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Technology (Ch. 8)

1 Measuring Technology Differences

We assumed that the aggregate production is of the Cobb-Douglas form:

$$\begin{aligned} Y &= AK^{\theta}(hL)^{1-\theta} = Ah^{1-\theta}K^{\theta}L^{1-\theta} \\ y &= Ak^{\theta}h^{1-\theta} \end{aligned}$$

where Y is total output, K is total physical capital, h is human capital per worker, and L is total number of workers (raw labor). Lets assume for now that A represents technology. Using this production function, allows us to measure the cross country differences in technology[1]:

$$\begin{aligned} \frac{y_i}{y_j} &= \left(\frac{A_i}{A_j}\right)\left(\frac{k_i^{\theta}h_i^{1-\theta}}{k_j^{\theta}h_j^{1-\theta}}\right) \\ \left(\frac{A_i}{A_j}\right) &= \frac{\left(\frac{y_i}{y_j}\right)}{\left(\frac{k_i^{\theta}h_i^{1-\theta}}{k_j^{\theta}h_j^{1-\theta}}\right)} \end{aligned}$$

It turns out that differences in technology across countries are quite large. Next we want to develop a theory of creation of new technology (innovation theory) in order to better understand where these differences in technologies across countries are coming from.

2 The Nature of Technological Progress

Technology is ideas.

- Technology is **nonrival** - the use of one person does not diminish the availability to others. This feature makes the transfer of technology between firms and countries easier.
- Technology is **nonexcludable** - hard to prevent someone from using it. This feature diminishes the incentives for creating new ideas[2].

[1] Provided that we know how to measure the inputs. As we have seen before, the most problematic input to measure is h.

[2] Patent laws exist exactly because ideas have this feature of nonexcludability.

3 Determinants of R & D

Most of the R & D is undertaken by private firms. Why do private firms engage in R & D? The main reason is that firms hope to increase their profits once they develop more efficient production techniques or new products. What factors determine the profitability of innovations?

1. Degree to which the innovation is protected, or how easy it is for other firms to copy its product.

2. Market size. For example, if the firm develops a new medicine, a major concern is how many people will be the potential users of the medicine. Access to international markets increases the potential rewards of innovations.

3. How long will the advantage of the new technology last. If other firms can develop something similar or even better, then the expected advantage of the innovation is smaller.

In the models presented below we do not address the effects of patent laws and protection of property rights on innovations. The topic is quite advanced and is addressed in advanced "Industrial Organization" courses. We will simply assume that a given fraction of the labor force in the economy is engaged in research and development.

4 One-Country Model of Innovation and Growth

Labor: we assume that the total labor force is fixed at the level of L. A fraction γ_A of the total labor is employed in the R&D sector and the rest are employed in the output producing sector, i.e.

$$L^A = \gamma_A L; \quad L^Y = (1 - \gamma_A) L$$

Output:

$$\begin{aligned} Y_t &= A_t (1 - \gamma_A) L \\ y_t &= A_t (1 - \gamma_A) \end{aligned}$$

Innovations:

$$\hat{A} = \frac{A_{t+1} - A_t}{A_t} = \frac{\gamma_A L}{\mu}$$

where μ is the price of innovations in units of labor. The net growth rate of output per worker is therefore

$$\hat{y} = \hat{A} = \frac{\gamma_A L}{\mu} \tag{1}$$

We see that when workforce is constant, the growth rate is constant.

