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ECON 560 **Spring 2012**

Problem set 3
Malthusian Model

1. (50 points). The following questions are based on the Malthusian discussed in class (and in the notes).
    a. Derive the equation of output per capita (y_t) and the law of motion of output per capita (y_{t+1} as a function of y_t) for this model.

<u>Output per capita:</u>

$$y_t = \frac{Y_t}{L_t} = \frac{A_t \Lambda^{\theta} L_t^{1-\theta}}{L_t} = A_t \left(\frac{\Lambda}{L_t} \right)^{\theta}$$

<u>Law of motion of output per capita:</u>

$$y_{t+1} = A_{t+1} \left(\frac{\Lambda}{L_{t+1}} \right)^{\theta} = \left(A_{t+1} / A_t \right) A_t \left(\frac{\Lambda}{g(y_t) L_t} \right)^{\theta} = \frac{A_{t+1} / A_t}{g(y_t)^{\theta}} y_t$$

    b. Suppose that in 16th century France, the technology level is fixed at $A_t = 5 \ \forall t$, the population growth function is $g(y_t) = 0.2\sqrt{y_t}$, the land share parameter is $\theta = 0.4$, and the land is $\Lambda = 1000$. Find the steady state level of output per worker (*y**) and population (*L**).

The steady state output per worker (*y**) is found by solving $g(y^*) = 1$. Thus

$$0.2\sqrt{y^*} = 1, \quad \sqrt{y^*} = 5$$

$$\boxed{y^* = 25}$$

Now, to find the steady state population we use the output per capita equation:

$$y^* = A \left(\frac{\Lambda}{L^*} \right)^{\theta}$$

$$25 = 5 \left(\frac{1000}{L^*} \right)^{0.4}$$

$$L^* = 17.89$$

c. Suppose that during the same time China was much more technologically advanced, with level of technology fixed at $A_t = 10 \ \forall t$. Find the steady state level of output per worker (*y**) and population (*L**).

The steady state output per worker (*y**) is found by solving $g(y^*) = 1$. Thus

$$0.2\sqrt{y^*} = 1, \ \sqrt{y^*} = 5$$

$$\boxed{y^* = 25}$$

Now, to find the steady state population we use the output per capita equation:

$$y^* = A\left(\frac{\Lambda}{L^*}\right)^{\theta}$$

$$25 = 10\left(\frac{1000}{L^*}\right)^{0.4}$$

$$L^* = 101.19$$

d. Based on your answer to part c, what does the Malthusian model predict about the cross country differences in standard of living in the long run? Are more technologically advanced countries enjoying higher standard of living than countries that are technologically backward?

The Malthusian model predicts that in the long run, technologically advanced countries will enjoy the same standard of living as technologically backward. The only according to the Malthusian model, the more advanced countries will have higher population.

e. Now suppose that France catches up technologically with China. That is, from period $\tau + 1$, the technology jumps to the same level as in China: $A_0 = 5, A_1 = 5, ..., A_\tau = 5, A_{\tau+1} = 10, A_{\tau+2} = 10, ...$ Find the new steady state level of output per worker (*y**) and population (*L**) and show the time paths of output per capita, population growth rate and total population in France.

The steady state output per worker (*y**) is found by solving $g(y^*) = 1$. Thus

$$0.2\sqrt{y^*} = 1, \ \sqrt{y^*} = 5$$

$$\boxed{y^* = 25}$$

Now, to find the steady state population we use the output per capita equation:

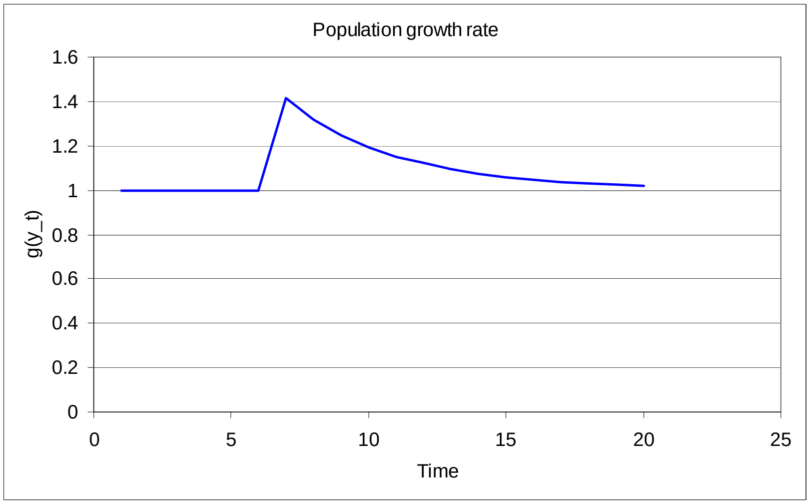
$$y^* = A\left(\frac{\Lambda}{L^*}\right)^{\theta}$$

