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Growth Accounting

So far we focused on accounting for cross country differences in GDP/capita. We decomposed those differences into differences in physical capital, labor, human capital, population growth, participation in the labor force, and productivity. Now we will apply the same accounting technique to compare the GDP/capita within the same country, over time. This is called **growth accounting**.

As before we model the production of aggregate output at time t with the Cobb-Douglas production function:

$$Y_t = A_t K_t^{\theta} (h_t L_t)^{1-\theta} \tag{1}$$

where A_t is the productivity (TFP), K_t is physical capital, h_t is human capital per worker, and L_t is raw labor (number of workers). The output per worker is

$$y_t^L = A_t h_t^{1-\theta} k_t^{\theta}$$

Output per capita is

$$y_t^N = \alpha_t A_t h_t^{1-\theta} k_t^{\theta}$$

where α_t is the number of workers in population. The growth accounting formula for output per worker is

$$\frac{y_{t+1}^L}{y_t^L} = \frac{A_{t+1} h_{t+1}^{1-\theta} k_{t+1}^{\theta}}{A_t h_t^{1-\theta} k_t^{\theta}} = \left(\frac{A_{t+1}}{A_t}\right)\left(\frac{h_{t+1}}{h_t}\right)^{1-\theta}\left(\frac{k_{t+1}}{k_t}\right)^{\theta}$$

This formula decomposes the growth of output per worker between time t and $t+1$ into the contribution of productivity, human capital and physical capital.

Similarly, if we want to account for the growth of GDP per capita, then the formula is

$$\frac{y_{t+1}^N}{y_t^N} = \left(\frac{\alpha_{t+1}}{\alpha_t}\right)\left(\frac{A_{t+1}}{A_t}\right)\left(\frac{h_{t+1}}{h_t}\right)^{1-\theta}\left(\frac{k_{t+1}}{k_t}\right)^{\theta}$$

Here we also have the term that represents the change in participation rate.

For small growth rates we can derive an approximation formula by taking $\ln' s$:

$$\ln\left(\frac{y_{t+1}^N}{y_t^N}\right) = \ln\left(\frac{\alpha_{t+1}}{\alpha_t}\right) + \ln\left(\frac{A_{t+1}}{A_t}\right) + (1-\theta)\ln\left(\frac{h_{t+1}}{h_t}\right) + \theta \ln\left(\frac{k_{t+1}}{k_t}\right)$$

Suppose that the growth rate of some variable x is small, that is

$$\hat{x} = \frac{x_{t+1} - x_t}{x_t} \text{ is small}$$

then

$$\ln\left(\frac{x_{t+1}}{x_t}\right) \approx \hat{x}$$

The proof is in the notes on growth rates. Thus, for small growth rates, we obtain the following approximation formula

$$\hat{y} = \hat{\alpha} + \hat{A} + (1-\theta)\hat{h} + \theta\hat{k}$$

or

$$\hat{A} = \hat{y} - \hat{\alpha} - \left[(1-\theta)\,\hat{h} + \theta\hat{k}\right]$$

When we perform accounting for the output per worker, the $\hat{\alpha}$ is dropped from the above formula.

Example: The next table shows the growth rates of output per worker $\hat{y}$, physical capital $\hat{k}$, and human capital $\hat{h}$, for three countries between the year 2005 and 2006. All the numbers are percentages.

Country	$\hat{y}$	$\hat{k}$	$\hat{h}$	$\hat{A}$
Argentina	1.17	1.59	0.74	
Australia	3.06	4.31	0.13	
Chile	2.00	1.47	0.74	

Example. Using the approximate growth accounting formula, find the growth rate in productivity in each of the above countries.

Solution: The growth rate of productivity is

$$\hat{A} = \hat{y} - \left[(1-\theta)\,\hat{h} + \theta\hat{k}\right]$$

Thus

Country	$\hat{y}$	$\hat{k}$	$\hat{h}$	$\hat{A}$
Argentina	1.17	1.59	0.74	$1.17 - \left[\frac{2}{3}\cdot 0.74 + \frac{1}{3}\cdot 1.59\right] = 0.147$
Australia	3.06	4.31	0.13	$3.06 - \left[\frac{2}{3}\cdot 0.13 + \frac{1}{3}\cdot 4.31\right] = 1.537$
Chile	2.00	1.47	0.74	$2.00 - \left[\frac{2}{3}\cdot 0.74 + \frac{1}{3}\cdot 1.47\right] = 1.017$

We can think of the term in the square brackets as the contribution of factor accumulation to growth. Thus, most of the growth in Argentina is due to factor accumulation, while most of the growth in the other two countries is due to the growth in productivity. The significance of these numbers follows from the fact that without growth in productivity, factor accumulation must stop at some point (see the Solow model). Thus, growth that is due only to factor accumulation is not sustainable.