Economics of Growth, ECON560 **San Francisco State University**
Michael Bar **Spring 2011**

Midterm Exam

Tuesday, February 22

1 hour, 15 minutes

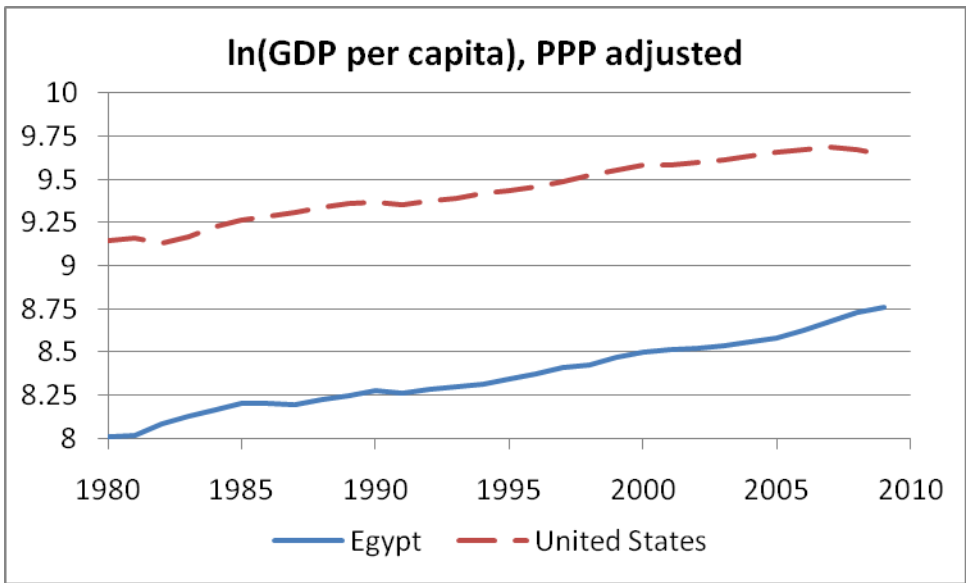
Name: _____

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (15 points). The next figure shows the natural log of GDP per capita (PPP adjusted) in Egypt and the U.S., during the 30 year period 1980-2009.



   a. Based on the figure (circle one answer),
      i. U.S. is growing faster than Egypt, on average, over the 30-year period.
      (ii.) The two countries grow at about the same rate, with Egypt growing slightly faster, on average, over the 30-year period.

   b. Briefly explain how you have reached your conclusion in the previous section.

The two graphs are roughly parallel, and a close look indicates that Egypt slightly closes the gap.

   c. Based on the figure, the approximate average annual growth rate of GDP per capita in Egypt, over the 30-year period is (circle one answer),
      i. 0.75%
      ii. 8%
      iii. 8.25%
      (iv.) 2.5%

$$x = \frac{0.75}{30} = 0.025 = 2.5\%$$

2. (10 points). Suppose that GDP per capita in Zimbabwe is growing at 1% per year, and total GDP is growing at 4% per year.
    a. What is the approximate growth rate of population in Zimbabwe?

$$growth\left(\frac{GDP}{POP}\right) \approx growth(GDP) - growth(POP)$$
$$1\% \approx 4\% - growth(POP)$$
$$\Rightarrow growth(POP) \approx 3\%$$

    b. Using the "rule of 70", how long will it take for the GDP per capita in Zimbabwe to double?

$$\frac{70}{1} = 70 \text{ years.}$$

3. (5 points). When comparing standard of living across countries, economists use the PPP exchange rate because (circle the correct answer)
    a. The market exchange rate tends to overvalue the standard of living in poor countries.
    b. The PPP exchange rate tends to undervalue the standard of living in poor countries.
    c. The market exchange rate does not account for the fact that non-traded goods are much cheaper in rich countries.
    (d.) The market exchange rate does not account for the fact that non-traded goods are much cheaper in poor countries.

4. (5 points). Suppose that output in some economy is produced with the following Cobb-Douglas production function $Y_t = A_t K_t^{0.35} L_t^{0.65}$, where Y_t is output, K_t is capital and L_t is labor, and A_t is total factor productivity. Circle the correct answer.
    a. The labor share is 35% of output.
    (b.) The labor share is 65% of output.
    c. The labor share depends on the capital per worker in the economy.
    d. The labor share depends on the total factor productivity in the economy.
    e. The capital share depends on the investment rate in the economy.