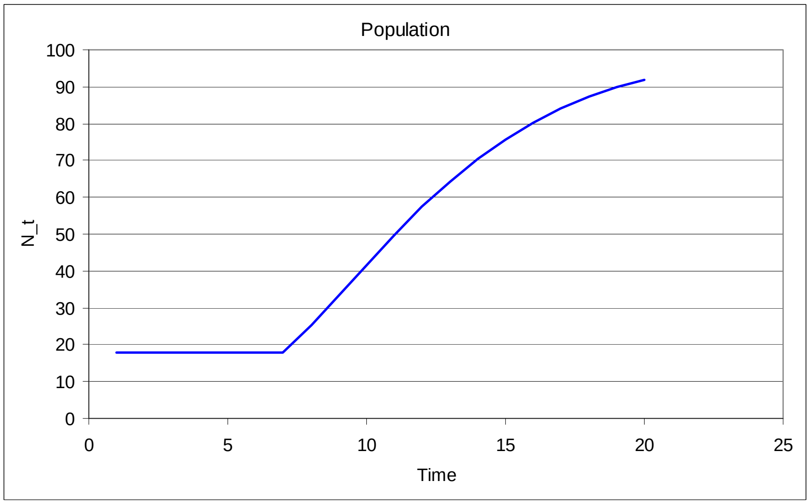
$$25 = 10\left(\frac{1000}{L^*}\right)^{0.4}$$

$$L^* = 101.19$$

Time paths:


Output per worker
y_t
60
50
40
30
20
10
0
0
5
10
15
20
25
Time


Population growth rate
g(y_t)
1.6
1.4
1.2
1
0.8
0.6
0.4
0.2
0
0
5
10
15
20
25
Time


Population
N_t
100
90
80
70
60
50
40
30
20
10
0
0
5
10
15
20
25
Time

f. Based on your answer in part e, what is the short run effect of the technological improvement on the standard of living in the Malthusian model?

In the short run, there is improvement in the standard of living as a result of a technological improvement. But eventually, population increases and reduces the output per capita to the subsistence level.

It is important to understand why this is happening in this model. The output per capita, when *A* is fixed, is given by

$$y_t = A\left(\frac{\Lambda}{L_t}\right)^{\theta}$$

In the short run, when *A* goes up, there is an increase in output per capita. But notice that output per capita depends also on the land per capita. We assumed that as long as y_t exceeds the subsistence level, the population will grow, and the term Λ / L_t will decline. This decline in resources per worker is the force that brings the level of output per worker back to subsistence after any temporary increase.

g. Based on your answer in part e, what is the long run effect of the technological improvement on the standard of living?

In the long run, the output per capita will return to the subsistence level. Thus, technological improvement leads to only temporary, but not permanent increase in the standard of living in the Malthusian model.

h. Now, suppose that the fertility behavior in the economy changed, so that the relationship between population growth and output per capita becomes $g(y_t) = 0.1\sqrt{y_t}$. Find the steady state output per worker in China.

