San Francisco State University **Michael Bar**
ECON 560 **Spring 2012**

Problem set 5

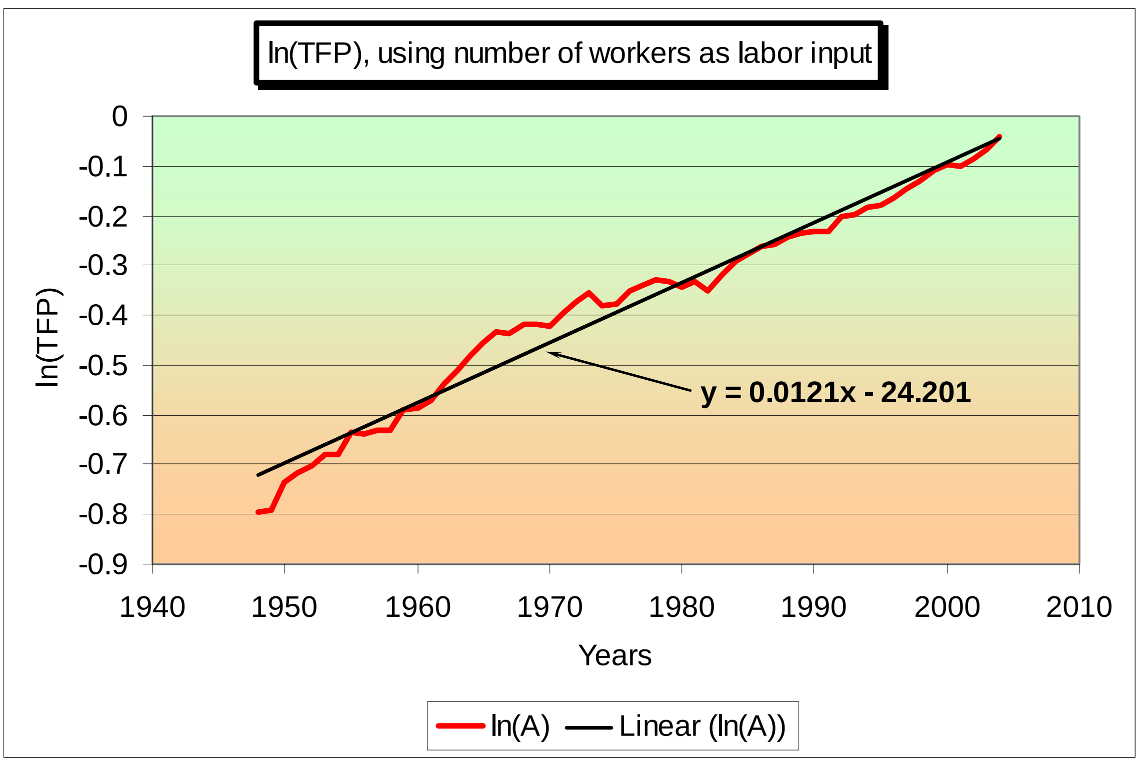
Measuring Productivity Growth

1. (20 points). In this question you need to use the "Data for HW5" posted on the course web page. The file contains data on real GDP, real capital stock and labor input in the U.S.
   a. Plot the graph of the ln(TFP) in the U.S. for the years 1948 – 2004, assuming that the capital share is $\theta = 0.35$ and using the number of workers as the labor input. Show the equation you used to obtain the time series of TFP. Add a linear trend line to the graph and display the slope and intercept on the chart. To add a trend line to the graph, right-mouse click on the graph and choose "Add Trendline". Choose the linear trend, and from the options choose "Display equation on chart". Excel will add a linear trend (linear regression line) to the chart and will display the estimated regression coefficients.

To obtain the TFP we need to use the following:

$$Y_t = A_t K_t^{\theta} L_t^{1-\theta}$$

$$\Rightarrow A_t = \frac{Y_t}{K_t^{\theta} L_t^{1-\theta}}$$



b. Based on the above graph, approximately, what is the average annual growth rate of productivity in the U.S. during the years 1948 – 2004?

