1-\theta}$$

where $Y$ is aggregate output, $A$ is productivity level, $K$ is capital stock, and $L$ is the number of workers (the labor input). In this formulation, $A$ captures the contribution of all factors, other than physical capital and labor, to the production of aggregate output. In these notes we take one step further and recognize that there are other aspects of labor, besides the number of workers, that comprise the labor input. Economists use the abstract name "Human Capital" to describe the quality of labor, which is a composition of workers' physical health, skill level, their level of education, etc. What we would like to do is to define

$$\text{Labor Services} = h \cdot L$$

where $h$ captures the human capital or the quality of workers, and $L$ is the number of workers. Then, the aggregate production function would be

$$Y = AK^{\theta}(hL)^{1-\theta} = Ah^{1-\theta}K^{\theta}L^{1-\theta}$$

Now, with this formulation, the productivity parameter $A$ captures the contribution of all factors, other than $h$, $K$ and $L$, to the production of aggregate output.

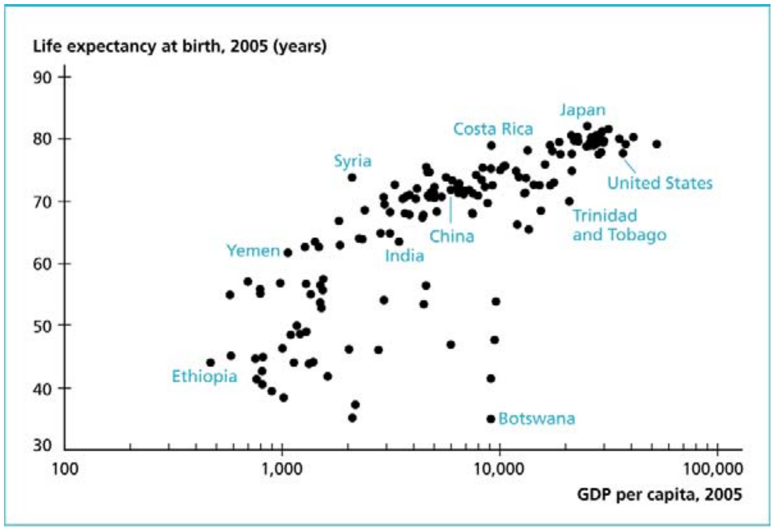
In what follows, we assume that human capital per worker is a function of health and education:

$$h\,(health, education)$$

There is however a problem with measuring this function. In particular, how do we obtain the human capital per worker in Japan for example, $h_{Japan}\,(health_{Japan}, education_{Japan})$? We don't even have data that represents health in Japan. In these notes we will settle for measuring just the part of the human capital that is in the form of education, and as we will see, this estimate is far from perfect.

## 1 Human Capital in the Form of Health

There is a positive correlation between different measures of health and GDP/ capita. For example, the next figure shows that there is a strong positive correlation between life expectancy (one of the most important measures of health) and GDP/capita.




*Sources:* Heston, Summers, and Aten (2006), World Bank (2007a).

This figure suggests that health and standard of living are very closely related. The problem that remains unsolved is how to translate the numbers of life expectancy into health and into human capital. For example, life expectancy in Japan is 80 years. Which value do we assign to health in Japan, $health_{Japan}$?