The steady state output per worker (*y**) is found by solving $g(y^*) = 1$. Thus

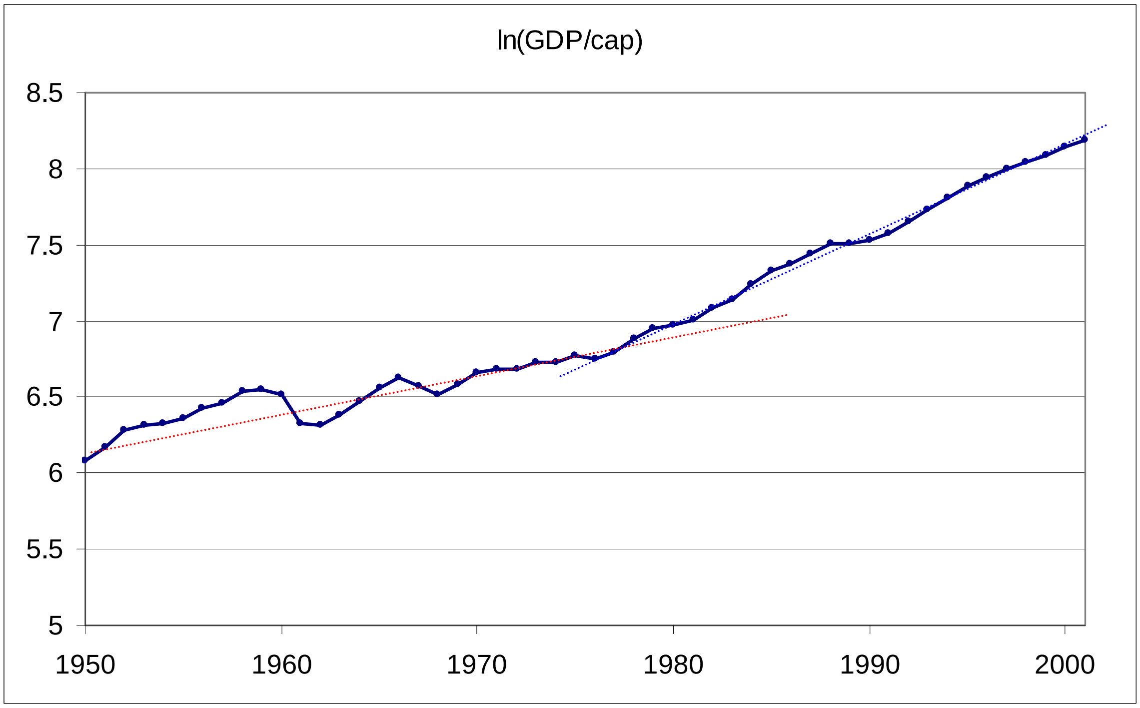
$$0.1\sqrt{y^*} = 1, \quad \sqrt{y^*} = 10$$

$$\boxed{y^* = 100}$$

i. According to the Malthusian model, what should be the effect of the one-child-law on the standard of living in China?

The Malthusian model predicts that a permanent increase in GDP per capita as a result of the law. When we changed the function *g* in the previous section such that for every income per capita the population growth declines, the result was an increase in the steady state income per capita.

The next graph shows the ln of GDP/capita in China since 1950.



It seems that until the late 70's the GDP per capita was growing at slower rate than since the late 70's. Remember that the slope of the ln(GDP/capita) tells us approximately how fast does it grow. The one-child law was enacted in 1979, and it looks like the prediction of the Malthusian model is supported by the data.

j. Compare the prediction of the Malthusian model with that of the Solow model concerning the once-and-for-all increase in technology. In particular, discuss what happens to the steady state output per worker in the Solow model as a result of the once-and-for-all increase in technology.

In the Malthusian model a once-and-foe-all increase in technology leads to only **temporary** increase in the standard of living. In the Solow model, on the other hand, a once-and-for-all increase in technology leads to **permanent** increase in the standard of living.

Recall the steady state level of output in the Solow model:

$$y^{SS} = A^{\frac{1}{1-\theta}}\left(\frac{s}{n+\delta}\right)^{\frac{\theta}{1-\theta}}$$

Higher A leads to higher y^{SS}, that is permanently higher output per worker.

Demography

2. (15 points). In this question you need to use the “Data for HW3” posted on the web.
   a. The spreadsheet called “LifeExp” contains data on age specific mortality rates, age specific survival rates, probability of being alive at any age, and ages specific fertility rates in Australia for years 1921 and 2003. The spreadsheet demonstrates how to calculate the life expectancy for 1921. Follow the example to calculate the life expectancy in Australia in 2003.

   $$LE = \sum_{i=0}^{\infty} \pi_i = 80.33$$

   b. Use the data on age specific fertility rates to compute the Total Fertility Rate (TFR) in Australia, in 1921.

   $$TFR = \sum_{i=0}^{\infty} F_i = 4$$

   c. Use the data on age specific fertility rates and the probabilities of being alive at any age, to compute the Net Reproduction Rate (NRR) in Australia in 1921. Assume that half of the newborns are girls.

   $$NRR = \frac{1}{2} \sum_{i=0}^{\infty} \pi_i F_i = 1.705$$

3. (10 points) The following table shows data for a country of Fantasia. Fantasians live for a maximum of five years. Also, all the people are women, there are no men.

