San Francisco State University **Michael Bar**
ECON 560 **Spring 2012**

Problem set 4

Human Capital

1. (20 points). The next table gives some data for China and U.S. for the year 2000.

	y	α	e	h	k	θ
U.S.	35586.88	0.514	12.247	?	148090.477	0.35
China	4005.487	0.5995	5.738	?	10133.6388	0.35

Here y is real GDP per capita, α is the fraction of workers in population, e is the years of education per worker, h is human capital per worker, k is physical capital per worker and θ is the capital share.

a. Assume that the marginal return to education in both countries is as described in the following table.

Years of schooling	1-4	5-8	9,10,…
Marginal return	1.134	1.101	1.068

Compute the level of human capital per worker in both countries.

$$h_{U.S.} = h_0 \cdot 1.134^4 \cdot 1.101^4 \cdot 1.068^{4.247} = h_0 \cdot 3.213$$
$$h_{China} = h_0 \cdot 1.134^4 \cdot 1.101^{1.738} = h_0 \cdot 1.955$$

b. Let *i* indicate the U.S. and *j* indicate China. Use the cross-country accounting formula in equation (1) in the notes on human capital to account for the ratio of real GDP per capita in the two countries in 2000. In other words, complete the next table:

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\frac{A_i}{A_j}$	$\frac{h_i^{1-\theta}}{h_j^{1-\theta}}$	$\frac{k_i^{\theta}}{k_j^{\theta}}$

The cross-country accounting formula is:

$$\frac{y_i}{y_j} = \frac{\alpha_i A_i h_i^{1-\theta} k_i^{\theta}}{\alpha_j A_j h_j^{1-\theta} k_j^{\theta}}$$

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\frac{A_i}{A_j}$	$\frac{h_i^{1-\theta}}{h_j^{1-\theta}}$	$\frac{k_i^{\theta}}{k_j^{\theta}}$
8.885	0.857	2.934	1.381	2.557

The ratio A_i / A_j is found as the residual in the formula, with the rest of the values known. It is **not possible** to separately find the values of A_i and A_j because we don't know the value of h_0.

c. Where is the biggest contribution to the difference between the GDP/cap in the two countries coming from: The difference in Productivity, difference in human capital per worker, the fraction of workers in population, difference in physical capital?

The biggest contribution to the ratio of GDP/cap in the two countries is coming from productivity differences.

d. If the only difference between the two countries was the productivity (technology, A_i / A_j), what would be the ratio of GDP/cap in the two countries?

Then the ratio y_i / y_j would have been 2.934.

Growth Accounting

2. (10 points). Assume that the aggregate output is produced according to $Y_t = A_t K_t^{\theta} (h_t L_t)^{1-\theta}$. Derive the approximate growth accounting formula for output per capita.

Output per worker is $y_t^L = A_t k_t^{\theta} h_t^{1-\theta}$ and output per capita is $y_t^N = \alpha_t A_t k_t^{\theta} h_t^{1-\theta}$, where α is the fraction of workers in population. Thus,

$$\frac{y_{t+1}^N}{y_t^N} = \frac{\alpha_{t+1} A_{t+1} k_{t+1}^{\theta} h_{t+1}^{1-\theta}}{\alpha_t A_t k_t^{\theta} h_t^{1-\theta}}$$

Taking logs and using and substituting $\hat{x} = \frac{x_{t+1} - x_t}{x_t}$ to denote growth rates, gives

$$\ln\left(1+\hat{y}^N\right) = \ln\left(1+\hat{\alpha}\right) + \ln\left(1+\hat{A}\right) + \theta \ln\left(1+\hat{k}\right) + \left(1-\theta\right)\ln\left(1+\hat{h}\right)$$

Using the approximation $\ln(1+g) \approx g$ for small *g*:

$$\boxed{\hat{y}^N = \hat{\alpha} + \hat{A} + \theta\hat{k} + (1-\theta)\hat{h}}$$

3. (20 points). The next table shows the growth rates of output per worker $\hat{y}$, physical capital $\hat{k}$, and human capital $\hat{h}$, for three countries between the year 2005 and 2006. All the numbers are percentages.

Country	$\hat{y}$	$\hat{k}$	$\hat{h}$	$\hat{A}$
Argentina	1.17	1.59	0.74	
Australia	3.06	4.31	0.13	
Chile	2.00	1.47	0.74	

a. Find the growth rate of productivity using the approximate growth accounting formula and under the assumption that $\hat{\alpha} = 0$ (the fraction of workers in population is not growing).