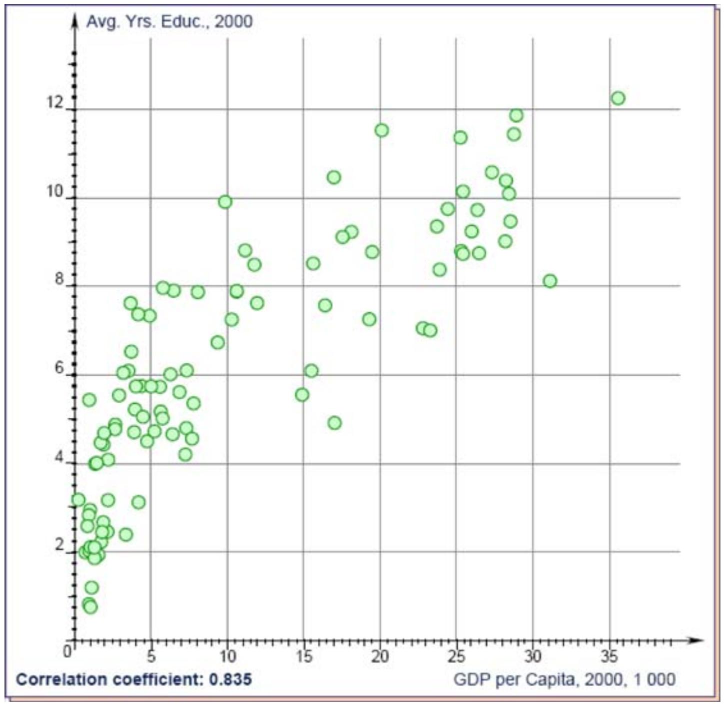
Suppose we managed to rank countries according to different measures of health, such as life expectancy, spread of disease, nutrition, and suppose that Japan gets the highest rank of 10, on the scale from 1 to 10. Then, we still don't know how to translate this number 10 into human capital $h$. Our only hope at this point is to ignore the health aspect of human capital and try to estimate the human capital in the form of education.

# 2 Human Capital in the Form of Education

In this section we consider only the human capital that is related to the skill level of workers, (not to their health), and we also assume that the skill level reflects the level of education. Thus, we assume that

$$h = h(e)$$

where $e$ is education level, measured in years of schooling, and $h$ is human capital. In this section we show how to estimate the level of $h$ for a given country. The next figure shows that there is a strong positive correlation between GDP per capita and average years of education across countries. How much of the differences in GDP/capita across countries can be accounted for by differences in education? In this chapter we attempt to answer this question.




Average years of education and GDP/cap in selected countries

## 2.1 Estimating human capital in the form of education $h(e)$

Economists use data on wages in order to estimate the return to schooling. The next table shows how the average wage increases in years of education in a sample of countries.

| Years of schooling | 1-4 | 5-8 | 9,10,... |
|---|---|---|---|
| Marginal return | 1.134 | 1.101 | 1.068 |

The data in the table is interpreted as follows. Suppose that the average wage of workers with no schooling is normalized to 1, i.e., $w(0) = 1$. Then each of the first 4 years of schooling increases the wage by 13.4%. Thus,

$$\begin{aligned} w(0) &= 1 \\ w(1) &= 1.134 \\ w(2) &= 1.134^2 = 1.286 \\ w(3) &= 1.134^3 = 1.46 \\ w(4) &= 1.134^4 = 1.65 \end{aligned}$$

This means that a person with 4 years of schooling will earn on average 65% more that a person with no schooling at all. Similarly, every year of schooling from 5th to 8th increases the expected wage by 10.1% and every additional year beyond 9th increases the expected wage by 6.8%, for example

$$\begin{aligned} w(5) &= 1.134^4 \cdot 1.101^1 = 1.82 \\ w(6) &= 1.134^4 \cdot 1.101^2 = 2.00 \\ w(8) &= 1.134^4 \cdot 1.101^4 = 2.43 \\ w(10) &= 1.134^4 \cdot 1.101^4 \cdot 1.068^2 = 2.77 \\ w(12) &= 1.134^4 \cdot 1.101^4 \cdot 1.068^4 = 3.16 \\ w(14) &= 1.134^4 \cdot 1.101^4 \cdot 1.068^6 = 3.61 \\ w(16) &= 1.134^4 \cdot 1.101^4 \cdot 1.068^8 = 4.11 \end{aligned}$$

Thus, the expected wage of a worker with 12 years of education is 3.16 times the expected wage of a worker with no schooling at all. The expected wage of a worker with 16 years of education is 4.11 times the expected wage of a worker with no schooling at all.

