San Francisco State University **Michael Bar**
ECON 560 **Spring 2012**

Problem set 2 - Solution

PPP Exchange Rates

1. The following table provides made up data on output per worker and prices in two countries. For the questions below, assume that TV's are traded goods while meals are not traded.

	TV's produced	Meals produced	Price of TV	Price of meal	GDP in local currency
U.S.	6	30	40 $	5 $	
India	2	10	1600 INR	1 INR	

   a. Compute the GDP in local currency in each country (i.e. complete the above table).

	TV's produced	Meals produced	Price of TV	Price of meal	GDP in local currency
U.S.	6	30	40 $	5 $	$6 \cdot 40 + 30 \cdot 5 = 390\$$
India	2	10	1600 INR	1 INR	$2 \cdot 1600 + 10 \cdot 1 = 3210 INR$

   b. Assuming that the law-of-one-price holds, find the market exchange rate in the two countries.

$$40\$ \cdot e \frac{INR}{\$} = 1600 INR$$

$$\Rightarrow e = \frac{1600}{40} = 40 \frac{INR}{\$}$$

   c. Using the market exchange rate, compute the ratio of GDP per worker in the two countries in common currency, i.e. compute $\frac{GDP_{U.S.}}{GDP_{IND}}$ (in common currency!). Does the result make sense?

$$GDP_{U.S.} = 390\$ \cdot 40 \frac{INR}{\$} = 15{,}600 INR$$

$$\frac{GDP_{U.S.}}{GDP_{IND}} = \frac{15{,}600 INR}{3210 INR} = 4.86$$

The result implies that the GDP per worker in the U.S. is almost 5 times that of India. When looking at the table of output per worker in part a we observe that the productivity of a U.S. worker in both goods is 3 times larger than the productivity of an Indian worker. The comparison of GDP using the market exchange rate gives ratio of U.S. GDP to India's GDP that is too high.

   d. Compute Purchasing Power Parity (PPP) exchange rate, assuming that the typical consumption bundle in each country is 1 TV and 5 meals.

The value of the bundle in \$: $1 \cdot 40 + 5 \cdot 5 = 65\$$
The value of the bundle in INR: $1 \cdot 1600 + 5 \cdot 1 = 1605\$$
PPP exchange rate:

$$65\$ \cdot e \frac{INR}{\$} = 1605 INR$$

$$e = 24.69 \frac{INR}{\$}$$

e. Using the PPP exchange rate, compute the ratio of GDP per worker in the two countries in common currency, i.e. compute $\frac{GDP_{U.S.}}{GDP_{IND}}$ (in common currency!). Does the result make sense?

$$GDP_{U.S.} = 390\$ \cdot 24.69 \frac{INR}{\$} = 9630 INR$$

$$\frac{GDP_{U.S.}}{GDP_{IND}} = \frac{9630 INR}{3210 INR} = 3$$

This is exactly what we expected.

Cross Country Growth Accounting

2. In this question you are required to decompose the cross country difference in GDP per capita, using the assumption that the aggregate GDP can be modeled with the Cobb-Douglas production function: $Y_t = A_t K_t^{\theta} L_t^{1-\theta}$, where Y_t is the total GDP, K_t is the total capital and L_t is the number of workers.
   a. What is the economic meaning of the parameter θ in this production function?

The parameter θ is the **capital share**, i.e. the fraction of total output that is paid to owners of physical capital under perfect competition.

b. What is the economic meaning of the parameter A_t in this production function?

The parameter A_t is the total factor productivity, i.e. it represents the contributions to output of all factors other than physical capital and labor (for example, technology level, efficiency, human capital, etc.)

c. Let N_t be the total population and α_t be the fraction of workers in population ($\alpha_t = L_t / N_t$). Show that output per worker and output per capita are given by:

$$[\text{Output per worker}]: y_t^L = A_t k_t^{\theta}$$

$$[\text{Output per capita}]: y_t^N = \alpha_t A_t k_t^{\theta}$$

$$y_t^L = \frac{Y_t}{L_t} = \frac{A_t K_t^{\theta} L_t^{1-\theta}}{L_t} = A_t K_t^{\theta} L_t^{-\theta} = A_t \left(\frac{K_t}{L_t} \right)^{\theta} = A_t k_t^{\theta}$$

$$y_t^N = \frac{Y_t}{N_t} = \alpha_t \frac{Y_t}{L_t} = \alpha_t A_t k_t^{\theta}$$

d. The following table presents data on two countries: U.S. – country *i*, and Indonesia – country *j*, for the year 2005.