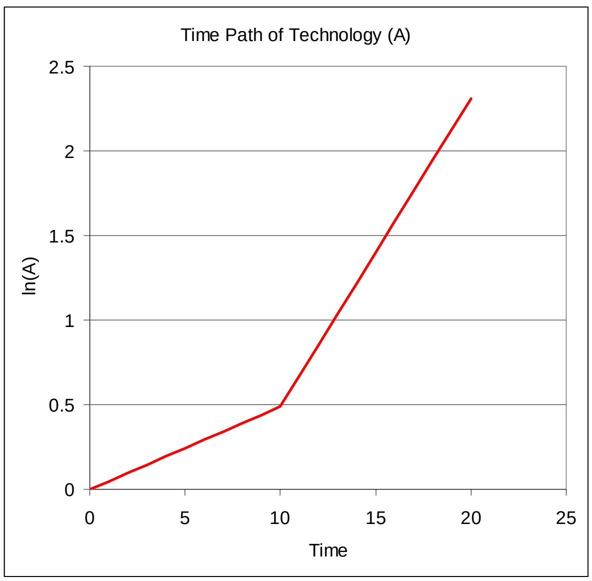
4.1 Experiments

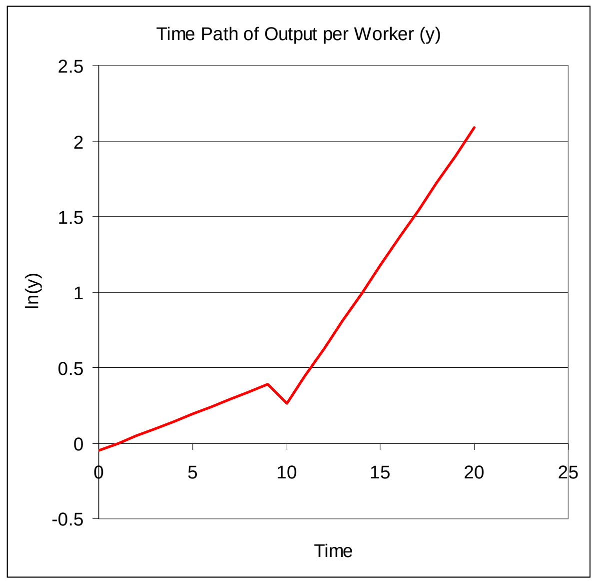
In what follows, we will hold the population constant.

4.1.1 Increase in R&D ($\gamma_A \uparrow$)

Suppose that at time t the economy increases the fraction of the workforce that is engaged in R&D. Higher γ_A leads to permanently higher growth rate of technology and higher growth rate of output per worker, as can be seen from equation (1). In the short run though, the *current* output per worker will be smaller than what it would otherwise be. Since the current output per worker is given by $y_t = A_t (1 - \gamma_A)$, then an increase in γ_A will decrease the fraction of the workforce engaged in the production of output, which leads to a decrease in y_t. However, since A is growing, $(A_t > A_{t-1})$, it is not necessary that the current output will fall (i.e., it is not necessary that $y_t < y_{t-1}$). Nevertheless, the current output y_t would have been larger if γ_A did not increase. Just like any investment, when we increase γ_A, we give up some of the current output in order to increase the productive capacity in the future.

The following figures show the time paths of technology and output per worker, when we increase γ_A from 5% to 20% in period 10 and keep γ_A at the new level forever.



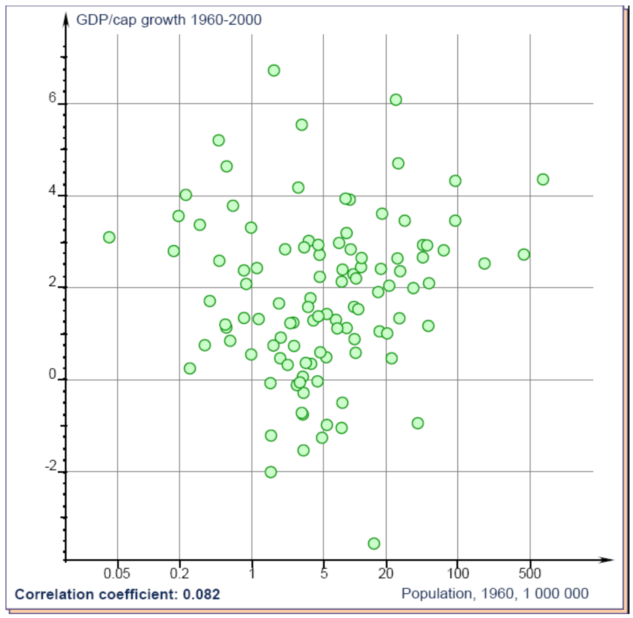


We see that in this example, the current output at the time of the increase in γ_A actually falls, but as was mentioned above, this need not be the case.

4.1.2 Higher population ($L \uparrow$)

Suppose that the level of population is constant before period t and then increases to a new level from period t on. Higher population leads to higher $\hat{y}$ and $\hat{A}$, i.e. permanently higher growth rate of technology and output per worker, as can be seen from equation (1). Unlike when γ_A increased, there is no sacrifice of output in the current period.

