Economics of Growth, ECON560
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Midterm Exam

Wednesday, April 5

1 hour, 15 minutes

Name: ___________________________________

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (30 points). Suppose that an economy is described by the Malthusian model. Consumers like to consume food (Y_t). Food production function is given by $Y_t = A_t \Lambda^{\theta} L_t^{1-\theta}$, $0 < \theta < 1$, where A_t is productivity level at time t, Λ is (fixed) land, and L_t is the number of workers, which is also the size of the population. Population evolves according to $L_{t+1} = g(y_t) L_t$, where $g(y_t)$ is the growth rate of population as a function of output per capita $y_t = Y_t / L_t$. It is assumed that there is some subsistence level of consumption per capita y^* such that $g(y_t) < 1$ when $y_t < y^*$, $g(y_t) > 1$ when $y_t > y^*$, and $g(y_t) = 1$ when $y_t = y^*$.
   a. (5 points). Suppose that in some country the technology level is fixed at $A_t = 100 \quad \forall t$, the population growth function is $g(y_t) = 0.1\sqrt{y_t}$, the land share parameter is $\theta = 0.5$, and the land is $\Lambda = 50{,}000$. Find the steady state level of output per worker (y^*) and population (L^*).

<u>Steady state output per worker</u>

$$g(y^*) = 1$$

$$0.1\sqrt{y^*} = 1$$

$$y^* = 100$$

<u>Steady state population level</u>

$$y^* = A\left(\frac{\Lambda}{L^*}\right)^{\theta}$$

$$100 = 100\left(\frac{50{,}000}{L^*}\right)^{1/2}$$

$$L^* = 50{,}000$$

b. (5 points). Suppose that the productivity level at time τ had doubled once-and-for-all, so that $A_t = 200$ for $t = \tau, \tau+1, \tau+2, ...$ Find the new steady state level of output per worker (y^*) and population (L^*).

Steady state output per worker (same as in previous section)

$$g(y^*) = 1$$

$$0.1\sqrt{y^*} = 1$$

$$y^* = 100$$

Steady state population level

$$y^* = A\left(\frac{\Lambda}{L^*}\right)^{\theta}$$

$$100 = 200\left(\frac{50{,}000}{L^*}\right)^{1/2}$$

$$\frac{1}{4} = \frac{50{,}000}{L^*}$$

$$L^* = 200{,}000$$

c. (5 points). Malthus observed that some educated families in the large cities restricted their fertility by delaying marriage and contraception. He called this behavior "preventive check". Suppose that the population growth function in the above economy changes to $g(y_t) = 0.05\sqrt{y_t}$. Find the new steady state of output per worker.

<u>Steady state output per worker</u>

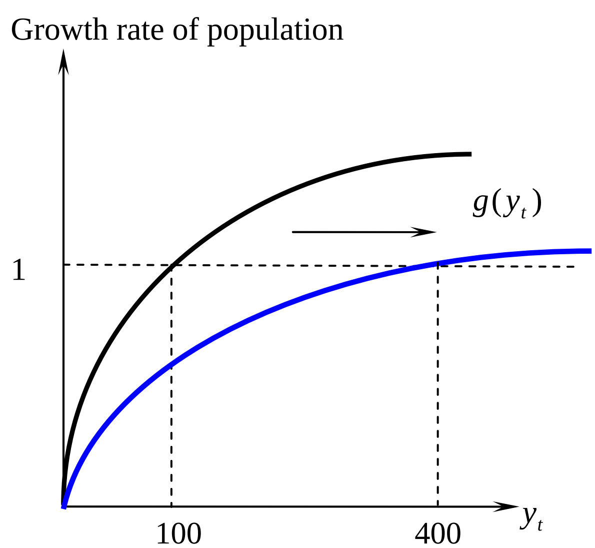
$$g(y^*) = 1$$

$$0.05\sqrt{y^*} = 1$$

$$\sqrt{y^*} = 20$$

$$y^* = 400$$

d. (5 points). In the following graph, illustrate the effect of the preventive check on the steady state output per worker. That is, clearly illustrate a shift of the curve (or a shift along the curve) and label the original and the new steady states of output per worker.



e. (10 points). Complete the following sentences using the words "temporary" or "permanent".

   i. In the Malthusian model, a once-and-for-all increase in productivity leads to a **__temporary____** improvement in the standard of living.

   ii. In the Solow model, a once-and-for-all increase in productivity leads to a **___permanent____** improvement in the standard of living.

2. (20 points). Suppose that in the country of Narnia, the infant mortality rate is 0.5, people who survive the first year of life will live until the age of 100 with certainty, and everybody dies the moment they reach the age of 100.
   a. (5 points). Calculate the life expectancy in Narnia.

$$\pi_i = 0.5, \quad \forall i = 0,...,99$$

$$LE = \sum_{i=0}^{99} 0.5 = 0.5 \cdot 100 = 50 \text{ years}$$

   b. (5 points). Suppose that women in Narnia are fertile during the ages of 20,...,49. The age specific fertility rates are given in the next table.