5. (20 points). Suppose that the aggregate GDP can be modeled with the Cobb-Douglas production function: $Y_t = A_t K_t^{\theta} L_t^{1-\theta}$, $0 < \theta < 1$, where Y_t is the total GDP, A_t is the Total Factor Productivity, K_t is the total capital and L_t is the number of workers.
    a. **Derive** the equation of output per worker (y^L) as a function of capital per worker (k).

$$y^L = \frac{Y}{L} = \frac{AK^{\theta}L^{1-\theta}}{L} = Ak^{\theta}$$

    b. **Write** the equation of output per capita, when the fraction of workers in population is α .

$$y^N = \alpha y^L = \alpha A k^{\theta}$$

c. The next table shows data for two countries: U.S. (i) and Angola (j). The variables y_i and y_j denote GDP per capita in U.S. and Angola respectively.

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\frac{A_i}{A_j}$	$\left(\frac{k_i}{k_j}\right)^{\theta}$
48	1.5	?	4

Based on the above table, if the only difference between the two countries was productivity, what would be the ratio of U.S. to Angolan GDP per capita?

$$\frac{y_i}{y_j} = \frac{A_i}{A_j} = \frac{48}{6} = 8$$

d. If the capital share is $\frac{1}{2}$, then U.S. capital per worker is ___ times greater than the Angolan capital per worker. Circle the correct answer.

i. 2
ii. 4
iii. 6
(iv.) 16

6. (20 points). Consider the Solow model discussed in class, and described as follows. Output is produced according to $Y_t = A_t K_t^{\theta} L_t^{1-\theta}$, $0 < \theta < 1$. Capital evolves according to $K_{t+1} = K_t(1-\delta) + I_t$, where δ is the depreciation rate and I_t is aggregate investment. People save a fraction *s* of their income. This fraction is exogenous. Thus, the total saving and total investment in this economy is $S_t = I_t = sY_t$. The population of workers grows at a constant rate of n, which is exogenous in this model. Thus, $L_{t+1} = (1+n)L_t$.
    a. Solve for the steady state capital per worker, output per worker, and consumption per worker (i.e. derive the expressions for k_{ss}, y_{ss}, c_{ss}).

Deriving the law of motion of capital per worker:

$$\frac{K_{t+1}}{L_{t+1}} = \frac{K_t(1-\delta)}{(1+n)L_t} + \frac{sAK_t^{\theta}L_t^{\theta}}{(1+n)L_t}$$

$$k_{t+1} = \frac{1-\delta}{1+n}k_t + \frac{sAk_t^{\theta}}{1+n}$$

Using the definition of a steady state: $k_{t+1} = k_t = k_{ss}$

$$k_{ss} = \frac{1-\delta}{1+n}k_{ss} + \frac{sAk_{ss}^{\theta}}{1+n}$$

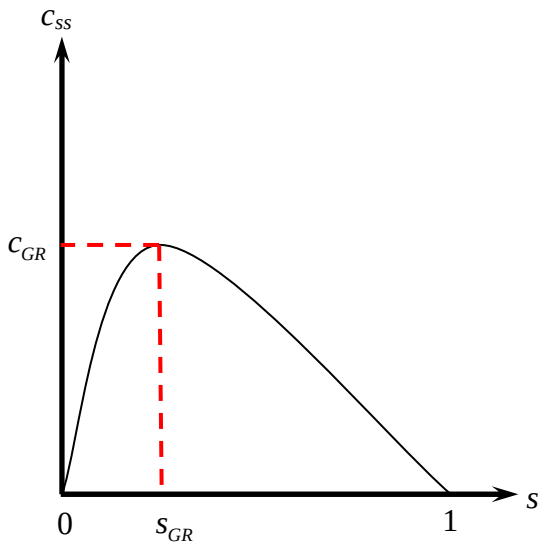
$$k_{ss}(1+n) = (1-\delta)k_{ss} + sAk_{ss}^{\theta}$$

$$k_{ss}(n+\delta) = sAk_{ss}^{\theta}$$

$$k_{ss}^{1-\theta} = \frac{sA}{n+\delta}$$

$$\boxed{\begin{aligned} k_{ss} &= \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\theta}} \\ y_{ss} &= Ak_{ss}^{\theta} \\ c_{ss} &= (1-s)y_{ss} \end{aligned}}$$

b. On a fully labeled diagram, describe the relationship between the saving rate (x-axis) and steady state consumption per worker (y-axis). Indicate on the graph the golden rule saving rate (s_{GR}) and the associated consumption per worker (c_{GR}).



c. Higher saving rate in the Solow model always leads to higher steady state consumption per worker. True/false, circle the correct answer and provide a brief proof.