We use the above relative wages in order to estimate the human capital in a country. We assume human capital in the form of education is proportional to the relative wage of that education group. Let $h_0$ represent the average human capital of a worker with no schooling. Then, the average human capital of a worker with 1 year of schooling is 13.4% bigger, and the average human capital of a worker with 16 years of schooling is 4.11 times that of a worker with no schooling. In general, we assume

$$h(e) = h_0 w(e)$$

Suppose that the average years of schooling in country $i$ is 12 years. Then the average human capital in country $i$ is

$$h_i = h_0 w(12) = 3.16 h_0$$

Suppose that the average schooling in country $j$ is 2 years. Then our estimate of the average human capital in country $j$ is

$$h_j = h_0 w(2) = h_0 \cdot 1.134^2 = 1.286 h_0$$

## 2.2 Accounting for cross-country income differences

Now we can use the estimated human capital in order to account for the cross country differences in GDP per capita.

### 2.2.1 Basic accounting

The most basic accounting assumes only that total output can be represented with Cobb-Douglas production function

$$Y = AK^\theta (hL)^{1-\theta} = Ah^{1-\theta} K^\theta L^{1-\theta}$$

where $h$ is the human capital associated with education. Output per worker is

$$y^L = \frac{Ah^{1-\theta} K^\theta L^{1-\theta}}{L} = Ah^{1-\theta} k^\theta$$

where $k$ is physical capital per worker. Output per capita is then

$$y^N = \alpha y^L = \alpha A h^{1-\theta} k^\theta$$

The accounting formula is then

$$\frac{y_i^N}{y_j^N} = \frac{\alpha_i A_i h_i^{1-\theta} k_i^\theta}{\alpha_j A_j h_j^{1-\theta} k_j^\theta} \tag{1}$$

This formula allows us to decompose the ratio of GDP per capita in two countries into 4 sources: (1) labor force as a fraction of population, (2) productivity, (3) human capital per worker, and (4) physical capital per worker.

Consider the example of China vs. U.S. Our data set contains data of human capital in some countries, in 1999. The rest of the data is for 2000. However, we do have some data on average years of education in 2000, so we can use the method described in the previous section in order to obtain an estimate of average human capital per worker in 2000.

| | $y$ | $k$ | $\alpha$ | $e$ | $h$ | $\theta$ |
|---|---|---|---|---|---|---|
| China ($j$) | 4005.487 | 10133.6388 | 0.5995 | 5.738 | $\mathbf{h_0 \cdot 1.9547}$ | 0.35 |
| U.S. ($i$) | 35586.88 | 148090.477 | 0.514 | 12.247 | $\mathbf{h_0 \cdot 3.2132}$ | 0.35 |

Using the method developed in the previous section, we can estimate the human capital in the two countries[1].

$$\begin{aligned} h_j &= h_0 \cdot 1.134^4 \cdot 1.101^{1.738} = h_0 \cdot 1.9547 \\ h_i &= h_0 \cdot 1.134^4 \cdot 1.101^4 \cdot 1.068^{4.247} = h_0 \cdot 3.213 \end{aligned}$$

Now, we are ready to do the accounting, using equation (1):

| $\frac{y_i}{y_j}$ | $\frac{\alpha_i}{\alpha_j}$ | $\frac{A_i}{A_j}$ | $\left(\frac{h_i}{h_j}\right)^{1-\theta}$ | $\left(\frac{k_i}{k_j}\right)^\theta$ |
|---|---|---|---|---|
| 8.884533 | 0.857381 | 2.934147284 | 1.381377146 | 2.556622 |

If the only difference between the two countries was education, this model suggests that GDP/capita in the U.S. would have been 38% higher than in China.

