Economics of Growth, ECON560
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Fall 2010

<u>Midterm Exam</u>

Thursday, October 28

1 hour, 15 minutes

Name: ___________________________________

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (35 points). Suppose that the economy of Great Britain (GB) is described by the Malthusian model. Consumers like to consume food (Y_t). Food production function is given by $Y_t = A_t \Lambda^{\theta} L_t^{1-\theta}$, $0 < \theta < 1$, where A_t is productivity level at time *t*, Λ is (fixed) land, and L_t is the number of workers, which is also the size of the population. Population evolves according to $L_{t+1} = g(y_t)L_t$, where $g(y_t)$ is the growth rate of population as a function of output per capita $y_t = Y_t / L_t$. It is assumed that there is some subsistence level of consumption per capita $y*$ such that $g(y_t) < 1$ when $y_t < y*$, $g(y_t) > 1$ when $y_t > y*$, and $g(y_t) = 1$ when $y_t = y*$.
   a. (5 points). Suppose that technology level in Great Britain is fixed at $A_t = 5 \;\; \forall t$, the population growth function is $g(y_t) = 0.2 y_t$, the land share parameter is $\theta = 0.5$, and the land is $\Lambda = 1000$. Find the steady state level of output per worker (*y**) and population (*L**).

<u>Steady state output per worker</u>

$$g(y^*) = 1$$

$$0.2 y^* = 1$$

$$y^* = 5$$

<u>Steady state population level</u>

$$y^* = A\left(\frac{\Lambda}{L^*}\right)^{\theta}$$

$$5 = 5\left(\frac{1000}{L^*}\right)^{1/2}$$

$$L^* = 1000$$

b. (5 points). Suppose that Great Britain discovered Australia and as a result its land doubled (i.e. the land becomes $\Lambda = 2000$). Find the new steady state level of output per worker (y*) and population (L*).

<u>Steady state output per worker</u>

$$g(y^*) = 1$$
$$0.2y^* = 1$$
$$y^* = 5$$

<u>Steady state population level</u>

$$y^* = A\left(\frac{\Lambda}{L^*}\right)^{\theta}$$
$$5 = 5\left(\frac{2000}{L^*}\right)^{1/2}$$
$$L^* = 2000$$

c. (5 points). Explain briefly why, in the Malthusian model, the standard of living did not increase **permanently** despite such a large increase in land.

Initially, there is an increase in output per worker due to the increase in land per worker (recall that output per worker is: $y_t = A_t\left(\frac{\Lambda}{L_t}\right)^{\theta}$). In the Malthusian model, as long as $y_t > y^*$, we have $g(y_t) > 1$, and the population is increasing. This occurs until output per worker returns to the subsistence level.

d. (5 points). Suppose that in addition to doubling of land (i.e. $\Lambda = 2000$), the productivity in Great Britain also doubled (i.e. $A_t = 10 \ \forall t$). Find the new steady state level of output per worker (y^*) and population (L^*).

Steady state output per worker

$$g(y^*) = 1$$
$$0.2y^* = 1$$
$$y^* = 5$$

Steady state population level

$$y^* = A\left(\frac{\Lambda}{L^*}\right)^{\theta}$$
$$5 = 10\left(\frac{2000}{L^*}\right)^{1/2}$$
$$\frac{1}{4} = \frac{2000}{L^*}$$
$$L^* = 8000$$

e. (5 points). Explain briefly why, in the Malthusian model, the standard of living did not increase **permanently** despite such a large increase in productivity.

Initially, there is an increase in output per worker due to the increase in productivity (recall that output per worker is: $y_t = A_t\left(\frac{\Lambda}{L_t}\right)^{\theta}$). In the Malthusian model, as long as $y_t > y^*$, we have $g(y_t) > 1$, and the population is increasing. This occurs until output per worker returns to the subsistence level.

f. (5 points). Suppose that in addition to the above changes ($\Lambda = 2000,\ A = 10$) the people changed their fertility behavior, so that the population growth, at any level of output per worker, is half of the original level (i.e. $g(y_t) = 0.1y_t$). Find the new steady state level of output per worker (y*) and population (L*).

