

Midterm Exam

Thursday, September 23

1 hour, 15 minutes

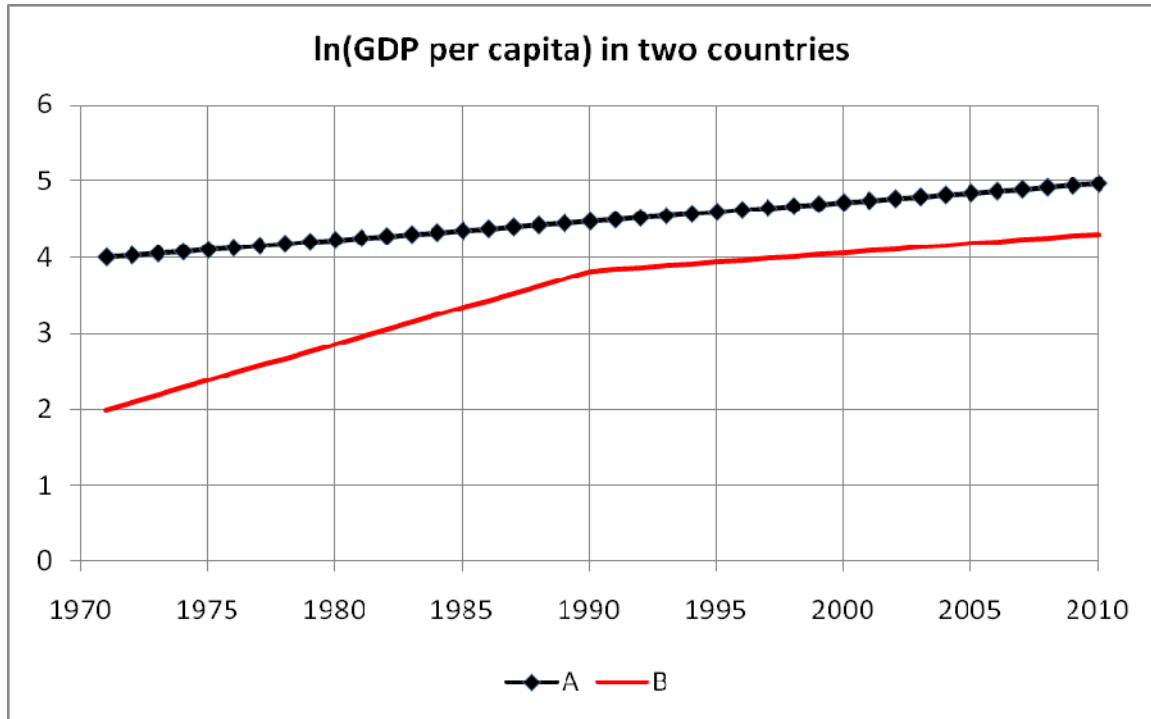
Name: _____

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (15 points). The next figure shows the natural log of GDP per capita in two countries, A and B, over the 40 year period 1971 – 2010.



- a. Based on the figure (circle one answer),
- Country A is growing faster than country B before 1990
 - Both countries grow at the same rate before 1990
 - Both countries grow at the same rate after 1990
 - Country B's GDP per capita is greater than A's before 1990.

- b. Based on the figure, the approximate annual growth rate of GDP per capita, in country A, over the entire period is (circle one answer),

- 1%
- 2.5%
- 4%
- 5%

$$x = \left(\frac{1}{40} \times 100 \right) \% = \frac{10}{4} \% = \frac{5}{2} \% = 2.5\%$$

- c. Based on your answer to part b, and given that population in country A grows at 1% per year, find the annual growth rate of GDP in country A.

$$\text{growth} \left(\frac{GDP}{POP} \right) \approx \text{growth}(GDP) - \text{growth}(POP)$$

$$2.5\% \approx \text{growth}(GDP) - 1\%$$

$$\text{growth}(GDP) = 3.5\%$$

2. (10 points). Explain briefly why it is important to adjust for purchasing power, when comparing standard of living across countries.

Without adjusting for purchasing power, we neglect the fact that non-traded goods are much cheaper in underdeveloped countries. Using the market exchange rate, which is determined by traded goods prices, therefore tends to undervalue the standard of living in under-developed countries.

