

Final Exam

Tuesday, May 17

2 hour, 30 minutes

Name: _____

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (10 points). Suppose that GDP per capita in Zimbabwe is growing at 1% per year, and total GDP is growing at 4% per year.
- a. What is the approximate growth rate of population in Zimbabwe?

$$\text{growth}\left(\frac{GDP}{POP}\right) \approx \text{growth}(GDP) - \text{growth}(POP)$$

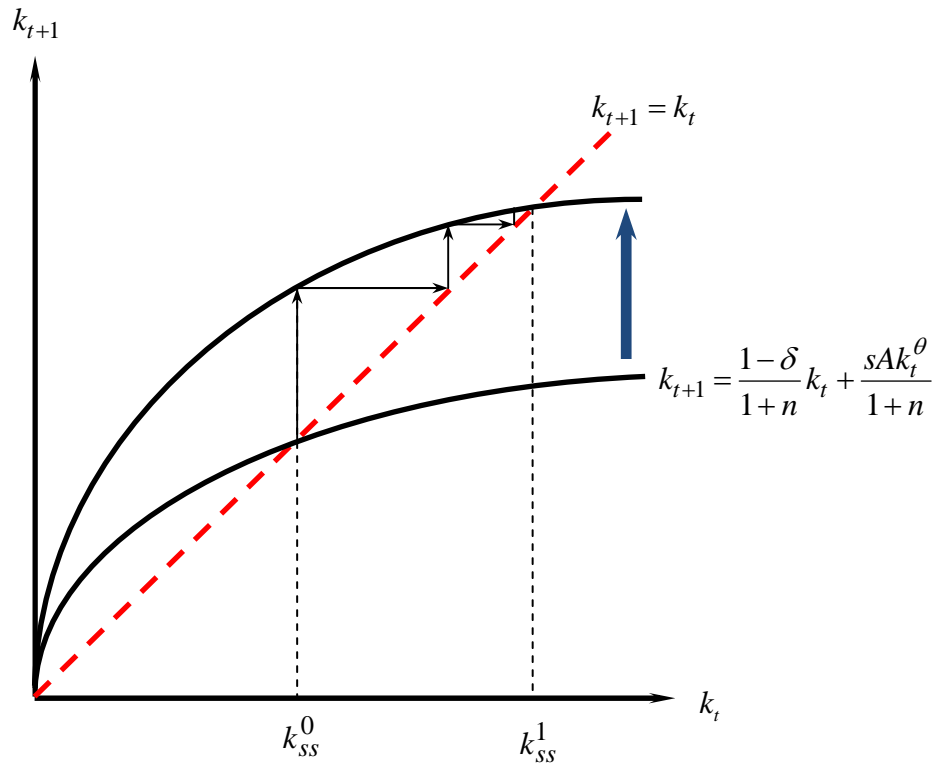
$$1\% \approx 4\% - \text{growth}(POP)$$

$$\Rightarrow \text{growth}(POP) \approx 3\%$$

- b. Using the “rule of 70”, how long will it take for the GDP per capita in Zimbabwe to double?

$$\frac{70}{1} = 70 \text{ years.}$$

2. (10 points). Suppose that the economy of China is described by the Solow model.
- Draw a fully labeled diagram of the law of motion of capital per worker in China, for given (fixed) productivity level. Indicate the initial steady state capital per worker k_{ss}^0 .



- On the above diagram, illustrate the impact of a once-and-for-all increase in productivity, and indicate the new steady state capital per worker k_{ss}^1 .

3. (10 points). Consider the model of Charles Jones 1999, where the technology evolves according to $\hat{A} = \frac{L_A^\lambda \cdot A^{-\phi}}{\mu}$. Here $L_A = \gamma_A L$ is the number of researchers, γ_A is the (constant) fraction of researchers in population, and $0 < \lambda, \phi < 1$. Output is produced according to $Y = A(1 - \gamma_A)L$.

- a. The term $A^{-\phi}$ represents the "fishing-out effect" of technology production. Briefly explain what this means.

The term $A^{-\phi}$ is a decreasing function of A , which means that if the stock of technology already in existence is large, it is harder to develop new technology because scientists need to cover a lot of material. The analogy to fishing is that after catching all the big and lazy fish in the lake, it is harder to catch new fish.

- b. Prove that a constant and positive growth in standard of living is possible in this model only as long as the number of researcher continues to grow.

First, we show that the output per worker grows at the same rate as technology.

$$y = \frac{Y}{L} = A(1 - \gamma_A)$$

$$\Rightarrow \hat{y} = \hat{A}$$

The implication follows from the fact that γ_A is constant.