	y	α	***k***	θ
U.S.	36805.74	0.52	162507.80	0.35
Indonesia	4450.616	0.49	15736.76	0.35

Using the cross-country accounting formula based on the Cobb-Douglas production function (i.e. the formula $\frac{y_i}{y_j} = \frac{\alpha_i A_i k_i^{\theta}}{\alpha_j A_j k_j^{\theta}}$), decompose the ratio of output per capita in the U.S. and Indonesia into: (i) ratio of labor force in population, (ii) ratio of TFPs, and (iii) ratio of capital per worker. Present your findings in the next table.

Accounting for cross-country differences in GDP/capita

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\frac{A_i}{A_j}$	$\frac{k_i^{\theta}}{k_j^{\theta}}$
8.27	1.07	3.41	2.26

e. Where is the biggest contribution to the difference between GDP/cap in the two countries coming from? (Difference in labor force as % of population, difference in productivity, or difference in capital per worker).

The biggest contribution to the ratio of GDP/capita between U.S and Indonesia is coming from differences in productivity.

f. If the only difference between U.S. and Indonesia was the fraction of workers in population, what would be the ratio of U.S. to Indonesian GDP?

$$\frac{y_i}{y_j} = 1.07$$

g. If the only difference between the U.S. and Indonesia was the stock of physical capital, what would be the ratio of U.S. to Indonesian GDP?

$$\frac{y_i}{y_j} = 2.26$$

h. If the only difference between the U.S. and Indonesia was the TFP, what would be the ratio of U.S. to Indonesian GDP?

$$\frac{y_i}{y_j} = 3.41$$

Solow Growth Model

3. This question is about the Solow model discussed in class: aggregate output is produced with $Y_t = A_t K_t^{\theta} L_t^{1-\theta}$, $0 < \theta < 1$, population of workers grows according to $L_{t+1} = (1+n) \cdot L_t$, capital stock evolves according to $K_{t+1} = (1-d)K_t + I_t$, and saving rate is *s*.

   a. Derive the equation of output per worker as a function of capital per worker.

$$y_t = \frac{Y_t}{L_t} = \frac{A_t K_t^{\theta} L_t^{1-\theta}}{L_t} = A_t K_t^{\theta} L_t^{-\theta} = A_t \left(\frac{K_t}{L_t}\right)^{\theta} = A_t k_t^{\theta}$$

Where k_t is capital per worker.

   b. Derive the law of motion of capital per worker.

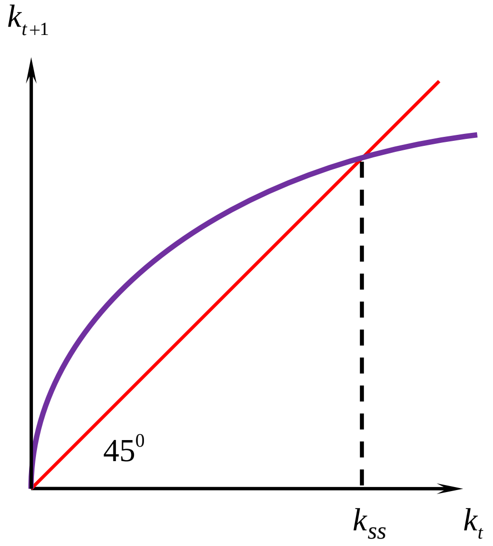
$$K_{t+1} = (1-\delta)K_t + I_t$$

$$\frac{K_{t+1}}{L_{t+1}} = \frac{(1-\delta)K_t}{L_{t+1}} + \frac{sA_t K_t^{\theta} L_t^{1-\theta}}{L_{t+1}}$$

$$k_{t+1} = \frac{(1-\delta)K_t}{L_t(1+n)} + \frac{sA_t K_t^{\theta} L_t^{1-\theta}}{L_t(1+n)}$$

$$k_{t+1} = \frac{(1-\delta)k_t}{1+n} + \frac{sA_t k_t^{\theta}}{1+n}$$

   c. Draw a graph of the law of motion of capital per worker, and indicate the steady state capital per worker (k_{ss}).



d. Assuming that *A* is fixed, derive the equation for the steady state level of capital per worker (k_{ss}), output per worker (y_{ss}) and consumption per worker (c_{ss}).

