Economics of Growth, ECON560
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Spring 2010

Final Exam

Wednesday, May 19

2 hour, 30 minutes

Name: ___________________________________

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (10 points). Suppose that the aggregate GDP can be modeled with the Cobb-Douglas production function: $Y_t = A_t K_t^{\theta} L_t^{1-\theta}$, $0 < \theta < 1$, where Y_t is the total GDP, A_t is the Total Factor Productivity, K_t is the total capital and L_t is the number of workers.
   a. **Derive** the equation of output per worker (y^L) as a function of capital per worker (k).

$$y^L = \frac{Y}{L} = \frac{AK^{\theta}L^{1-\theta}}{L} = Ak^{\theta}$$

   b. **Write** the equation of output per capita, when the fraction of workers in population is α .

$$y^N = \alpha y^L = \alpha A k^{\theta}$$

c. The next table shows data for two countries: U.S. (i) and Indonesia (j). The variables y_i and y_j denote GDP per capita in U.S. and Indonesia respectively.

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\frac{A_i}{A_j}$	$\left(\frac{k_i}{k_j}\right)^\theta$
9	1.5	?	2

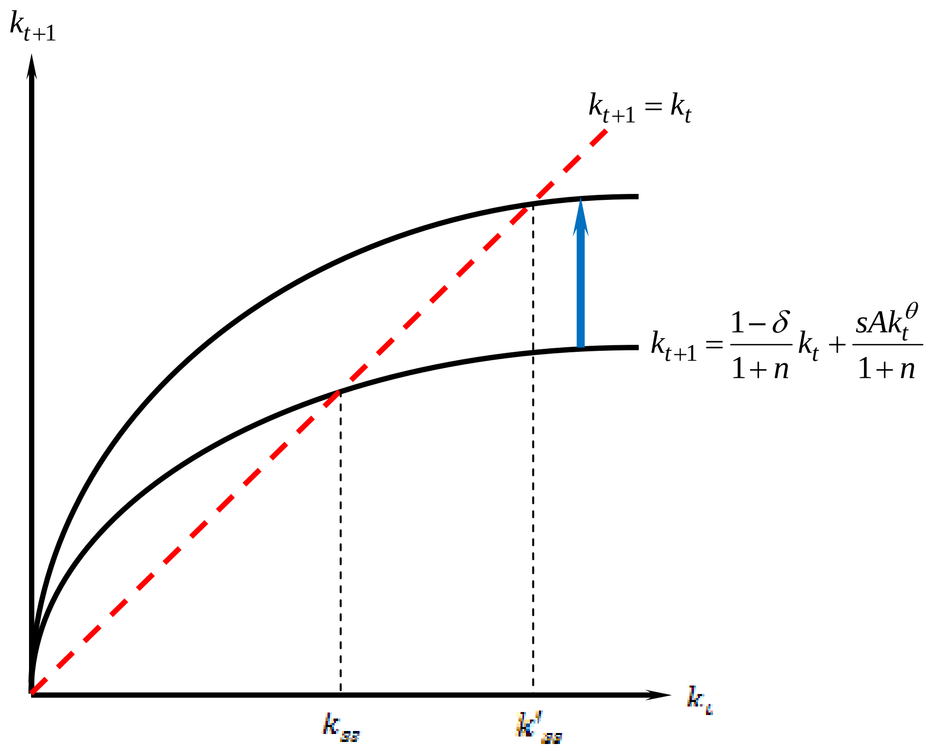
Based on the above table, if the only difference between the two countries was productivity, what would be the ratio of U.S. to Indonesian GDP per capita?