Next, since these growth rates are constant, it follows that:

$$\hat{y} = \hat{A} = \text{constant} = \frac{L_A^\lambda \cdot A^{-\phi}}{\mu}$$

In approximate growth rates the above can be written as:

$$0 = \lambda \hat{L}_A - \phi \hat{A}$$

$$\hat{A} = \frac{\lambda}{\phi} \hat{L}_A$$

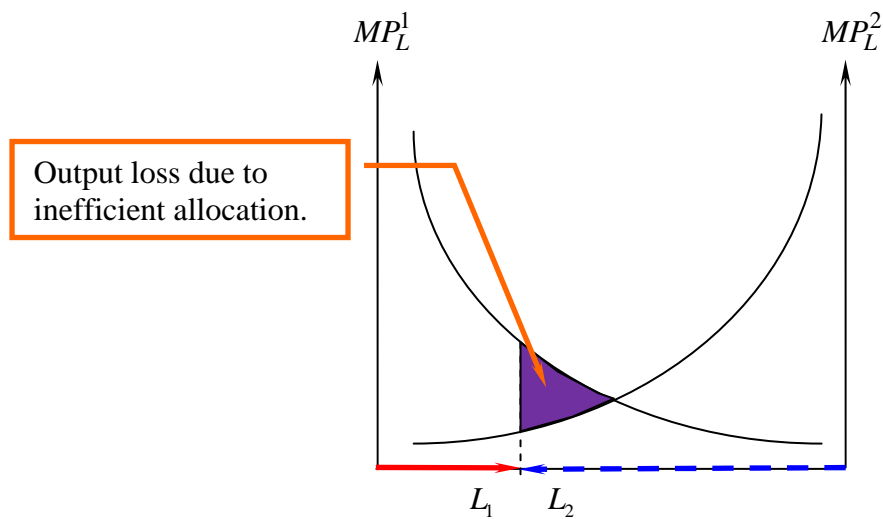
Thus, $\hat{A} > 0$ is possible only if $\hat{L}_A > 0$.

4. (10 points). Give one example for the following types of inefficiency:
- a. (i) Unproductive activities, and (ii) Idle Resources.

Unproductive activities : Theft, smuggling, burglary, civil war,...

Idle Resources: Unemployment.

- b. (5 points). In the following diagram, illustrate overallocation of labor to industry 2. Highlight the output loss due to inefficient allocation of resources.



5. (15 points). Vincent and Jules are producers of clay pots. Vincent and Jules can make at most two pots per day and sell each pot at the market price of \$10 per pot. Their personal cost of making and delivering the pots to the market is \$6 for the first pot and \$8 for the second.
- a. (5 points). How many pots will each of them sell to the market when each of them runs his business privately? Explain briefly.

Pot number	0	1	2
Cost per pot	0	\$6	\$8
Revenue per pot	0	\$10	\$10
Profit per pot	0	\$4	\$2

From the above table we see that each of them will sell 2 pots per day, because each pot creates positive net profit.

- b. (5 points). How many pots will each of them sell if they are forced to share their revenues? To answer this question you are required to construct a payoff matrix and find the Nash Equilibrium (or equilibria, if there is more than one).

		Jules		
		0	1	2
Vincent	0	0, 0	5, -1	10, -4
	1	-1, 5	4, 4	9, 1
	2	-4, 10	1, 9	6, 6

When they share the revenue, producing two pots per day is not in their best interest. Given that Jules makes two pots, Vincent's best response is not to produce anything. By symmetry, given that Vincent produces 2 pots, Jules doesn't want to produce anything. Thus, $\{2, 2\}$ is not a Nash equilibrium. It can be seen that $\{0, 0\}$ is the only Nash equilibrium in this game.

- c. (5 points). What do we learn from the above example about the inefficiencies embodied in communal (socialist) industry?

With shared revenues, workers have incentives to shirk (free ride). Once the fruits of the work are shared, every worker knows that given that others work hard, he/she does not have to. Since all workers face the same incentive, the result is underproduction and poverty.

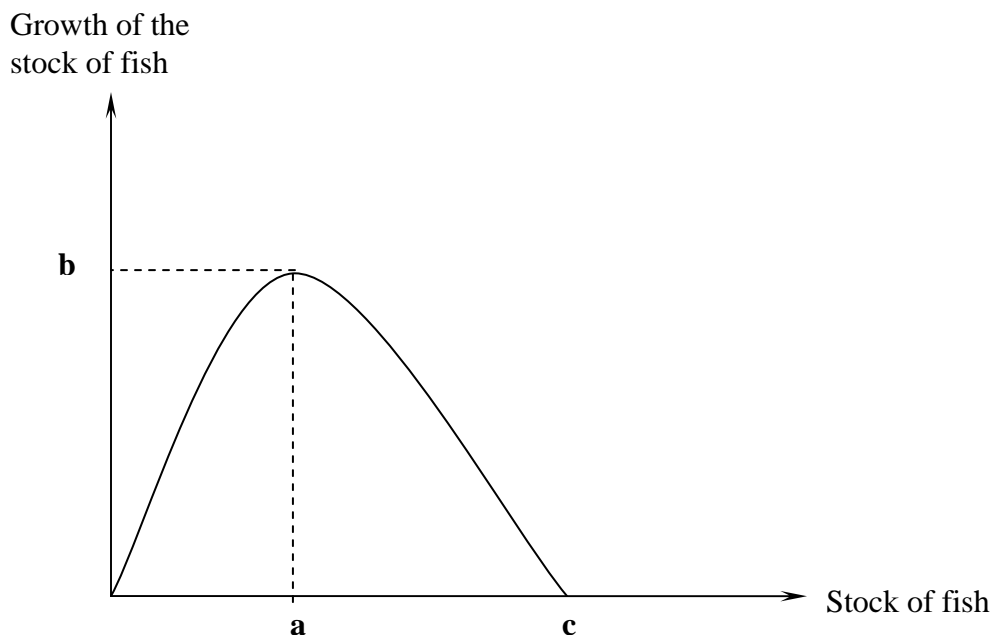
6. (5 points). Give one example of nonrenewable natural resource.