This model predicts that countries with higher population, everything else held constant, should exhibit higher growth rate of output per worker. This prediction however is not supported by the data. The next figure shows a scatter plot of population size in 1960 and the growth rate of GDP per capita since 1960 in selected countries. If the prediction of the model was consistent with the data, we would see a positive correlation between the population size in 1960 and the growth rate of GDP per capita since 1960. What we see however, is almost no correlation between the two variables. This means, that countries with larger population, do not necessarily grow faster than countries with smaller population.



Population size vs. growth of GDP/cap

In the real world however, the technology used in one country is not necessarily developed in that country. Technologies cross borders. In the next section we investigate a model in which one country is technological leader and the other countries copy its technology (followers). We will see that higher population of the follower does not lead to higher growth rate of output per worker, but does lead to higher standard of living.

5 Two-Country Model

Consider two countries, country 1 is the "leader" and country 2 is the "follower" (copier). Let $\gamma_{A,1}$ be the fraction of the labor force of the leader that is engaged in R&D and let $\gamma_{A,2}$ be the fraction of the labor force of the follower that is engaged in copying new technology. We assume that $\gamma_{A,2} < \gamma_{A,1} < 0.5$. It will become clear later, why we need these assumptions.

Labor: L_1, L_2. We assume that the labor force in the follower country is "not too big": $\gamma_{A,2}L_2 < \gamma_{A,1}L_1$. That is, we assume that the leader employs more workers in R&D than the follower. That is the reason why he is the leader in the first place.

Output per worker:

$$\begin{aligned} y_{1t} &= A_{1t}\left(1-\gamma_{A,1}\right) \\ y_{2t} &= A_{2t}\left(1-\gamma_{A,2}\right) \end{aligned}$$

Innovations (in the "leader" country):

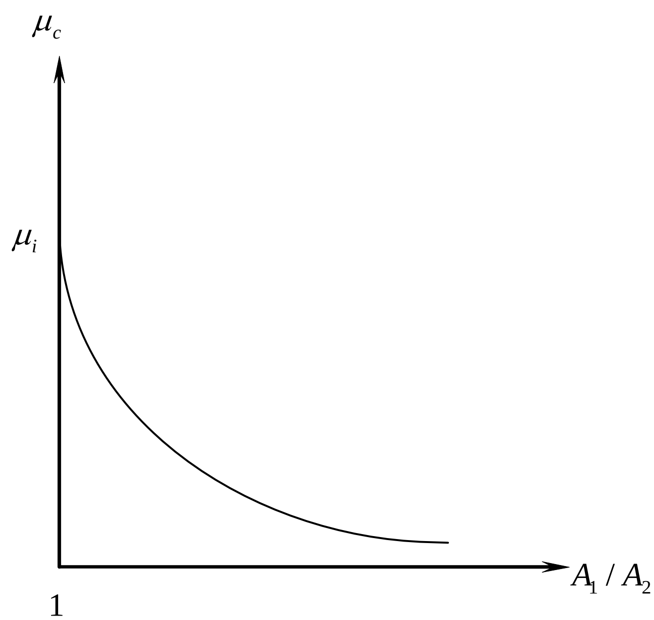
$$\hat{A}_1 = \frac{A_{1t+1} - A_{1t}}{A_{1t}} = \frac{\gamma_{A,1} L_1}{\mu_i}$$

Here μ_i is the cost of innovation.

Copying (in the "follower" country). The "follower" does not innovate. We assume that the cost of copying from the leader is given by

$$\mu_c = \frac{\mu_i}{A_1/A_2} \tag{2}$$

Notice that the cost of copying are decreasing in A_1/A_2. This fraction indicates how far technologically the follower is behind the leader. If the fraction A_1/A_2 is very big, this means that the follower is very far behind the leader and the cost of adopting technology (which is new for the follower, but very old for the leader) is low. Also notice that if the follower is closer to the leader, as $A_2 \to A_1$, the cost of copying approaches the cost of innovating μ_i. The graph of μ_c is depicted in the following figure.