Country	$\hat{y}$	$\hat{k}$	$\hat{h}$	$\hat{A}$
Argentina	1.17	1.59	0.74	1.17 - [(2/3)·0.74 + (1/3)·1.59] = 0.147
Australia	3.06	4.31	0.13	3.06 - [(2/3)·0.13 + (1/3)·4.31] = 1.537
Chile	2.00	1.47	0.74	2.00 - [(2/3)·0.74 + (1/3)·1.47] = 1.017

b. Is the growth in standard of living in Argentina sustainable? In your answer refer to the sources of growth.

The growth rate of productivity in Argentina is very low, so most of the growth comes from factor accumulation (physical capital per worker and human capital per worker). From the Solow we learned, under very general assumptions that without productivity growth there cannot be growth in standard of living based only on factor accumulation. So the growth in standard of living in Argentina is not sustainable since it is not propelled by productivity growth.

4. (10 points). Why do patent laws exist? (Be precise about the property of innovations which necessitates the patent laws).

Ideas are **nonexcludable**, which means that once an idea becomes public it is hard to prevent others from using it without paying. Patent laws exist because of this property of ideas. The purpose of patent laws is to protect the rights of the person who originated the idea to collect the benefits of the idea.

Suppose I invented the light bulb, and published my invention in a journal. Without patent laws every manufacturer can read my article and produce light bulbs. Most of the goods that we see around us are excludable, i.e. you need to pay for it to consume it. Ideas on the other had are different, and without patent laws can be used by anyone. There are some economists today that describe the conditions under which patent laws would not be needed. See for example:
http://levine.sscnet.ucla.edu/general/intellectual/against.htm

5. (25 points). Consider a model with one leader (country 1), and two followers (countries 2 and 3). In the leader country the labor force is 10, the fraction of the labor force in R&D is 20%, and the cost of innovation is 10. In country 2, the labor force is 10, the fraction of the labor force in R&D is 5%. In country 3, the labor force is 20, the fraction of the labor force in R&D is 5%. Assume that the cost of copying function is the same one as in the notes.
   a. Compute the steady state ratios of technologies and output per worker:

$$\left(\frac{A_1}{A_2}\right)_{ss}, \left(\frac{A_1}{A_3}\right)_{ss}, \left(\frac{y_1}{y_2}\right)_{ss}, \left(\frac{y_1}{y_3}\right)_{ss}$$

$$\left(\frac{A_1}{A_2}\right)_{ss} = \frac{\gamma_{A,1}L_1}{\gamma_{A,2}L_2} = \frac{0.2\cdot 10}{0.05\cdot 10} = 4$$

$$\left(\frac{A_1}{A_3}\right)_{ss} = \frac{\gamma_{A,1}L_1}{\gamma_{A,3}L_3} = \frac{0.2\cdot 10}{0.05\cdot 20} = 2$$

$$\left(\frac{y_1}{y_2}\right)_{ss} = \frac{\gamma_{A,1}L_1(1-\gamma_{A,1})}{\gamma_{A,2}L_2(1-\gamma_{A,2})} = \frac{0.2\cdot 10\cdot 0.8}{0.05\cdot 10\cdot 0.95} = 3.368$$

$$\left(\frac{y_1}{y_3}\right)_{ss} = \frac{\gamma_{A,1}L_1(1-\gamma_{A,1})}{\gamma_{A,3}L_3(1-\gamma_{A,3})} = \frac{0.2\cdot 10\cdot 0.8}{0.05\cdot 20\cdot 0.95} = 1.684$$

b. Which of the followers is better off in terms of output per worker?

The follower with the bigger population is better off (country 3). We see that $y_1/y_3 < y_1/y_2$, so country 3 is closer to the leader. The leader has output per worker which is 3.368 times that of country 2, but only 1.684 times that of country 3.

c. Now suppose that the leader increases its R&D, so that the fraction labor force in R&D is 40%. Compute the steady state ratios of technologies and output per worker:

$$\left(\frac{A_1}{A_2}\right)_{ss}, \left(\frac{A_1}{A_3}\right)_{ss}, \left(\frac{y_1}{y_2}\right)_{ss}, \left(\frac{y_1}{y_3}\right)_{ss}$$