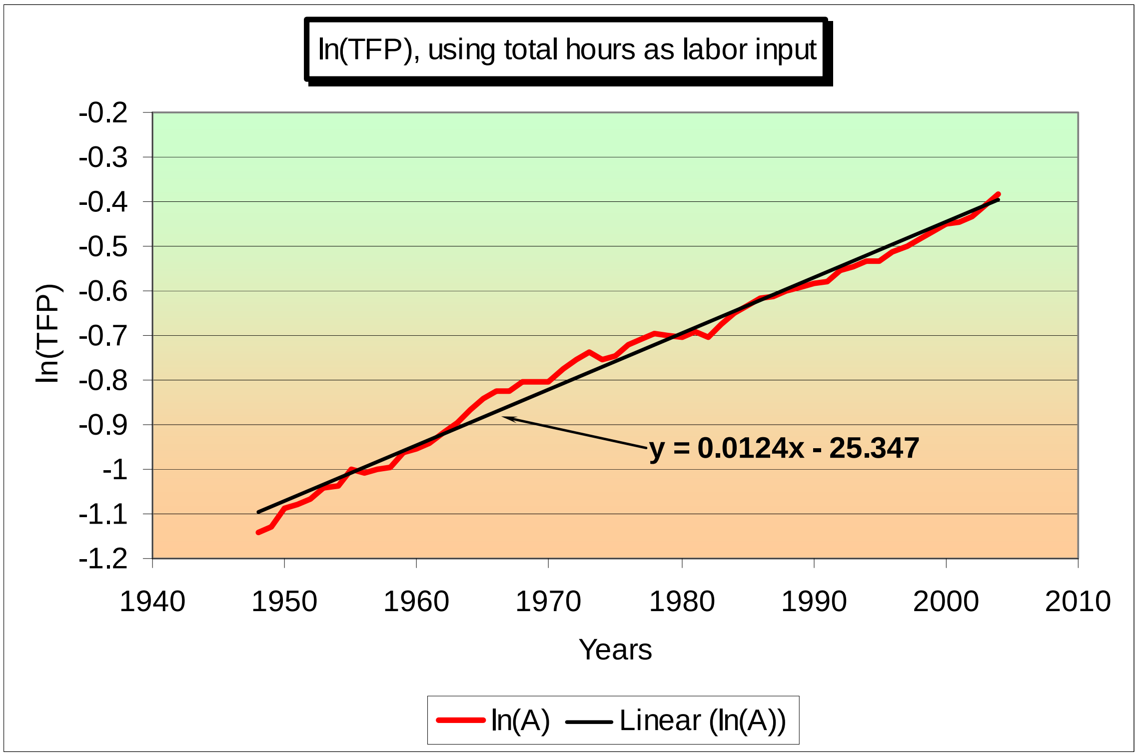
Approximately, 1.2% per year.

c. Some economists argue that the number of workers is not a good measure of labor input because the hours worked may change during the business cycle. Repeat part a using the aggregate hours as the labor input. Show the equation you used to obtain the time series of TFP.

To obtain the TFP we need to use the following:

$$Y_t = A_t K_t^{\theta} H_t^{1-\theta}$$

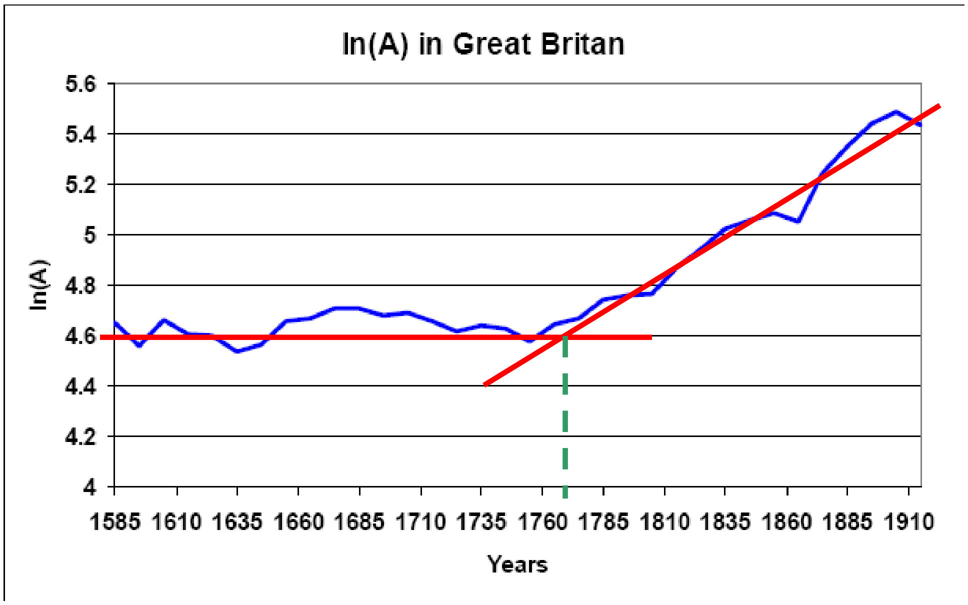
$$\Rightarrow A_t = \frac{Y_t}{K_t^{\theta} H_t^{1-\theta}}$$



d. Based on the above graph, approximately, what is the average annual growth rate of productivity in the U.S. during the years 1948 – 2004? Compare this with part b – is there a big difference in the estimated growth rate of productivity when you use different measures of labor input?

Approximately, 1.2%. Using the number of workers as labor input or the total hours as the labor input does not matter much for the measured average growth rate of productivity.

2. (5 points). The next graph shows the ln of productivity in Great Britain. Use a ruler and a pencil to estimate the onset (i.e. beginning) of the industrial revolution. Report your estimate (the year).



The ln(TFP) clearly exhibits two regimes: (1) slow or no growth at all, and (2) faster growth. We can draw two trends through the data to fit the two regimes. This gives us an estimate of the regime switch at around 1770, as shown in the graph. Notice that we already used this technique for China, in question 1 of HW3.

Improved specification of technology production function

3. (15 points). Suppose that the technology production function is

$$\hat{A} = \frac{L_A^{\lambda} A^{-\phi}}{\mu}$$

Where $0 < \lambda < 1$ and $0 < \phi < 1$, L_A is the number of researchers in the economy and μ is the cost of technological improvement in terms of researchers.

a. Suppose that A is growing at constant rate. Derive the approximate relationship between the growth rate of productivity ($\hat{A}$) and the growth rate in the number of researchers ($\hat{L}_A$). Show your derivations.

$$1 = \frac{L_{A,t+1}^{\lambda} A_{t+1}^{-\phi}}{L_{A,t}^{\lambda} A_t^{-\phi}} = (1+\hat{L}_A)^{\lambda}(1+\hat{A})^{-\phi}$$

$$0 = \lambda \ln(1+\hat{L}_A) - \phi \ln(1+\hat{A})$$

This approximately

$$\hat{A} = \frac{\lambda}{\phi}\hat{L}_A$$

b. How would you estimate the ratio λ / ϕ from the data?

One way to estimate λ / ϕ is to use the fact that TFP has been growing at the rate of 1.2% (see question 1 in this assignment), while the average growth rate of researchers was 5%. Thus, $\lambda / \phi = 1.2/5 = 0.24$.

c. According to this theory, will the growth of productivity continue forever? Explain why?

No. According to this theory the growth in productivity requires growth in the number of researchers. How can the number of researchers grow? First, the fraction of the labor force that is engaged in research can grow, but only up to certain limit. Somebody has to produce output, so we cannot have the entire labor force engaged in R&D. Second, the labor force itself can grow simply because the population grows. The growth of population is not limitless however, and we observe that in many developed countries (e.g. Japan, Italy) the population is shrinking. Based on this technology production function we must conclude that the growth of productivity must slow down and eventually stop.