Thus, the growth rate of technology of the follower is given by:

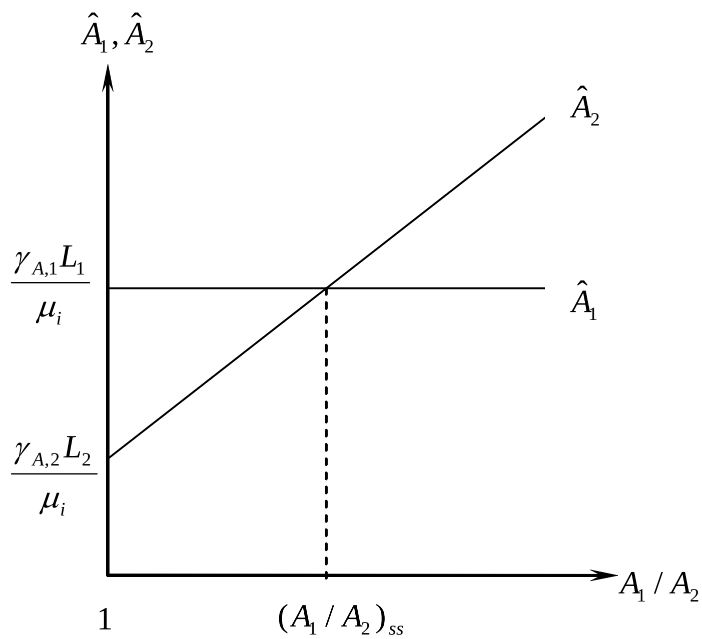
$$\hat{A}_2 = \frac{A_{2t+1} - A_{2t}}{A_{2t}} = \frac{\gamma_{A,2} L_2}{\mu_c}$$

Substituting from equation (2) gives

$$\hat{A}_2 = \frac{\gamma_{A,2} L_2}{\mu_i} \left(\frac{A_1}{A_2} \right)$$

5.1 Equilibrium (steady state)

We will assume that the population of the follower is "not too big", such that $\gamma_{A,2} L_2 < \gamma_{A,1} L_1$. With this assumption we can show that both countries must exhibit the same growth rate of technology and output. In the next figure we plot the graphs of $\hat{A}_1$ and $\hat{A}_2$ as functions of A_1/A_2.



Notice that our assumption that the population of the follower is not too big guarantees that if the follower has the same technology as the leader ($A_1/A_2 = 1$) then the follower grows slower than the leader. If it wasn't for this assumption, the follower would become the leader.

Now observe that if $A_1/A_2 < (A_1/A_2)_{ss}$, then the leader is growing faster than the follower and if $A_1/A_2 > (A_1/A_2)_{ss}$, then the follower grows faster than the leader. Therefore, in the long run, both the follower and the leader will grow at the same rate. Unlike in the one country model, higher population of the follower does not lead to faster growth rate of output per worker. In fact, if we have a world with one leader and many followers, then all the countries in the world, including the leader, will grow at the same rate in the steady state.

To find the steady state, we need to solve $\hat{A}_1 = \hat{A}_2$,

$$\frac{\gamma_{A,1}L_1}{\mu_i} = \frac{\gamma_{A,2}L_2}{\mu_i}\left(\frac{A_1}{A_2}\right)_{ss}$$

Which gives

$$\left(\frac{A_1}{A_2}\right)_{ss} = \frac{\gamma_{A,1}L_1}{\gamma_{A,2}L_2} \tag{3}$$

Thus, the steady state ratio of productivity in the two countries is equal to the ratio of the labor employed in R&D in the two countries. The steady state output ratio is

$$\left(\frac{y_1}{y_2}\right)_{ss} = \left(\frac{A_1}{A_2}\right)_{ss}\left(\frac{1-\gamma_{A,1}}{1-\gamma_{A,2}}\right) \tag{4}$$

Substituting (3) in (4) gives

$$\left(\frac{y_1}{y_2}\right)_{ss} = \frac{\gamma_{A,1} L_1}{\gamma_{A,2} L_2} \left(\frac{1-\gamma_{A,1}}{1-\gamma_{A,2}}\right) \tag{5}$$

Interestingly, being the leader does not guarantee higher standard of living. For the leader to have higher output per worker than the follower, we need to have