Age (from last Birthday)	Population in 2000	Age specific fertility rates	Probability of surviving to next age
0	100	0	1
1	100	0.8	1
2	100	0.8	1
3	100	0	0.5
4	100	0	0

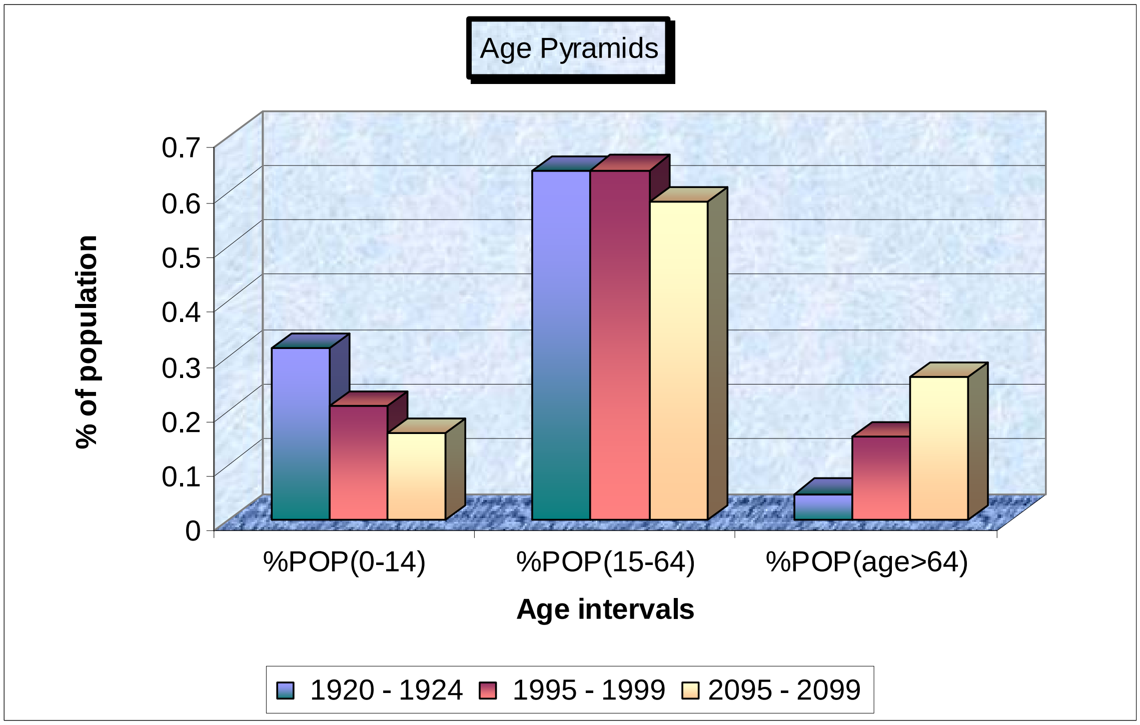
Calculate the population of Fantasia in 2001.

Age (from last Birthday)	Population in 2000	Age specific fertility rates	Probability of surviving to next age	Population In 2001
0	100	0	1	$0.8 \cdot 100 + 0.8 \cdot 100 = 160$
1	100	0.8	1	$1 \cdot 100 = 100$
2	100	0.8	1	$1 \cdot 100 = 100$
3	100	0	0.5	$1 \cdot 100 = 100$
4	100	0	0	$0.5 \cdot 100 = 50$
Total	**500**			**510**

4. (20 points). In this question you need to use the Excel file named “Data for HW3”. This question is similar to the previous one, except that here you need to predict the future population in a real country, and the data is real data, not made up.
   a. Use the spreadsheet named “Population” to predict the size of the population and population growth rate in Australia, in the year 2100, assuming that the age specific mortality and fertility will remain at their levels in 1995-1999. Do you predict that Australia’s population will shrink or increase, in the absence of migration?

The predicted population in the end of this century (2095 – 2099) is 10,151,150 (in thousands), which is less than today’s population. So we predict that population will shrink.

b. Create a diagram with columns that represent the fractions of population at 3 different age groups: (1) % of population of $0 \le age \le 14$, (2) % of population of $15 \le age \le 64$, and (3) % of population of $age > 64$, for years 1920-1924, 1995-1999, and 2095-2099. The first group is children, the second group is working age population, and the third group is old age. This diagram is called "age pyramid".



c. Based on your answer in part b, comment on the aging of Australia's population.