Efficiency

4. (15 points). Suppose that productivity in the U.S. is 3 times that of China, i.e. $A_i / A_j = 3$, where *i* is U.S. and *j* is China. Suppose that the productivity in the U.S. is growing at 1.2% per year and technologically, China is 10 years behind the U.S.

d. Find the ratio of efficiency in U.S. vs. China (E_i / E_j) under the assumption that efficiency in the U.S. did not change much in the last 10 years.

$$\frac{A_i}{A_j} = \frac{T_i}{T_j} \times \frac{E_i}{E_j}$$

$$\frac{A_i}{A_j} = 3$$

$$\frac{T_i}{T_j} = (1.012)^{10} = 1.127$$

$$\Rightarrow \frac{E_i}{E_j} = \frac{3}{1.127} = 2.663$$

e. If the only difference between the two countries was efficiency, what would be the ratio of productivity of the two countries?

2.663

f. Based on the above, where is most of the difference in productivity coming from, the difference in technology or the difference in efficiency?

Most of the difference is coming from efficiency.

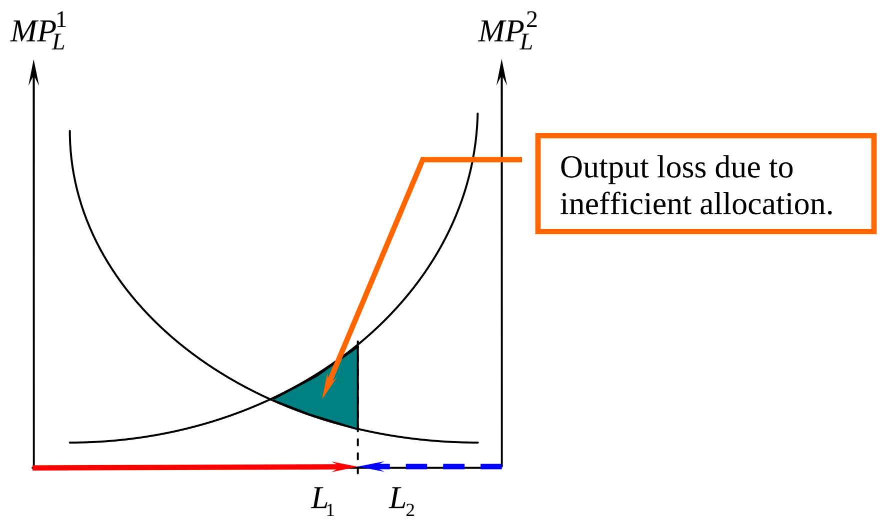
5. (5 points). In 1910 the productivity of textile workers differed a lot across countries. What evidence do we have that this difference in productivity is mainly due to efficiency and not technology?

The countries under investigation used the same machines and similar material, so technology cannot explain the differences in productivity. In some countries (U.S., Canada, England) the industry was organized better, so each worker operated more machines. Thus, the differences in productivity are mainly due to **efficiency**.

6. (5 points). Table 10.2 in the textbook shows that there are big differences in productivity of selected industries in the U.S., Japan and Germany. Why is it argued that most of the differences are due to efficiency differences and not technology differences?

The three countries above are at the frontiers of technology and technology tends to move fast between them. Therefore, the conclusion is that the differences in productivity have to come from differences in efficiency, not technology.

7. (5 points). Draw a diagram with marginal product curves of two industries and overallocation to industry 1. Highlight the output loss due to inefficient allocation of resources.



8. (20 points). List the 4 types of inefficiency discussed in the book, and give an example from the textbook for **one** of the types.

Types of inefficiency:

i. Unproductive activities (e.g. burglary, war,.. any use of factors of production to take output from others, instead of producing output).
ii. Idle resources (e.g., unemployment, underemployment).
iii. Misallocation of resources (e.g., employing too many workers in some industry and too little in another).
iv. Technology blocking (e.g., after 15 century China prohibited building big ships).

There are many examples in the textbook (chapter 10) for each type of inefficiency.

9. (10 points). Read the lyrics of *"Maria Maria"* by Carlos Santana (http://www.lyrics007.com/Santana%20Lyrics/Maria%20Maria%20Lyrics.html). Find 3 examples of inefficiency and determine to which type of inefficiency they belong.

The three examples in the song are: (1) looting, (2) shooting and (3) pick pocking on the corner. This type of inefficiency is "Unproductive Activities", i.e. the use of labor to take output from others, rather than to produce output.