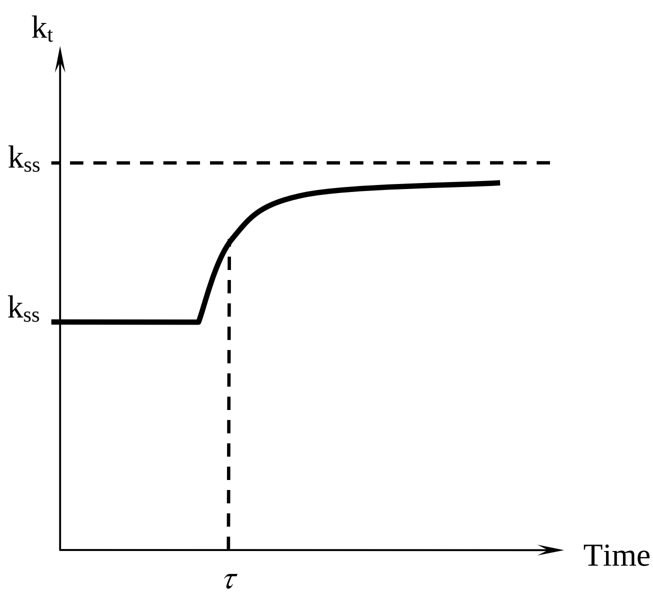
$$\frac{y_i}{y_j} = \frac{A_i}{A_j} = \frac{9}{1.5 \cdot 2} = 3$$

d. If the capital share is $\frac{1}{3}$, then U.S. capital per worker is ___ times greater than the Indonesian capital per worker. Circle the correct answer.

i. 2
ii. 3
iii. 4
iv. 8

2. (10 points). Suppose China introduces a one-child-law at time τ, and therefore the population growth rate slows down once and for all. Assuming that China was initially at a steady state, use the Solow model to describe the impact of the policy on the capital per worker. In your answer use two **fully labeled graphs**: (i) the law of motion of capital per worker and (ii) a time path of capital per worker.





3. (10 points). The next table shows how the average wage increases in years of education in a sample of countries.

Years of schooling	1-4	5-8	9,10,…
Marginal return	1.134	1.101	1.068

a. Suppose that workers in Japan have on average 10 years of education, while workers in Indonesia have on average 8 years of education. Assuming that the quality of education in both countries is the same, calculate the ratio of human capital per worker in the two countries. (Simplify your answer, but there is no need to provide the exact numerical answer).

$$\frac{h_{Japan}}{h_{Indonesia}} = \frac{h_0 \cdot 1.134^4 \cdot 1.101^4 \cdot 1.068^2}{h_0 \cdot 1.134^4 \cdot 1.101^4} = 1.068^2$$

b. Based on the following table, where is the biggest contribution to the differences in income per capita of countries *i* and *j* coming from: (circle the correct answer).

   i. Differences in the fraction of workers in population
   ii. Differences in productivity
   iii. Differences in human capital per worker
   iv. Differences in physical capital per worker

$\frac{y_i}{y_j}$	$\frac{\alpha_i}{\alpha_j}$	$\frac{A_i}{A_j}$	$\frac{h_i^{1-\theta}}{h_j^{1-\theta}}$	$\frac{k_i^{\theta}}{k_j^{\theta}}$
12	1	?	2	2

c. Based on the above table, if the only difference between countries *i* and *j* was in productivity, what would have been the ratio of GDP/capita of the two countries then?

$$\frac{y_i}{y_j} = \frac{A_i}{A_j} = 3$$

4. (10 points). Assume that the Total Factor Productivity (*A*) depends on technology (*T*) and efficiency (*E*) as follows: $A = T \cdot E$.
    a. Suppose that productivity in the U.S. is 6 times that of Haiti, i.e. $A_i / A_j = 6$, where *i* is U.S. and *j* is Haiti. Suppose that the technology in the U.S. is growing at 1% per year, and technologically, Haiti is 70 years behind the U.S. Find the approximate ratio of technology in U.S. vs. Haiti (T_i / T_j). Hint: recall the rule of 70.