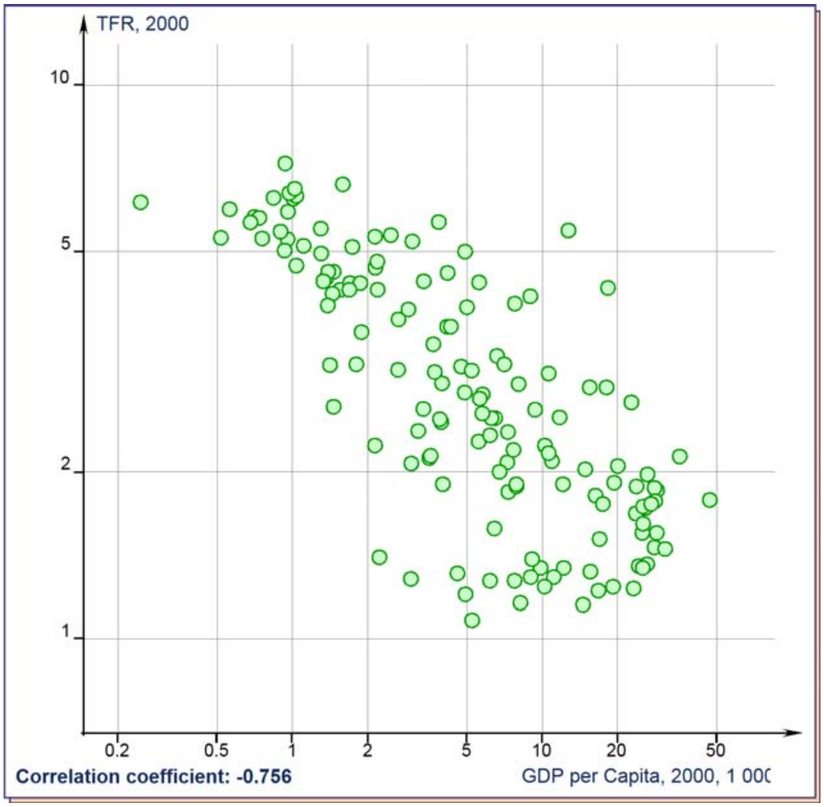
We see that the fraction of old people ($age > 64$) increases while the fraction of children ($0 \le age \le 14$) decreases. Thus, the population in Australia is aging. This phenomenon is widespread across many developed countries.

d. Can you think of any economic implications of the aging of population?

One economic implication is on the social security program. If the social security system in Australia is "pay-as-you-go", then this means that the current working population pays the pensions of the current retirees. Our demographic prediction shows that in the future the fraction of retirees will increase relative to the fraction of workers, which means that pensions might have to decline, unless the output per worker grows fast enough.

Another economic implication is for the health services. Aging population might require greater burden of health services on the economy.

5. (10 points). For this question you need to use the data plotter from the textbook resources. From the course webpage choose "Textbook Resources", then "Classroom Resources', then 'Data Plotter", and finally click on "Click Here to Launch the Data Plotter".
    a. Use the Data Plotter to produce the scatter of Total Fertility Rate in 2000 (Y axis) against GDP per capita in 2000 (X axis) across countries. Choose the Ratio scale for both axes.



    b. Based on your graph in the previous part, is there a positive or negative relationship between fertility and GDP per capita across countries? Support your answer by reporting the correlation coefficient between the two variables.

There is a negative relationship between GDP/capita and TFR. The correlation coefficient is -0.756.

6. (10 points). Observe the next table and answer the following questions.

TABLE 4.1
Demographic Data for India

Period	Total Fertility Rate	Life Expectancy at Birth	Net Rate of Reproduction
1955–1960	5.92	42.6	1.75
1965–1970	5.69	48.0	1.87
1975–1980	4.83	52.9	1.73
1985–1990	4.15	57.4	1.61
1995–2000	3.45	62.1	1.43

Source: United Nations Population Division (2002).

a. Between the years 1955-1980 both the total fertility rate and life expectancy in India changed. At the same time the net reproduction rate stayed roughly constant. Provide a clear and brief explanation how this is possible. In your explanation use the formula of NRR.

The net reproduction rate depends both on fertility and mortality, as the formula indicates

$$NRR = \frac{1}{2}\sum_{i=0}^{\infty} \pi_i F_i$$

On the one hand fertility declined, which reflected by $F_i \downarrow$, and on the other hand mortality declined as well (life expectancy increased), so the probability of being alive at certain age increased, which is reflected by $\pi_i \uparrow$. As the data shows, it turns out that the two forces canceled each other.

b. After 1980, the life expectancy increased, but nevertheless the net reproduction rate declined. Provide a clear and brief explanation how this is possible. In your explanation, use the formula of NRR.

Although life expectancy increased, the decline in fertility was significant ($F_i \downarrow$) and was the dominating factor that drove the decline in NRR.