Oil, gold.

7. (5 points). Give one example of renewable natural resource.

Plants, animals, atmosphere.

8. (5 points). The next picture shows the growth of the stock of fish in the lake as a function of the size of the stock. Indicate on the graph the following magnitudes:
- a. Optimal stock of fish
 - b. Maximal sustainable yield
 - c. Carrying capacity



9. (15 points). There are 10 people in a village. Each person can work in the local lake or in the city. The wage in the city is \$55. The revenue R , the average revenue AR , as a function of number of fishermen L is given in the next table. Assume that all the fishermen have the same skill, so that everybody will catch the same amount of fish.

L	R	AR	MR
1	150	150	150
2	280	140	130
3	390	130	110
4	480	120	90
5	550	110	70
6	600	100	50
7	630	90	30
8	640	80	10
9	630	70	-10
10	600	60	-30

- a. (5 points). If the lake is a common property, find the allocation of workers between the city and the lake and calculate the total income of the villagers.

If the lake is **common**, then 10 villagers will work in the lake ($AR(10) \geq 55$).

The total income of the villagers: 600 .

- b. (5 points). If the lake is privately owned by one of the villagers, find the allocation of workers and the total income of the villagers. Briefly explain how you got the answer.

If the lake is privately owned, 5 villagers will work in the lake ($MR(5) \geq 55$) and 5 will work in the city.

The total income of villagers: $550 + 5 \cdot 55 = 550 + 275 = 825$.

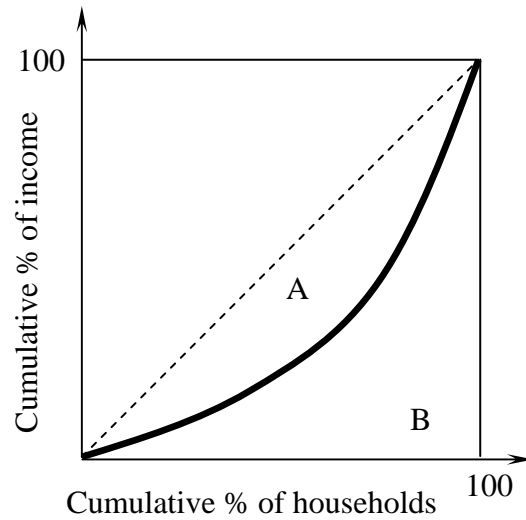
- c. (5 points). Find the size of a fishing permit that would result in optimal allocation of workers between fishermen and city workers. Briefly explain how you got the answer.

$$110 - T \geq 55 \Rightarrow T \leq 55$$

$$100 - T < 55 \Rightarrow T > 45$$

$$45 < T \leq 55$$

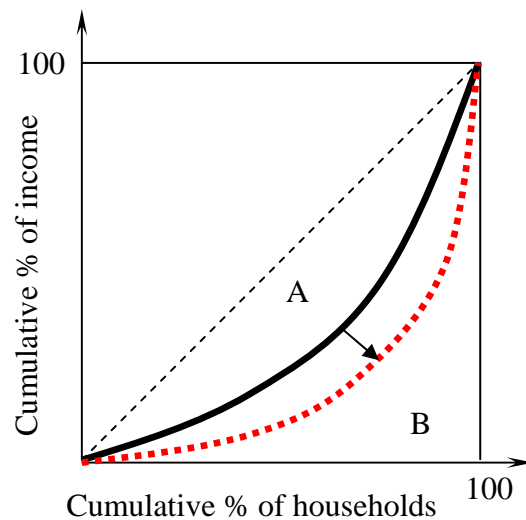
10. (10 points). The following diagram describes the income inequality in a given country.



- a. (5 points). Suppose that $B = 0.35$. Calculate the Gini coefficient for this country.

$$Gini = \frac{A}{A+B} = \frac{0.15}{0.15+0.35} = \frac{0.15}{0.5} = 0.3$$

- b. (5 points). Suppose that income inequality in the above country increased. Draw on the above graph the new Lorenz curve, which represents this change.



11. (5 points). Suppose that people derive utility not only from their own income x_i , but also from their **relative** position to the average consumption \bar{x} . Thus, the utility function of individual i is given by: $U\left(x_i, \frac{x_i}{\bar{x}}\right)$. With such preferences, it is possible that some individuals will not benefit from growth, even if their own real income increased. True/false, circle the correct answer and provide a brief explanation.

Even if own income x_i grows, it is possible that relative income x_i / \bar{x} declines (if the average grows faster than own income). Therefore, if the degree of envy is high enough, i.e. if the individuals care enough about the second component in the utility function, it is possible that their utility will decrease even if x_i grows.