Formally, one needs to solve the following equation:

$$T_i = T_j (1 + 0.01)^{70}$$

$$\frac{T_i}{T_j} = (1 + 0.01)^{70}$$

If technology in the U.S. grows at 1% per year, it must have doubled over the last 70 years. Thus technology in the U.S. now is twice what it was 70 years ago and the same as Haiti now, i.e.

$$\frac{T_i}{T_j} \approx 2$$

    b. Find the ratio of efficiency in U.S. vs. Haiti (E_i / E_j).

$$\underbrace{\frac{A_i}{A_j}}_{6} = \underbrace{\frac{T_i}{T_j}}_{2} \times \underbrace{\frac{E_i}{E_j}}_{?}$$

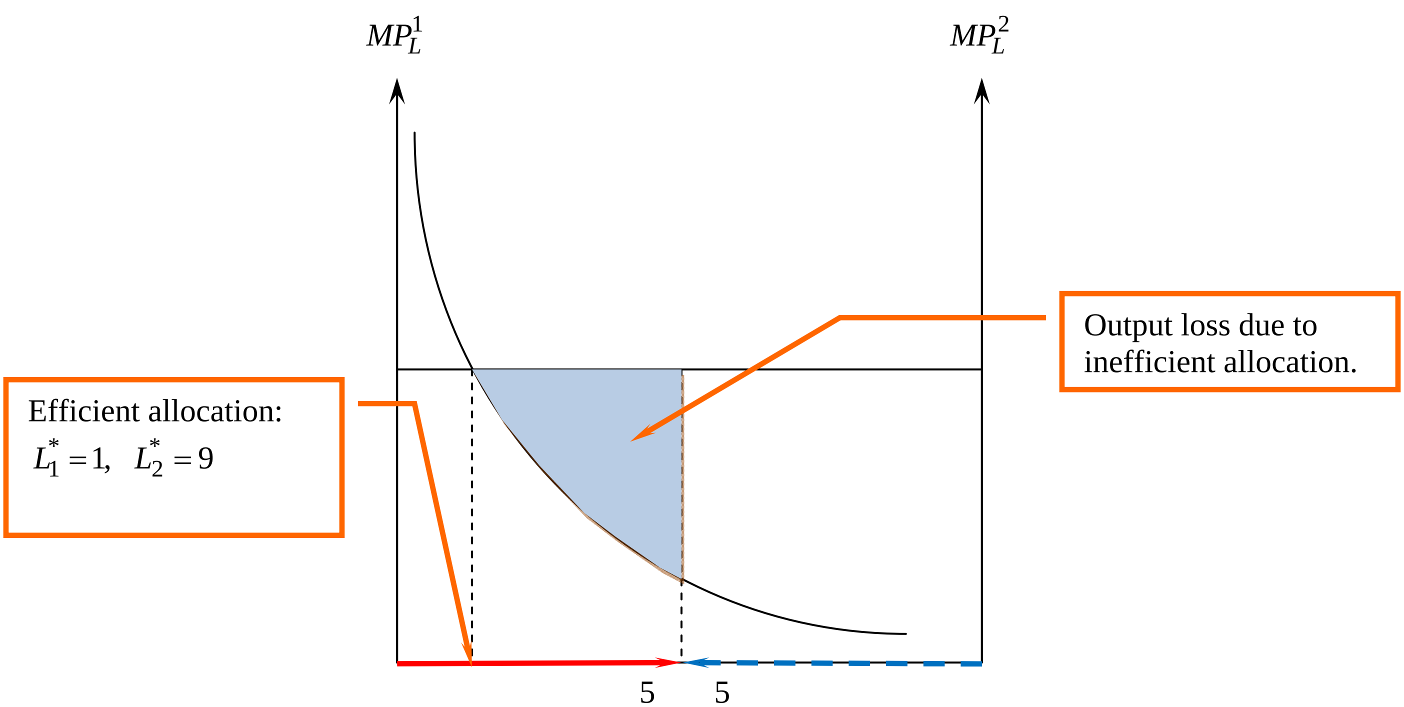
$$\frac{E_i}{E_j} = 3$$

5. (10 points). Suppose that the total output, as a function of labor, in industries 1 and 2 is given by $Y_1 = A_1 L_1^{1/2}$ and $Y_2 = A_2 L_2$, and the total labor available for these two industries is $L_1 + L_2 = 10$. Also suppose that the productivities are given by $A_1 = 2,\ A_2 = 1$. Currently, 5 workers are allocated to sector 1 and 5 workers are allocated to sector 2 (i.e. $L_1 = L_2 = 5$). Determine whether this allocation is (i) efficient, (ii) overallocation to sector 1, or (iii) overallocation to sector 2, and illustrate your answer graphically.

