San Francisco State University Michael Bar

Income Inequality - supplemental notes

In these notes we propose two additional arguments in favor of income redistribution, which are not mentioned in the textbook. The two arguments are nicknamed (1) *Life Lottery* and (2) *Envy*.

1 Life Lottery

The first idea is related to the topic of insurance. People, before birth face the uncertainty about their future wealth, and income redistribution works like insurance. It is a well known result in economics, that people that are risk averse would prefer insurance over facing the risk.

More formally, a person is *risk averse* if he prefers the expected value of a lottery over the lottery itself. For example, suppose that a lottery consists of a coin toss, so that with probability 0.5 the person gets \$1000 and with probability 0.5 he gets nothing. The expected value of the prize is $0.5 \cdot 1000 + 0.5 \cdot 0 =$ \$500. A risk averse person, if offered the choice between taking the \$500 with certainty or participating in the lottery, would prefer the \$500 with certainty. It is easy to prove that a person is risk averse if and only if his utility is a strictly concave function of wealth. That is, if wealth is denoted by w, then $U(w)$ is a strictly concave function. In the following example, we use a specific utility function: $U(w) = \sqrt{w}$

There is substantial evidence that people are for the most part risk averse; the large fraction of nations' incomes that are spent on various kinds of insurance (health insurance, life insurance, auto insurance, etc.) are difficult to argue with. Studies with animals, such as rats, suggest that animals in general are risk averse. In particular, when rats are offered two mazes, one with certain amount of food with certainty, and another with double the amount but half of the times, they prefer the expected return over the lottery.

The model

Suppose that there are two countries, A and B. In both countries there is equal probability of being poor (with \$0) and being rich (with \$1000). Country A does not have any income redistribution, while country B taxes the rich at 40% and gives it to the poor. In other words, in country B there is equal probability of having \$400 and \$600. Suppose that a risk averse person is asked pre-birth, in which country he would want to be born. Which country would he choose?

We assume here (without proof) that the preferences of people over lotteries are given by expected utility. Thus, the expected utility in country A is $0.5\sqrt{0} + 0.5\sqrt{1000} = 15.81$ and in country B $0.5\sqrt{400} + 0.5\sqrt{600} = 22.25$. The intuition is simple. A strictly concave utility function exhibits diminishing marginal returns to wealth. Thus, giving up some wealth in the good state (when you have \$1000) does not reduce your utility as much as the gain that you have from increasing the income in the bad state (when you have 0\$).

2 Envy

People derive utility from their own consumption x, and also from their position relative to the rest of the economy $x/\bar{x}$. The utility of person i is

$$U\left(x_i, \frac{x_i}{\bar{x}}\right)$$

A specific example would be

$$U\left(x_i, \frac{x_i}{\bar{x}}\right) = \ln\left(x_i \cdot \left(\frac{x_i}{\bar{x}}\right)^e\right)$$

where e is the envy parameter. If $e = 0$, then people care only about their own consumption. If $e > 0$ then people care about their own consumption and also about their *relative status.* Thus, if economic growth increases the average consumption, without increasing the consumption of person i, that person is actually worse off.

Suppose that the average income grows at rate g. What should be the growth rate of individual consumption, so that individual i will be no worse off? The utility can be written as

$$\begin{aligned} U\left(x_i, \frac{x_i}{\bar{x}}\right) &= \ln\left(x_i \cdot \left(\frac{x_i}{\bar{x}}\right)^e\right) \\ &= \ln(x_i) + e\ln(x_i) - e\ln(\bar{x}) \\ &= (1+e)\ln(x_i) - e\ln(\bar{x}) \end{aligned}$$

Suppose that individual $i's$ consumption grows at rate γ. Then, individual i will tolerate the average growth if

$$\begin{aligned} (1+e)\ln(x_i(1+\gamma)) - e\ln(\bar{x}(1+g)) &\geq (1+e)\ln(x_i) - e\ln(\bar{x}) \\ (1+e)\ln(x_i) + (1+e)\ln(1+\gamma) - e\ln(\bar{x}) - e\ln(1+g) &\geq (1+e)\ln(x_i) - e\ln(\bar{x}) \\ (1+e)\ln(1+\gamma) - e\ln(1+g) &\geq 0 \end{aligned}$$

Using the approximation that $\ln(1+\gamma) \approx \gamma$ and $\ln(1+g) \approx g$ for small γ and g, we get approximately

$$\begin{aligned} (1+e)\gamma &\geq eg \\ \gamma &\geq \left(\frac{e}{1+e}\right)g \end{aligned}$$

If there is no envy, then any positive growth rate of individual consumption is enough to make the individual better off. If, on the other hand $e = 0.5$, then individual i is better off only if $\gamma \geq g/3$, i.e. only if his income is growing faster than 1/3 of the average. If $e = 0.25$, then the individual is happy when $\gamma \geq g/5$, i.e. only if his income is growing faster than 1/5 of the average.

The implications of this exercise is that the degree of envy in the economy determines the necessary income redistribution so that the average growth would be politically acceptable. In particular, the income of the median voter has to grow at lest as fast as $\left(\frac{e}{1+e}\right)g$.