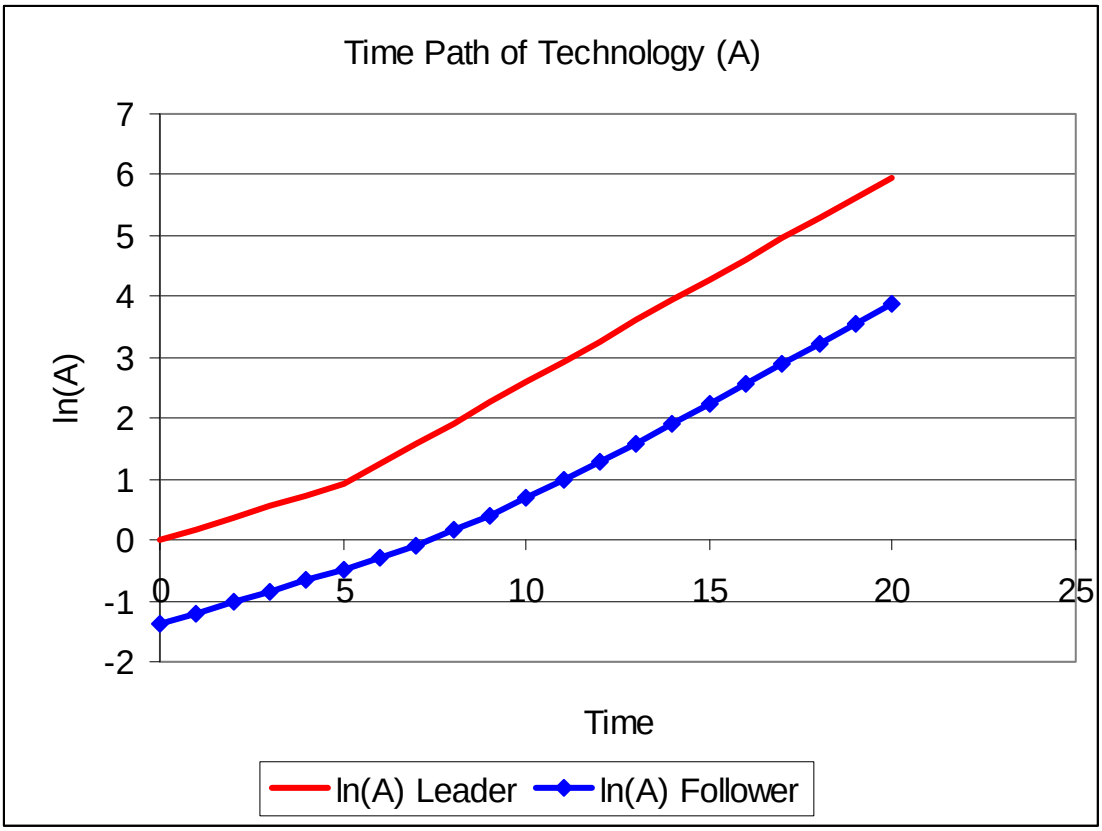
$$\left(\frac{A_1}{A_2}\right)_{ss} = \frac{\gamma_{A,1}L_1}{\gamma_{A,2}L_2} = \frac{0.4\cdot 10}{0.05\cdot 10} = 8$$

$$\left(\frac{A_1}{A_3}\right)_{ss} = \frac{\gamma_{A,1}L_1}{\gamma_{A,3}L_3} = \frac{0.4\cdot 10}{0.05\cdot 20} = 4$$

$$\left(\frac{y_1}{y_2}\right)_{ss} = \frac{\gamma_{A,1}L_1(1-\gamma_{A,1})}{\gamma_{A,2}L_2(1-\gamma_{A,2})} = \frac{0.4\cdot 10\cdot 0.6}{0.05\cdot 10\cdot 0.95} = 5.053$$

$$\left(\frac{y_1}{y_3}\right)_{ss} = \frac{\gamma_{A,1}L_1(1-\gamma_{A,1})}{\gamma_{A,3}L_3(1-\gamma_{A,3})} = \frac{0.4\cdot 10\cdot 0.6}{0.05\cdot 20\cdot 0.95} = 2.526$$

d. Use the Excel file that accompanies these notes in order to plot the time paths of technology in country 1 and country 2 that result from the change in part c. Assume that the change takes place in period 5.