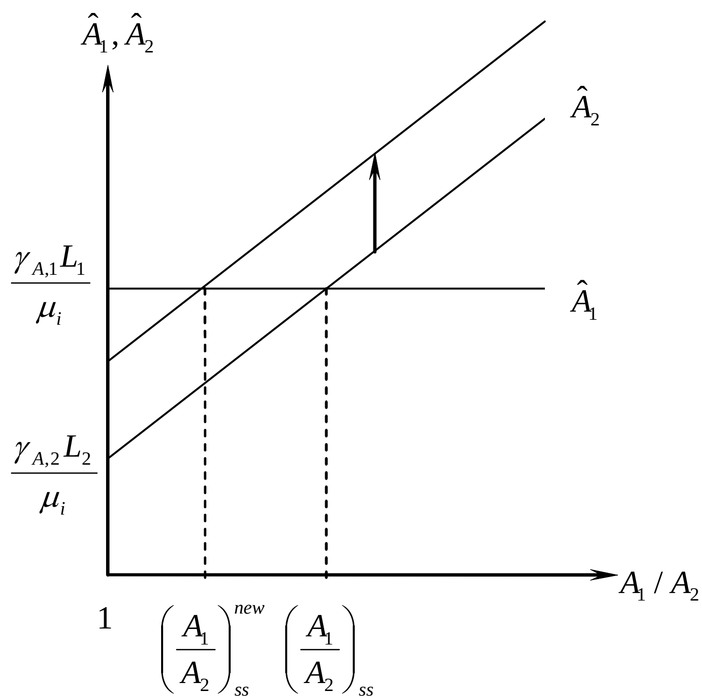
$$\frac{\gamma_{A,1} L_1}{\gamma_{A,2} L_2} \left(\frac{1-\gamma_{A,1}}{1-\gamma_{A,2}}\right) > 1$$

Suppose $L_1 = L_2$ and $\gamma_{A,1} = 0.8,\ \gamma_{A,2} = 0.5$. In this case $y_1 < y_2$, so the follower enjoys higher standard of living.

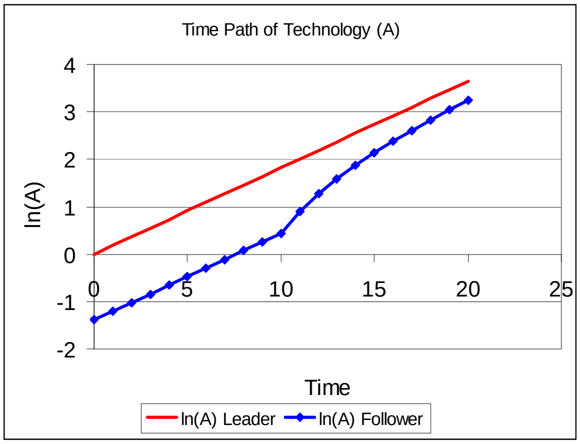
5.2 Experiments

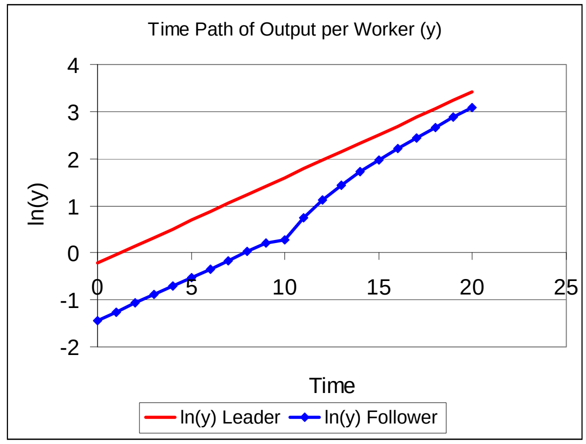
5.2.1 Increase in R&D in the follower country $(\gamma_{A,2} \uparrow)$

As can be seen in the next graph, the steady state ratio of $(A_1/A_2)_{ss}$ is smaller, so the follower is "catching up" technologically with the leader.



It is not clear however what will happen to $(y_1/y_2)_{ss}$. Observe that in equation (4) on the one hand $(A_1/A_2)_{ss} \downarrow$, but on the other hand the second term increases. From equation (5) we see that when $\gamma_{A,2}$ goes up, the term $\gamma_{A,2}\left(1-\gamma_{A,2}\right)$ can either go up or down. In order to guarantee that $(y_1/y_2)_{ss}$ is decreasing in $\gamma_{A,2}$ we need to assume that $\gamma_{A,2} < 0.5$. See appendix for proof. With this assumption, the ratio of output per worker is also smaller, so the follower is "catching up" with the leader in terms of standard of living. The next figure shows the time paths of technology and output per worker in the two countries. Notice that in the long run both countries will grow at the same rate, but the ratios of A_1/A_2 and y_1/y_2 are smaller[3].