Fertility F_i	Age (i)
0.4	20, 21,...,29
0.3	30,31,...,39
0.1	40,41,...,49

Calculate the total fertility rate (TFR) the net reproduction rate (NRR) in Narnia.

$$TFR = 10 \cdot 0.4 + 10 \cdot 0.3 + 10 \cdot 0.1 = 8 \text{ children per woman.}$$

$$NRR = 0.5 \cdot 0.5 \cdot [10 \cdot 0.4 + 10 \cdot 0.3 + 10 \cdot 0.1] = 2 \text{ Daughters per surviving woman}$$

c. (5 points). Suppose that the age specific fertility in Narnia has declined, and is given in the next table.

Fertility F_i	Age (i)
0.2	20, 21,...,29
0.15	30,31,...,39
0.05	40,41,...,49

In addition, suppose that infant mortality became 0. Calculate the total fertility rate (TFR) the net reproduction rate (NRR) in Narnia.

$TFR = 10 \cdot 0.2 + 10 \cdot 0.15 + 10 \cdot 0.05 = 4$ children per woman.

$$NRR = 1 \cdot 0.5 \cdot [10 \cdot 0.2 + 10 \cdot 0.15 + 10 \cdot 0.05] = 2$$

d. (5 points). Briefly explain why the net reproduction rate is the same in sections b and c, despite the fact that age specific fertility rates in section c are lower. In your explanation, use the formula for net reproduction rate.

Recall that the net reproduction rate formula is $NRR = \frac{1}{2}\sum_{i=0}^{\infty}\pi_i F_i$, where π_i is the probability that a woman is alive at age i and F_i fertility at age i. The *NRR* is increasing in both arguments, and the numbers in this example are chosen to be such that $\pi_i \uparrow$, and $F_i \downarrow$ cancel each other.

3. (10 points). The next table shows how the average wage increases in years of education in a sample of countries.

Years of schooling	1-4	5-8	9,10,…
Marginal return	1.134	1.101	1.068

Suppose that workers in Japan have on average 12.43 years of education, while workers in Indonesia have on average 8 years of education. Assuming that the quality of education in both countries is the same, calculate the ratio of human capital per worker in the two countries. (Simplify your answer, but there is no need to provide the exact numerical answer).

$$\frac{h_{Japan}}{h_{Indonesia}} = \frac{h_0 \cdot 1.134^4 \cdot 1.101^4 \cdot 1.068^{4.43}}{h_0 \cdot 1.134^4 \cdot 1.101^4} = 1.068^{4.43}$$

4. (10 points). Suppose that workers in country *i* have 4 times as much human capital as the workers in country *j*, and that capital share in both countries is $\theta = 0.5$. If the only difference between the two countries was human capital of workers, the ratio of GDP per capita would have been (circle the correct answer):

   a. $\frac{y_i}{y_j} = 8$

   b. $\frac{y_i}{y_j} = 4$

   c. $\frac{y_i}{y_j} = 2$

   d. $\frac{y_i}{y_j} = 0.5$

5. (20 points). Consider the model of technological leader and follower, briefly described as follows. The production of output in the leader country is according to $Y_1 = A_1(1-\gamma_{A,1})L_1$, where Y is output, A is technology level, γ_A is the fraction of population engaged in R&D, and L_1 is total labor (= total population). The production of new technology is described by $\hat{A}_1 = \gamma_{A,1}L_1 / \mu_i$ where μ_i is the cost of innovation. The follower country has the same production function of output as the leader: $Y_2 = A_2(1-\gamma_{A,2})L_2$. The copying of technology from the follower is according to $\hat{A}_2 = \gamma_{A,2}L_2 / \mu_c$ where $\mu_c = \dfrac{\mu_i}{A_1 / A_2}$.

   a. (5 points). Suppose that in country 1 (the leader) the labor force is 10, the fraction of the labor force in R&D is 20%, and the cost of innovation is 10. In country 2 (the follower), the labor force is 10, the fraction of the labor force in R&D is 10%. Compute the steady state ratios of technologies and output per worker: $(A_1 / A_2)_{ss}, (y_1 / y_2)_{ss}$.

$$\left(\frac{A_1}{A_2}\right)_{ss} = \frac{\gamma_{A,1}L_1}{\gamma_{A,2}L_2} = \frac{0.2 \cdot 10}{0.1 \cdot 10} = 2$$

$$\left(\frac{y_1}{y_2}\right)_{ss} = \frac{\gamma_{A,1}L_1(1-\gamma_{A,1})}{\gamma_{A,2}L_2(1-\gamma_{A,2})} = \frac{0.2 \cdot 10 \cdot (1-0.2)}{0.1 \cdot 10 \cdot (1-0.1)} = \frac{16}{9}$$