3. (10 points). Suppose that output is produced with the following Cobb-Douglas production function $Y_t = A_t K_t^{0.3} L_t^{0.7}$, where Y_t is output, K_t is capital and L_t is labor. Prove that if labor wage is its marginal product, then 70% of the output is paid to labor.

$$w_t = MPL_t = 0.7 A_t K_t^{0.3} L_t^{0.7-1}$$
$$w_t \cdot L_t = 0.7 A_t K_t^{0.3} L_t^{0.7-1} \cdot L_t = 0.7 Y_t$$

4. (20 points). Suppose that the aggregate GDP can be modeled with the Cobb-Douglas production function: $Y_t = A_t K_t^\theta L_t^{1-\theta}$, $0 < \theta < 1$, where Y_t is the total GDP, A_t is the Total Factor Productivity, K_t is the total capital and L_t is the number of workers.
- a. **Derive** the equation of output per worker (y^L) as a function of capital per worker (k).

$$y^L = \frac{Y}{L} = \frac{AK^\theta L^{1-\theta}}{L} = Ak^\theta$$

- b. **Write** the equation of output per capita, when the fraction of workers in population is α .

$$y^N = \alpha y^L = \alpha Ak^\theta$$

- c. The next table shows data for two countries: U.S. (i) and Haiti (j). The variables y_i and y_j denote GDP per capita in U.S. and Haiti respectively.

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\frac{A_i}{A_j}$	$\left(\frac{k_i}{k_j}\right)^\theta$
18	1	?	3

Based on the above table, if the only difference between the two countries was productivity, what would be the ratio of U.S. to Indonesian GDP per capita?

$$\frac{y_i}{y_j} = \frac{A_i}{A_j} = \frac{18}{3} = 6$$

- d. If the capital share is $\frac{1}{2}$, then U.S. capital per worker is ____ times greater than the Haitian capital per worker. Circle the correct answer.
- i. 2
 - ii. 3
 - iii. 9
 - iv. 18

5. (20 points). Consider the Solow model discussed in class, and described as follows. Output is produced according to $Y_t = A_t K_t^\theta L_t^{1-\theta}$, $0 < \theta < 1$. Capital evolves according to $K_{t+1} = K_t(1 - \delta) + I_t$, where δ is the depreciation rate and I_t is aggregate investment. People save a fraction s of their income. This fraction is exogenous. Thus, the total saving and total investment in this economy is $S_t = I_t = sY_t$. The population of workers grows at a constant rate of n , which is exogenous in this model. Thus, $L_{t+1} = (1 + n)L_t$.
- a. Solve for the steady state capital per worker, output per worker, and consumption per worker (i.e. derive the expressions for k_{ss}, y_{ss}, c_{ss}).

Deriving the law of motion of capital per worker:

$$\frac{K_{t+1}}{L_{t+1}} = \frac{K_t(1 - \delta)}{(1 + n)L_t} + \frac{sAK_t^\theta L_t^\theta}{(1 + n)L_t}$$

$$k_{t+1} = \frac{1 - \delta}{1 + n}k_t + \frac{sAk_t^\theta}{1 + n}$$

Using the definition of a steady state: $k_{t+1} = k_t = k_{ss}$

$$k_{ss} = \frac{1 - \delta}{1 + n}k_{ss} + \frac{sAk_{ss}^\theta}{1 + n}$$

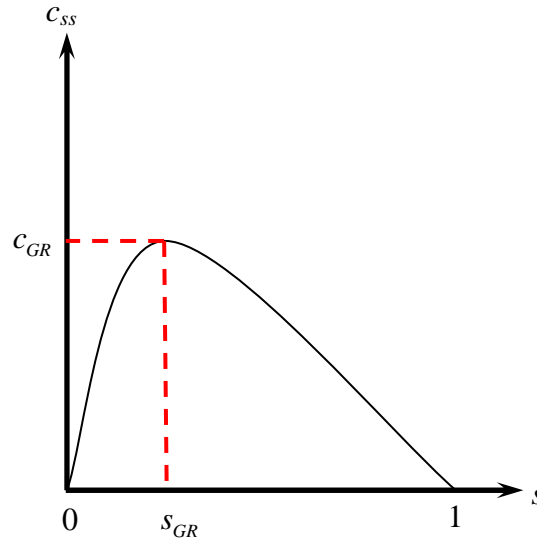
$$k_{ss}(1 + n) = (1 - \delta)k_{ss} + sAk_{ss}^\theta$$

$$k_{ss}(n + \delta) = sAk_{ss}^\theta$$

$$k_{ss}^{1-\theta} = \frac{sA}{n + \delta}$$

$k_{ss} = \left(\frac{sA}{n + \delta} \right)^{\frac{1}{1-\theta}}$ $y_{ss} = Ak_{ss}^\theta$ $c_{ss} = (1 - s)y_{ss}$

- b. The next figure describes the relationship between the steady state consumption per worker and saving rate in some economy. Indicate on the graph the golden rule saving rate (s_{GR}) and the associated consumption per worker (c_{GR}).



- c. Higher saving rate in the Solow model always leads to higher steady state consumption per worker. True/false circle the correct answer and provide a brief proof.