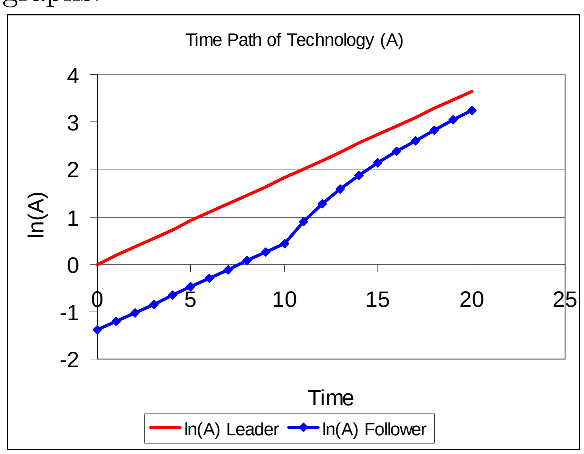


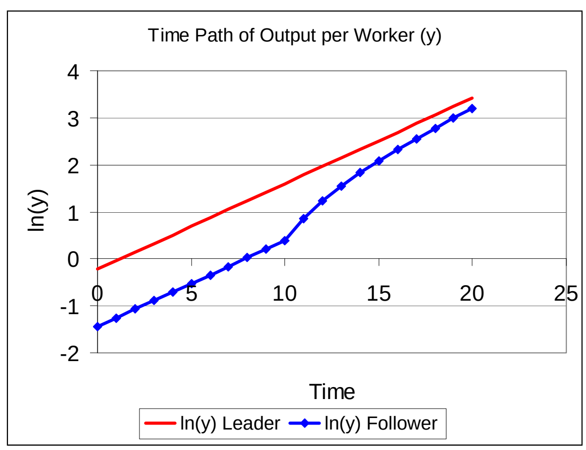


Hence, in the two-country model, a once-and-for-all increase in γ_A leads to temporary increase in the growth rate, until the new steady state is reached, and in the long run the follower grows as fast as the leader. There is a slowdown in the growth of y_t at the period in which γ_A increased (here period 10). The output per worker is not as high in period 10 as it would have been without the increase in γ_A.

5.2.2 Higher population in the follower country ($L_2 \uparrow$)

The effect of an increase in L_2 on the steady state is similar to the effect of increase in γ_A. In the new steady state the ratios of A_1/A_2 and y_1/y_2 are smaller, so the follower will catch up. There is no sacrifice of current output per worker though, as can be seen in the next graphs.



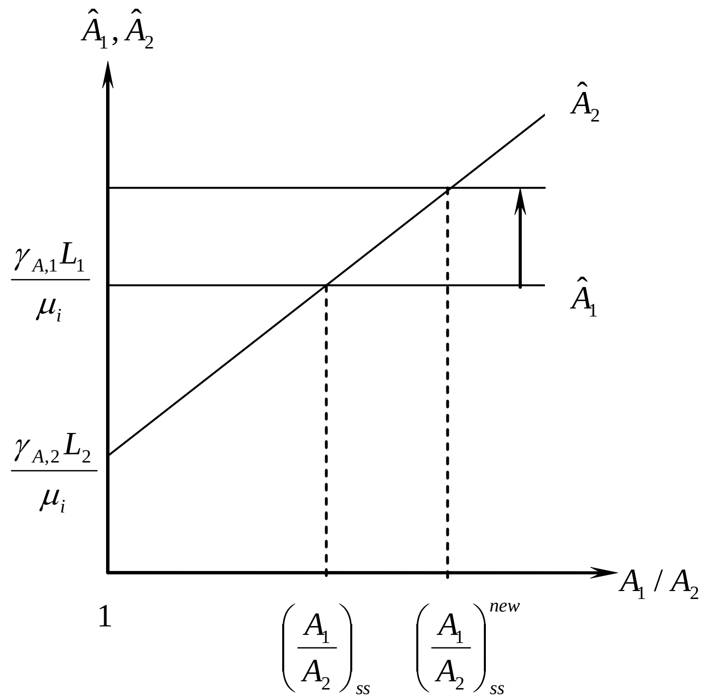


Thus, increasing the population of the follower is another way he can catch up with the leader. This prediction of the model is not supported by the data.