Higher saving rate increases the steady state output per worker, but decreases the fraction of output consumed:

$$c_{ss} = \underbrace{(1-s)}_{\downarrow}\overbrace{y_{ss}}^{\uparrow}$$

It is not obvious which of the two opposite forces is stronger, but obviously, if saving rate becomes 100%, consumption will become zero, which means that beyond certain saving rate consumption per worker must decline.

7. (10 points). In this question use the cross-country accounting formula, based on the steady state of the Solow model. Here i and j are two countries.

$$\frac{y_i}{y_j} = \frac{\alpha_i}{\alpha_j}\left(\frac{A_i}{A_j}\right)^{\frac{1}{1-\theta}} \frac{\left(\frac{s_i}{n_i+\delta}\right)^{\frac{\theta}{1-\theta}}}{\left(\frac{s_j}{n_j+\delta}\right)^{\frac{\theta}{1-\theta}}}$$

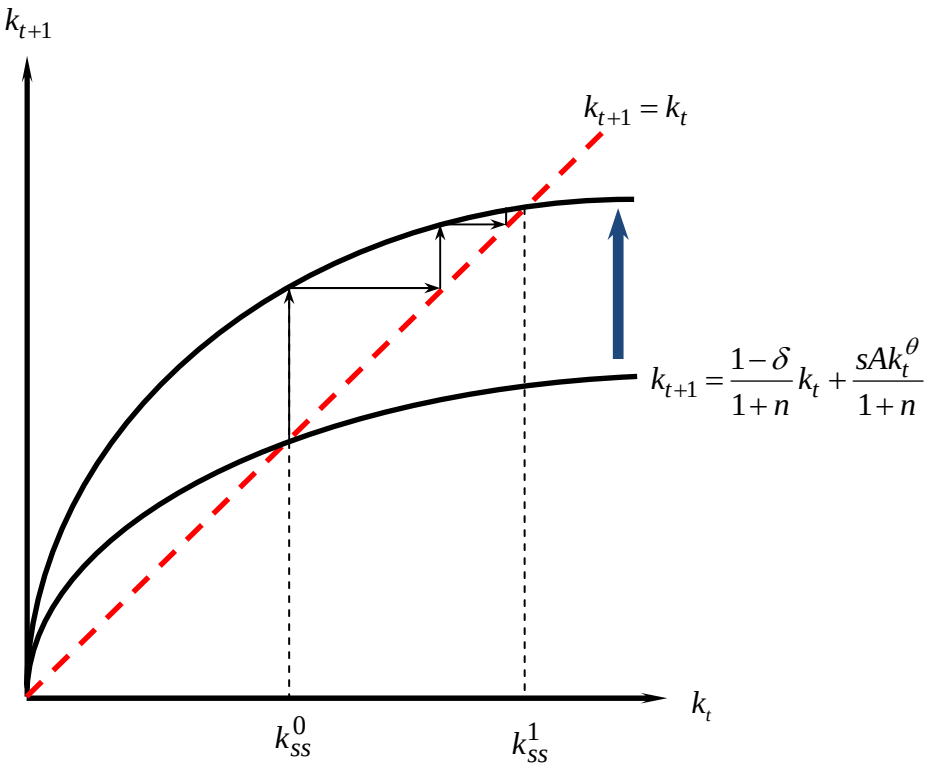
a. According to this model, if the only difference between the two countries was the total factor productivity, what would have been the ratio of GDP per capita in the two countries?

$$\frac{y_i}{y_j} = \left(\frac{A_i}{A_j}\right)^{\frac{1}{1-\theta}}$$

b. According to this model, if the only difference between the two countries was the population growth rate, what would have been the ratio of GDP per capita in the two countries?

$$\frac{y_i}{y_j} = \left(\frac{n_j+\delta}{n_i+\delta}\right)^{\frac{\theta}{1-\theta}}$$

8. (15 points). Suppose that the economy of China is described by the Solow model.
   a. Draw a fully labeled diagram of the law of motion of capital per worker in China, for given (fixed) productivity level. Indicate the initial steady state capital per worker k_{ss}^0.



   b. On the above diagram, illustrate the impact of a once-and-for-all increase in productivity, and indicate the new steady state capital per worker k_{ss}^1.

   c. An IMF economist argued (based on the Solow model) that the effects of productivity growth are identical to the effects of higher saving rate, and that China can grow forever just by increasing the saving rate. Explain briefly, why the IMF economist is wrong.

The IMF economist neglects the fact that saving rate cannot increase without bounds. There is an obvious restriction that people cannot consume zero and save all their income. The bounds of productivity and technology on the other hand, are still not known. At least with our current knowledge, scientists believe that we are nowhere near exhausting the technological potential existing in the universe.