Steady state output per worker

$$g(y^*) = 1$$

$$0.1y^* = 1$$

$$y^* = 10$$

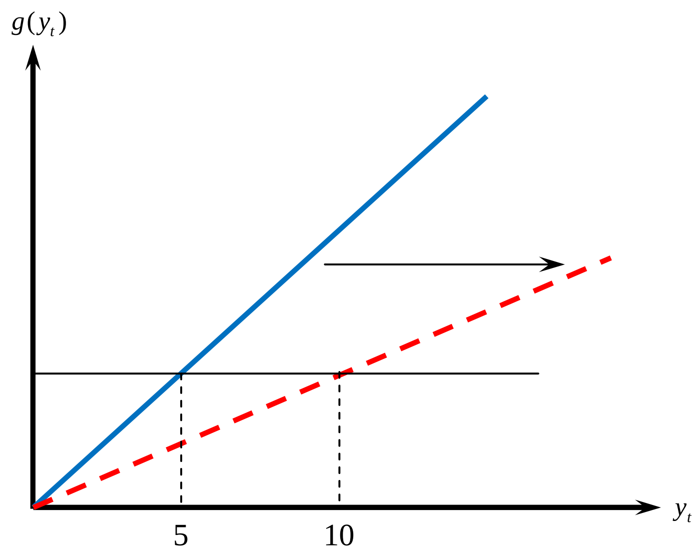
Steady state population level

$$y^* = A\left(\frac{\Lambda}{L^*}\right)^{\theta}$$

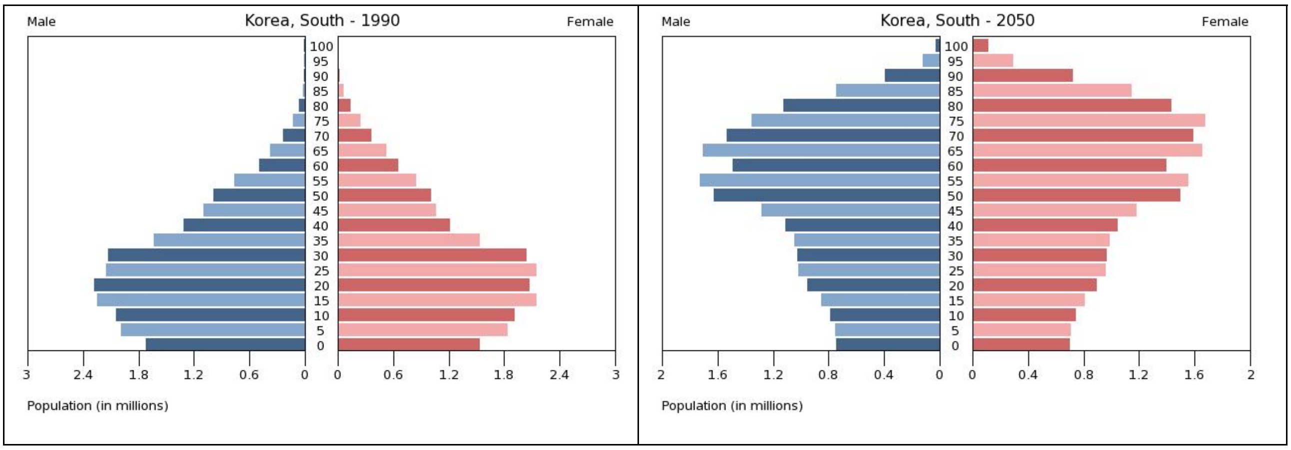
$$10 = 10\left(\frac{2000}{L^*}\right)^{1/2}$$

$$L^* = 2000$$

g. (5 points). Draw the graph of $g(y_t)$ before and after the change in fertility behavior, and illustrate what happened to the steady state output per capita.



2. (10 points). The following “age pyramids” show the age distribution (vertical axis) in Korea in 1990 and 2050 (predicted).



   a. (5 points). Based on these graphs, the population in Korea is getting younger/older. Circle the correct answer, and briefly explain how you reached your conclusion.

The 1990 diagram shows that most of the population is below 40, while the 2050 (predicted) diagram shows that most of the population is above 40. The bars indicate the size of the population in each age. We see that in the 1990 diagram, the longest bars are for ages 10 – 40, while in the 2050 (predicted) diagram, the longest bars are for ages 45 - 80.

   b. (5 points). Explain briefly why some people propose to increase the age of retirement in developed countries, like Korea. Relate your explanation to the above diagrams.