Higher saving rate increases the steady state output per worker, but decreases the fraction of output consumed:

$$c_{ss} = \underbrace{(1-s)}_{\downarrow} \overbrace{y_{ss}}^{\uparrow}$$

It is not obvious which of the two opposite forces is stronger, but obviously, if saving rate becomes 100%, consumption will become zero, which means that beyond certain saving rate consumption per worker must decline.

6. (10 points). In this question use the cross-country accounting formula, based on the steady state of the Solow model. Here i and j are two countries.

$$\frac{y_i}{y_j} = \frac{\alpha_i \left(\frac{A_i}{A_j} \right)^{\frac{1}{1-\theta}} \left(\frac{s_i}{n_i + \delta} \right)^{\frac{\theta}{1-\theta}}}{\left(\frac{s_j}{n_j + \delta} \right)^{\frac{\theta}{1-\theta}}}$$

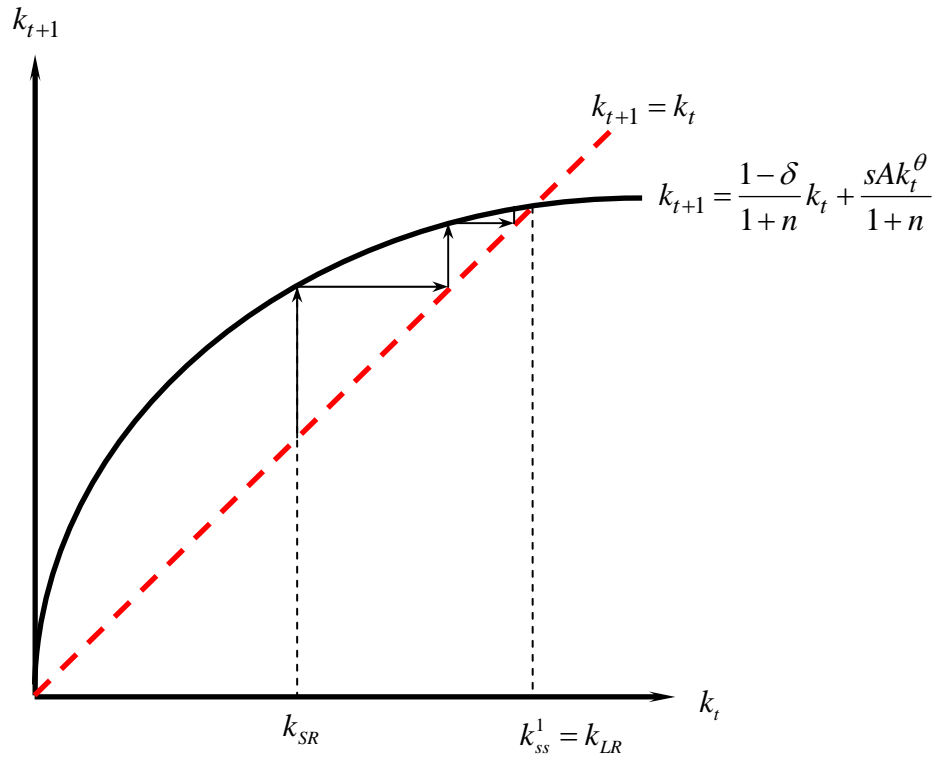
- a. According to this model, if the only difference between the two countries was the saving rate, what would have been the ratio of GDP per capita in the two countries?

$$\frac{y_i}{y_j} = \left(\frac{s_i}{s_j} \right)^{\frac{\theta}{1-\theta}}$$

- b. According to this model, if the only difference between the two countries was the population growth rate, what would have been the ratio of GDP per capita in the two countries?

$$\frac{y_i}{y_j} = \left(\frac{n_j + \delta}{n_i + \delta} \right)^{\frac{\theta}{1-\theta}}$$

7. (15 points). Suppose economy of Haiti is described well by the Solow model.
- Suppose that productivity in Haiti is not growing. In the next diagram, draw the law of motion of capital per worker, and show the initial steady state capital per worker k_{ss}^1 .



- Suppose an earthquake destroyed a large fraction of physical capital in Haiti. Illustrate in the above diagram the short run and long run impact of the earthquake on the capital per worker in Haiti, assuming that Haiti was initially in the steady state from part a.

In the short run, the capital per worker falls to some level k_{SR} . Notice that this change does not shift the law of motion of capital per worker (which shifts when A, s, n change). In the long run the capital per worker will recover to the initial steady state level.

- What will be the long run growth rate in standard of living in Haiti? Explain briefly.

Since productivity is not growing in Haiti, the long run growth rate, according to the Solow model, is **zero**. We proved in class that without growth in productivity the economy converges to a steady state, with no growth in capital per worker, output per worker or consumption per worker.