### 2.2.2 Solow accounting

The production function is given by $Y = Ah^{1-\theta} K^\theta L^{1-\theta}$, and this can be written as

$$Y = \tilde{A} K^\theta L^{1-\theta}$$

where $\tilde{A} = Ah^{1-\theta}$. Thus, under the assumptions of the Solow model, the steady state capital per worker is

$$k_{ss} = \left(\frac{s\tilde{A}}{n+\delta}\right)^{\frac{1}{1-\theta}}$$

[1] We do have data of human capital in the U.S. in 1999, and it is $h_0 \cdot 3.299$. Our method of estimating the human capital from average schooling gives very similar result.

and steady state output per worker is

$$y_{ss} = \tilde{A}\left(\frac{s\tilde{A}}{n+\delta}\right)^{\frac{\theta}{1-\theta}} = \tilde{A}^{\frac{1}{1-\theta}}\left(\frac{s}{n+\delta}\right)^{\frac{\theta}{1-\theta}} = A^{\frac{1}{1-\theta}}h\left(\frac{s}{n+\delta}\right)^{\frac{\theta}{1-\theta}}$$

Thus, the cross country accounting formula under the assumptions of the Solow model and assuming that both countries are in their steady states:

$$\frac{y_i}{y_j} = \frac{\alpha_i A_i^{\frac{1}{1-\theta}} h_i \left(\frac{s_i}{n_i+\delta}\right)^{\frac{\theta}{1-\theta}}}{\alpha_j A_j^{\frac{1}{1-\theta}} h_j \left(\frac{s_j}{n_j+\delta}\right)^{\frac{\theta}{1-\theta}}}$$

**Example.** Suppose that the only difference between the China and the U.S. was average schooling. What would be the ratio of the GDP/cap of two countries in the steady state?

**Solution:**

$$\frac{h_i}{h_j} = \frac{3.213243253}{1.954693094} = 1.643860749$$

**Conclusion**

We learned how to estimate the human capital in the form of education. Our estimates suffer from many deficiencies. For example, we don't have a way of estimating quality of education (recall that we assumed that one year of education in Japan creates the same human capital as one year of education is Ethiopia). Furthermore, we estimated the human capital that is associated with education, and not all aspects of human capital. For example, we don't have a good way of estimating human capital associated with health. So given our methodology of estimating human capital in the form of education, we see that *this* human capital cannot account for much of the differences in cross country GDP/cap.

## 2.3 Workers are Capitalists

We can use the same methodology as described in section 2.1 in order to decompose the total payments to labor services into the part that is paid to raw labor and a part that is paid to human capital. The following table contains data on relative wages and fraction of population by years of schooling, in two groups of countries: developing and developed.

| Years of schooling | Wage Relative to No Schooling | % of POP developing | % of POP Developed |
|---|---|---|---|
| 0 | 1 | 0.344 | 0.037 |
| 4 | 1.65 | 0.226 | 0.117 |
| 8 | 2.43 | 0.119 | 0.134 |
| 10 | 2.77 | 0.163 | 0.265 |
| 12 | 3.16 | 0.083 | 0.166 |
| 14 | 3.61 | 0.035 | 0.151 |
| 16 | 4.11 | 0.03 | 0.13 |

The average total wage in the developing countries is:

$$1 \cdot 0.344 + 1.65 \cdot 0.226 + ... + 4.11 \cdot 0.03 = 1.96951$$

The wage paid to raw labor is 1. Thus, the wage paid to human capital is 0.96951. Thus, a fraction $1/1.96951 = 51\%$ is paid to raw labor and the rest 49% are paid to human capital. Similarly, in developed countries, the average total wage is

$$1 \cdot 0.037 + 1.65 \cdot 0.117 + ... + 4.11 \cdot 0.13 = 2.89369$$

The wage paid to raw labor is 1. Thus, the wage paid to human capital is 1.89369. The fraction of total wages that is paid to the human capital is

$$\frac{1.89369}{2.89369} = 65.44\%$$

Therefore, even in developing countries, half of the wage is paid to human capital, while in developed countries 65% of the wages are paid to human capital. Thus, in a sense, workers are capitalists, since they possess human capital and most of their paycheck is a return to human capital.

The development of countries increases the share of wages that are paid to human capital. In developed countries about 2/3 of the GDP is paid to labor, and 1/3 is paid to physical capital. If 65% of the payments to labor represent payments to human capital, this means that the share of human capital in GDP in developed countries is 43%. This means that human capital's share of the entire output is greater than the share of the physical capital in developed countries. The development thus decreases the conflict between the classes of workers and capitalists, since the workers are essentially capitalists themselves.