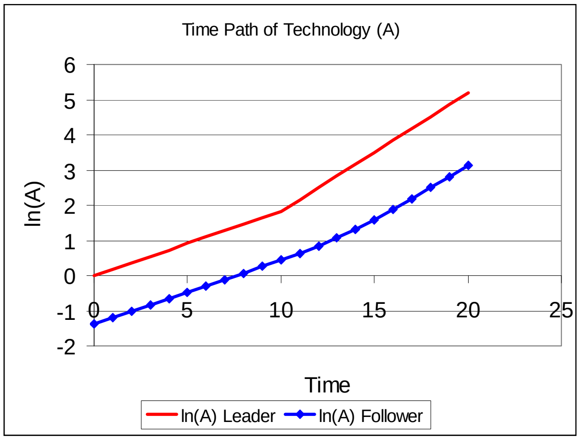
[3] Recall that $\ln(y_1/y_2) = \ln(y_1) - \ln(y_2)$, so if the distance between the logs is smaller, then the ratios of the original variables is smaller.

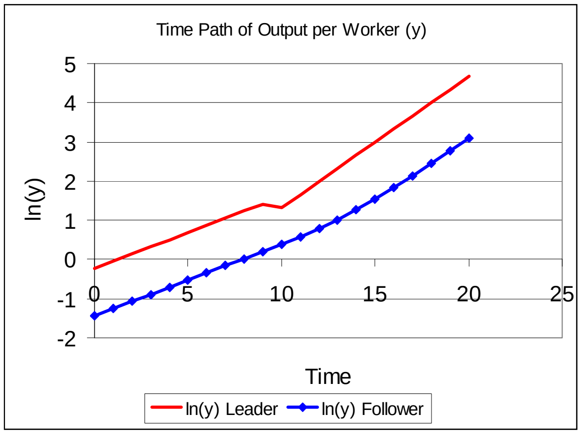
5.2.3 Increase in R&D in the leader country $(\gamma_{A,1} \uparrow)$

The next graph show the effect on the steady state ratio $(A_1/A_2)_{ss}$. The the follower will fall farther behind the leader technologically.



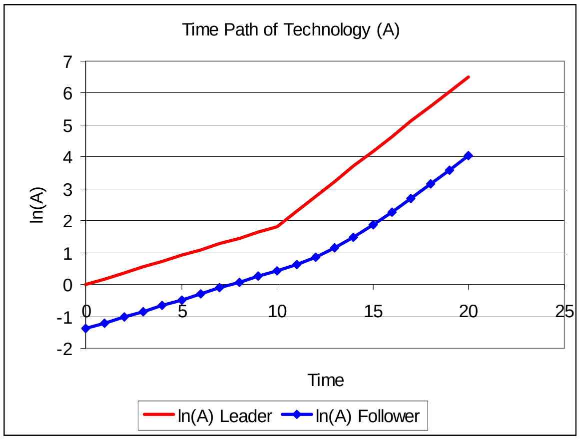
However, since the leader starts growing faster, the follower will grow faster as well, i.e. the ratios of A_1/A_2 and y_1/y_2 are larger. In the long run, the leader and the follower will grow at the same rate. This is illustrated in the following graphs.

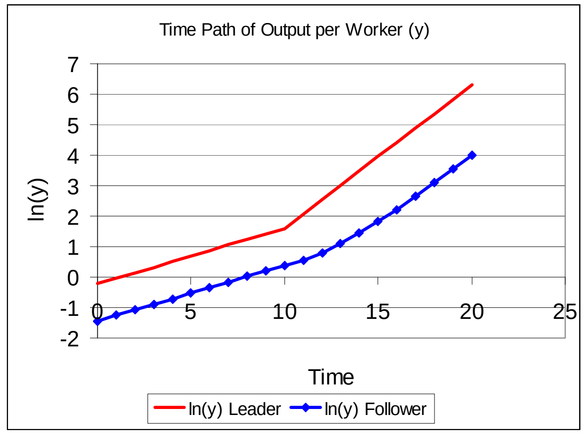




5.2.4 Higher population in the leader country $(L_1 \uparrow)$

This is similar to the increase in $\gamma_{A,1}$, except that now there is no sacrifice in current consumption of the leader.