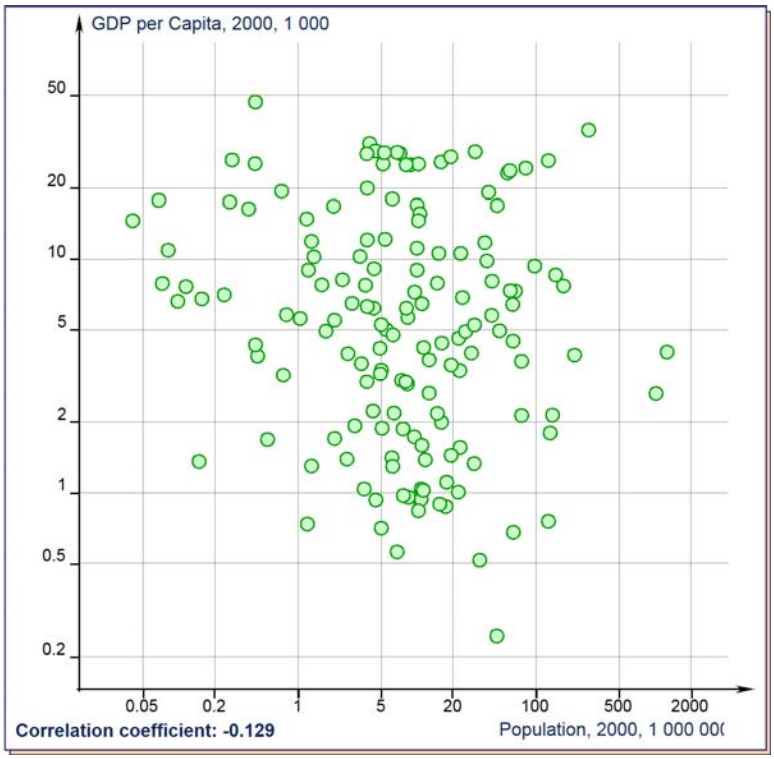


e. In the steady state, the leader grows faster than the followers both in technology and output per capita. True/false, circle the correct answer and prove it.

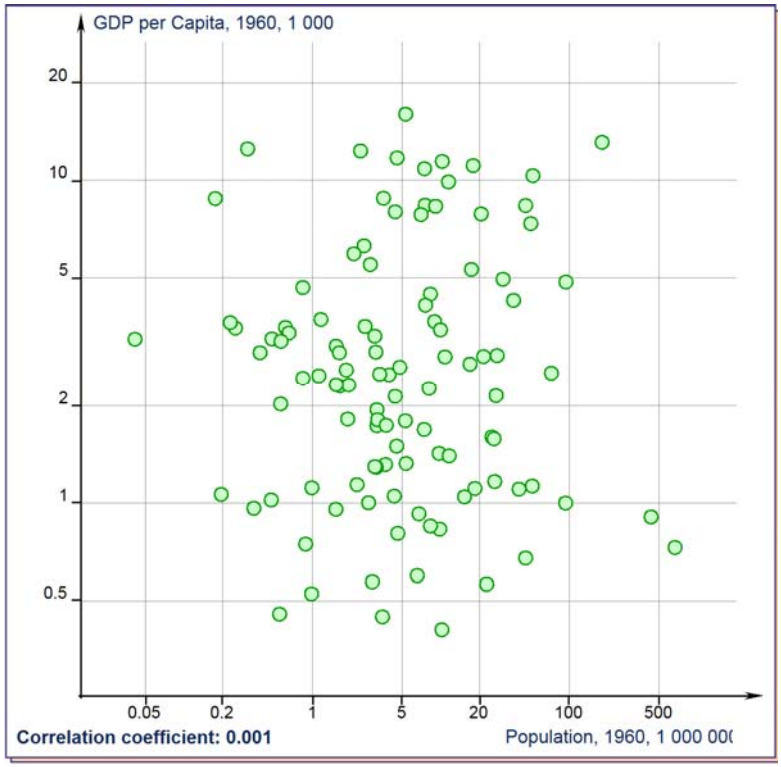
In the steady state the ratios (A_1 / A_2), (y_1 / y_2) are constant by definition, so the leader and the follower must grow at the same rate, both in technology and output per worker.

6. (5 points). According to the two-country model, countries with higher population should have higher/lower output per worker. Circle the correct answer without explaining.

7. (15 points). Is this prediction (in question 6) supported by the data?
   a. To answer this question use the data plotter provided by the textbook web page (the link to the web page is on the course web page) to plot a scatter diagram of GDP/capita in 2000 vs. Population in 2000. For the A axis choose population in 2000, for the Y axis choose GDP/cap in 2000. For both axes choose "ratio" scale. This just means logarithmic scale. (I will not attempt to explain here why this scale is better, but it has something to do with regressions. Come to my office hours if you want to know the details).



b. Repeat part a for the year 1960.



c. Based on the two graphs in part a and b, answer the main question: “Is this prediction (in question 6) supported by the data?”.

The prediction that countries with higher population should have higher GDP/cap is not supported by the data. If the data exhibited positive correlation, then that would have supported the prediction of the model. But in the first graph we observe small negative correlation and nearly zero correlation in the second graph.