Efficient allocation equalizes the marginal products in the two sectors:

$$MP_L^1 = MP_L^2$$
$$\frac{1}{2} A_1 L_1^{-1/2} = A_2$$
$$\frac{1}{2} 2 L_1^{-1/2} = 1$$
$$L_1^* = 1, \quad L_2^* = 9$$

Thus, the current allocation of $L_1 = L_2 = 5$ is not efficient, and we have an overallocation of labor to sector 1.



6. (10 points). In the following table, provide one example for each type of inefficiency:

Type of inefficiency	Example
1. Unproductive Activities	Theft, burglary, rent seeking
2. Idle Resources	Unemployment
3. Technology Blocking	Microsoft attempted to suppress Java programming language, and Netscape browser

7. (20 points). Michael and Scott are taking ECON 690 and are required to submit a **joint** project for the course. Suppose that each can choose one of three effort levels: {0, 1, 2}, and the costs (in $) of these effort levels are {0, 50, 200}. The grade that they will both receive is a function of the **sum of their efforts**, and each grade has a monetary value (in $) as described in the next table:

Total Effort	0	1	2	3	4
Grade	F	D	C	B	A
$ Benefit from grade	0	100	200	300	400

a. Describe the payoff matrix for the game in which both students are choosing between effort levels {0, 1, 2} as strategies.

		Michael 0	Michael 1	Michael 2
	0	0, 0	100, 50	200, 0
Scott	1	50, 100	150, 150	250, 100
	2	0, 200	100, 250	200, 200

b. Find the Pareto Optimal outcome of the game of section a, and find the grade and monetary payoffs of the two students.

Pareto Optimal outcome is (2, 2), meaning that both students put their best effort. The payoffs are (200, 200) and they both get the grade "A". With this outcome, the total payoff is maximized and it is impossible to make any of them better off without making the other worse off. For example, the outcome (1, 2) is not PO because by moving to (2, 2) the two of them gain extra $50 which can be divided between them to make both better off.

c. Is the Pareto Optimal allocation you found in part b, also Nash equilibrium? If not, then find the Nash equilibrium (or Nash equilibria if there is more than one), and describe the equilibrium payoffs and grades.

NO. The unique Nash Equilibrium is (1, 1), with payoffs (150, 150) and grade "C" to both. To see that (2, 2) is not NE, notice that given that Michael plays 2, Scott wants to deviate and play 1. Also observe that "1" is a dominant strategy for both students, and therefore it must be always a best response.

d. What does this example reveal about incentives in a socialist regime with shared property?

Shared property diminishes the incentives to put your best effort, just like shared grade. Each worker in a communal industry (e.g. "The Great Leap Forward" plan by Mao), benefits from efforts of others, and therefore has a negative incentive to work hard.

8. (20 points). Suppose that the quantity of fish that grows each year in Lake Superior depends on the existing stock of fish as follows: $G_t = S_t(100 - S_t)$.
   a. Calculate the **carrying capacity** of fish in this lake.

The carrying capacity is the maximal stock which can be sustained. In this case it is $S_{\max} = 100$. With more fish than that, the growth is negative.

   b. Calculate the **optimal stock** of fish in the lake.

The optimal stock maximizes growth, and thus allows for maximal harvest. To find it, we solve:

$$\max_S G = S(100 - S) = 100S - S^2$$

The first order condition is:

$$100 - 2S = 0$$

$$S^* = 50$$

   c. Calculate the **maximum sustainable yield**.

The maximum sustainable yield:

$$G^* = S^*\left(100 - S^*\right) = 50(100 - 50) = 2500$$

   d. Suppose that due to overfishing, the government wants to reduce the number of fishermen in the lake. Suggest a policy that will achieve this goal.

Fishing permit. It increases the cost of fishing, and reduces the number of people fishing in the lake.

   e. What is the difficulty with applying similar policy you suggested in the last section to fight global warming?

Atmosphere, as opposed to most lakes, does not belong to a single country. Thus, a permit that reduces pollution and abuse of atmosphere require coordination and agreement of many countries.