The most popular social security system in developed countries is “pay-as-you-go”, in which the current workers pay the pensions of the currently retired. As we we see in the above diagrams, the ratio of workers to retired people is predicted to dramatically decline in the next several decades. Notice that in the 1990 diagram, there are relatively few people aged abpve 65, compared to the 2050. Without increasiong the age of retirement, the retired people might experience a signifficant decline in their pensions.

3. (25 points) Age specific fertility rates (F_i) and probability of being alive at age i (π_i) in India are given in the following table.

	1950		**2010**	
Age	F_i	π_i	F_i	π_i
0 - 19	0	0.7	0	0.95
20 - 29	0.3	0.5	0.2	0.9
30 - 39	0.2	0.4	0.1	0.8
40 - 49	0.1	0.3	0	0.7

a. (5 points). Calculate the total fertility rate (TFR) in India for the year 1950.

$$TFR = 10 \cdot 0.3 + 10 \cdot 0.2 + 10 \cdot 0.1 = 6$$

b. (5 points). Calculate the net reproduction rate (NRR) in India in 1950, assuming that half of the babies are girls.

$$NRR = \frac{1}{2}\left[10 \cdot 0.3 \cdot 0.5 + 10 \cdot 0.2 \cdot 0.4 + 10 \cdot 0.1 \cdot 0.3\right] = 1.3$$

c. (5 points). Calculate the total fertility rate (TFR) in India for the year 2010.

$$TFR = 10 \cdot 0.2 + 10 \cdot 0.1 = 3$$

d. (5 points). Calculate the net reproduction rate (NRR) in India in 2010, assuming that half of the babies are girls.

$$NRR = \frac{1}{2}\left[10 \cdot 0.2 \cdot 0.9 + 10 \cdot 0.1 \cdot 0.8\right] = 1.3$$

e. (5 points). Explain briefly why the Net Reproduction Rate did not change between the years 1950 and 2010, despite the fact that Total Fertility Rate during these years decreased dramatically.

The Net Reproduction Rate is a joint measure of fertility and mortality:

$$NRR = \frac{1}{2}\sum_{i=0}^{\infty} \pi_i F_i$$

During the period under discussion, fertility and mortality both declined, which means that: $\pi_i \uparrow$, $F_i \downarrow$. In other words, women are having fewer children, but there is also a higher chance that they survive through their child bearing years. These two opposing forces just happen to cancel each other out.

4. (15 points). The next table shows how the average wage increases in years of education in a sample of countries.

Years of schooling	1-4	5-8	9,10,…
Marginal return	1.134	1.101	1.068

   a. (5 points). Suppose that workers in Japan have on average 13 years of education, while workers in Indonesia have on average 8 years of education. Calculate the ratio of human capital per worker in the two countries. (Simplify your answer, but there is no need to provide the exact numerical answer).

$$\frac{h_{Japan}}{h_{Indonesia}} = \frac{h_0 \cdot 1.134^4 \cdot 1.101^4 \cdot 1.068^5}{h_0 \cdot 1.134^4 \cdot 1.101^4} = 1.068^5$$

   b. (5 points). Based on the following table, where is the biggest contribution to the differences in income per capita of countries *i* and *j* coming from: (circle the correct answer).
      i. Differences in the fraction of workers in population
      ii. Differences in productivity
      iii. Differences in human capital per worker
      iv. Differences in physical capital per worker

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\frac{A_i}{A_j}$	$\frac{h_i^{1-\theta}}{h_j^{1-\theta}}$	$\frac{k_i^{\theta}}{k_j^{\theta}}$
12	1	?	1.5	2

You first need to calculate the ratios of productivities:

$$\frac{A_i}{A_j} = \frac{y_i / y_j}{\frac{\alpha_i}{\alpha_j}\frac{h_i^{1-\theta}}{h_j^{1-\theta}}\frac{k_i^{\theta}}{k_j^{\theta}}} = \frac{12}{1.5 \cdot 2} = 4$$

   c. (5 points). Based on the above table, if the only difference between countries *i* and *j* was in productivity, what would have been the ratio of GDP/capita of the two countries then?

$$\frac{y_i}{y_j} = \frac{A_i}{A_j} = 4$$