5.3 Summary of the results and discussion

The leader in this model behaves just like the one-country model. That is, higher $\gamma_{A,1}$ or higher population lead to *permanently* higher growth rates of technology and output per worker. The follower (or the followers) behave somewhat differently. Higher $\gamma_{A,2}$ or higher population lead to only *temporary* increase in the growth rate of technology and output per worker. In the long run, the follower grows at the same rate as the leader. Higher population of the follower, although does not lead to higher growth rate, still enables the follower to catch up with the leader[4], i.e. smaller ratios of A_1/A_2 and y_1/y_2.

If the world was like this model, with one technological leader and all the other countries were followers, then we would expect to see that in the long run all the countries grow at the same rate (as the leader). If the leader expands its R&D, then the leader starts growing faster and the rest of the countries eventually grow at the same faster rate. When we look at the data, we observe that there are very large differences in the growth experience of countries. One reason why the countries differ in their growth experience is that in the real world there is no single leader in technology, but instead different countries are leaders in different fields.

The model also predicts that higher population has an advantage for the followers, as can be seen from equations (3) and (4). Suppose that two followers have the same γ_A, but one of them has higher population. Then the one with the higher population would be closer to the leader in terms of technology and output per worker. There is no support in the data for this prediction. In the data, more populated countries do not necessarily enjoy higher standard of living. There are other theories of innovation that look at factors such as population density and education, and not just the size of the population.

The general lesson from this model is that an increase in R&D in one country has two effects: (1) the country itself starts growing faster, and (2) the new technologies will spill to other countries and they will start growing faster as well. Therefore, the model is optimistic that even the most backward countries will eventually copy the technology and grow faster.

[4] If we maintain the assumptions that $\gamma_{A,2} < \gamma_{A,1} < 0.5$.

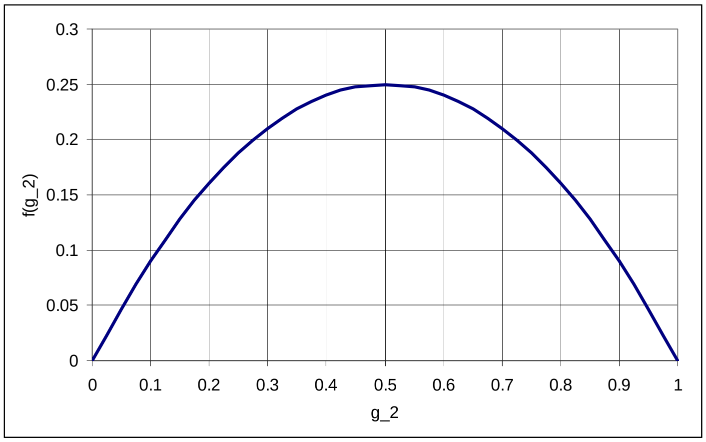
6 Appendix

We want to prove that if $\gamma_{A,2} < 0.5$, then $(y_1/y_2)_{ss}$ is decreasing in $\gamma_{A,2}$.

Proof. Recall that

$$\left(\frac{y_1}{y_2}\right)_{ss} = \frac{\gamma_{A,1}L_1}{\gamma_{A,2}L_2}\left(\frac{1-\gamma_{A,1}}{1-\gamma_{A,2}}\right)$$

If we change $\gamma_{A,2}$ the only term that changes is $f\left(\gamma_{A,2}\right) = \gamma_{A,2}\left(1-\gamma_{A,2}\right)$. The function f is strictly concave with maximum at $\gamma_{A,2} = 0.5$. To see this notice that $f'\left(\gamma_{A,2}\right) = 1 - 2\gamma_{A,2}$ and $f''\left(\gamma_{A,2}\right) = -2$. The maximum of f is attained at $\gamma_{A,2} = 0.5$. The next figure shows the graph of f.



Thus, as long as $\gamma_{A,2} < 0.5$ any increase in $\gamma_{A,2}$ will lead to increase in f. ■