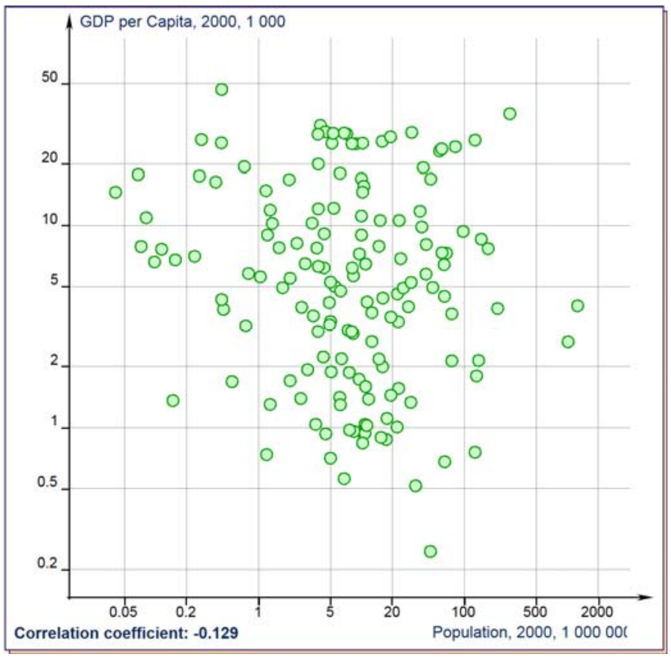
b. (5 points). Now consider another follower, country 3, which is identical to country 2, except it has population of 16. Compute the steady state ratios of technologies and output per worker for the leader and country 3. Which follower is better off in terms of output per worker?

$$\left(\frac{A_1}{A_3}\right)_{ss} = \frac{\gamma_{A,1}L_1}{\gamma_{A,3}L_3} = \frac{0.2\cdot 10}{0.1\cdot 16} = \frac{20}{16} = \frac{5}{4}$$

$$\left(\frac{y_1}{y_3}\right)_{ss} = \frac{\gamma_{A,1}L_1(1-\gamma_{A,1})}{\gamma_{A,3}L_3(1-\gamma_{A,3})} = \frac{0.2\cdot 10\cdot(1-0.2)}{0.1\cdot 16\cdot(1-0.1)} = \frac{10}{9}$$

Country 3 is better off than country 2, because we see that that $(y_1/y_3)_{ss} < (y_1/y_2)$, i.e. country 3 is closer to the leader than country 2.

c. (5 points). The following data supports the prediction of this model, that countries with higher population should also have higher output per worker. True/false, circle the correct answer and provide a brief explanation.



If the data supported the prediction of the model, the correlation between population size and GDP per capita should have been positive. Instead, we see that the correlation is negative.

d. (5 points). In the steady state, the leader grows faster than the followers both in technology and output per capita. True/false, circle the correct answer and prove it.

In the steady state, the ratios $(A_1 / A_2), (y_1 / y_2), (A_1 / A_3), (y_1 / y_3)$ are constant by definition of the steady state. Therefore, the leader and all the followers must grow at the same rate, both in technology and output per worker.

6. (10 points). Consider the model of Charles Jones 1999, where the technology evolves according to $\hat{A} = \frac{L_A^{\lambda} \cdot A^{-\phi}}{\mu}$. Here $L_A = \gamma_A L$ is the number of researchers, γ_A is the (constant) fraction of researchers in population, and $0 < \lambda, \phi < 1$. Output is produced according to $Y = A(1-\gamma_A)L$.

   a. The term $A^{-\phi}$ represents the "fishing-out effect" of technology production. Briefly explain what this means.

The term $A^{-\phi}$ is a decreasing function of *A*, which means that if the stock of technology already in existence is large, it is harder to develop new technology because scientists need to cover a lot of material. The analogy to fishing is that after catching all the big and lazy fish in the lake, it is harder to catch new fish.

   b. Prove that a constant and positive growth in standard of living is possible in this model only as long as the number of researcher continues to grow.

First, we show that the output per worker grows at the same rate as technology.

$$y = \frac{Y}{L} = A(1-\gamma_A)$$

$$\Rightarrow \hat{y} = \hat{A}$$

The implication follows from the fact that γ_A is constant.

Next, since these growth rates are constant, it follows that:

$$\hat{y} = \hat{A} = \text{constant} = \frac{L_A^{\lambda} \cdot A^{-\phi}}{\mu}$$

In approximate growth rates the above can be written as:

$$0 = \lambda \hat{L}_A - \phi \hat{A}$$

$$\hat{A} = \frac{\lambda}{\phi} \hat{L}_A$$

Thus, $\hat{A} > 0$ is possible only if $\hat{L}_A > 0$.