The system can be in steady state if A is fixed.

$$k_{t+1} = \frac{(1-\delta)k_t}{1+n} + \frac{sAk_t^{\theta}}{1+n}$$

$$k = \frac{(1-\delta)k}{1+n} + \frac{sAk^{\theta}}{1+n}$$

$$k(1+n) = (1-\delta)k + sAk^{\theta}$$

$$n+\delta = sAk^{\theta-1}$$

$$k_{ss} = \left(\frac{sA}{n+\delta}\right)^{\frac{1}{1-\theta}}$$

$$y_{ss} = A(k_{ss})^{\theta}$$

$$c_{ss} = (1-s)y_{ss}$$

e. According to this model, countries with higher saving rate will enjoy higher steady state consumption per worker. True/[✓ False], circle the correct answer and provide a proof.

The statement cannot be true because the highest saving rate is 100%, which leads to steady state consumption of zero.

4. In this question you are required to perform cross country accounting using the Solow model.
   a. Write the cross country accounting formula based on the Solow model.

$$\frac{y_i}{y_j} = \frac{\alpha_i}{\alpha_j}\left(\frac{A_i}{A_j}\right)^{\frac{1}{1-\theta}} \frac{\left(\frac{s_i}{n_i+\delta}\right)^{\frac{\theta}{1-\theta}}}{\left(\frac{s_j}{n_j+\delta}\right)^{\frac{\theta}{1-\theta}}}$$

   b. Write the cross country accounting formula based on the Solow model, for the case in which the two countries have the same depreciation rate and population growth rate.

$$\frac{y_i}{y_j} = \frac{\alpha_i}{\alpha_j}\left(\frac{A_i}{A_j}\right)^{\frac{1}{1-\theta}}\left(\frac{s_i}{s_j}\right)^{\frac{\theta}{1-\theta}}$$

   c. The following table presents the data on U.S. and China, where all the data is for the year 2005, except that *i* is the average investment rate over the years 1970 – 2005.

	y	s	i	α	k	θ
U.S.	36805.74	13.87	19.48	0.52	162507.80	0.35
China	5955.26	49.04	27.64	0.60	23000.92	0.35

Assume that the population growth rate and depreciation in both countries is the same. Perform the cross country accounting using the Solow cross-country accounting formula (from section b), and present your results in the next table.

Accounting for cross-country differences in GDP/capita

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\left(\frac{A_i}{A_j}\right)^{\frac{1}{1-\theta}}$	$\left(\frac{s_i}{s_j}\right)^{\frac{\theta}{1-\theta}}$
6.18	0.88	13.87	0.51

d. If the only difference between the two countries was the saving rate, China's GDP per capita would be roughly twice that of the U.S., according to the Solow model. [✓ True]/False, circle the correct answer and briefly explain.

$$\frac{y_i}{y_j} = 0.51 \Rightarrow \frac{y_j}{y_i} = \frac{1}{0.51} = 1.97$$

e. Repeat the accounting in section d, but instead of using the saving rate in 2005, use the investment rate over the years 1970 – 2005. Report your results in the next table.

Accounting for cross-country differences in GDP/capita

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\left(\frac{A_i}{A_j}\right)^{\frac{1}{1-\theta}}$	$\left(\frac{i_i}{i_j}\right)^{\frac{\theta}{1-\theta}}$
6.18	0.88	8.48	0.83

f. Explain why using investment rate over the 35 year period 1970 – 2005, makes more sense than the saving rate in 2005, when applying the Solow growth accounting framework to real world countries. (Hint: (i) does the assumption of the Solow model that economies are closed, hold in the real world? (ii) Was the capital stock at 2005 built with investment in 2005, or with investment over several decades prior to 2005? These two questions are supposed to lead you to the answer).

The two economies that we are comparing, U.S. and China, are not closed economies, so national saving is not equal to domestic investment. Since the Solow model is a theory of capital formation, and capital is augmented through investment, it is more correct to use investment rate this context. Also, the current stock of physical capital was created over many years, and therefore it makes more sense to use the average investment rate rather than the investment rate in the last year.

5. Based on the examples in this assignment and in class, what is the most important factor behind income differences between poor and rich countries?
    a. Differences in labor force participation rate.
    b. Differences in saving rates.
    c. Differences in physical capital per worker.
    [✓ d.] Differences in productivity.