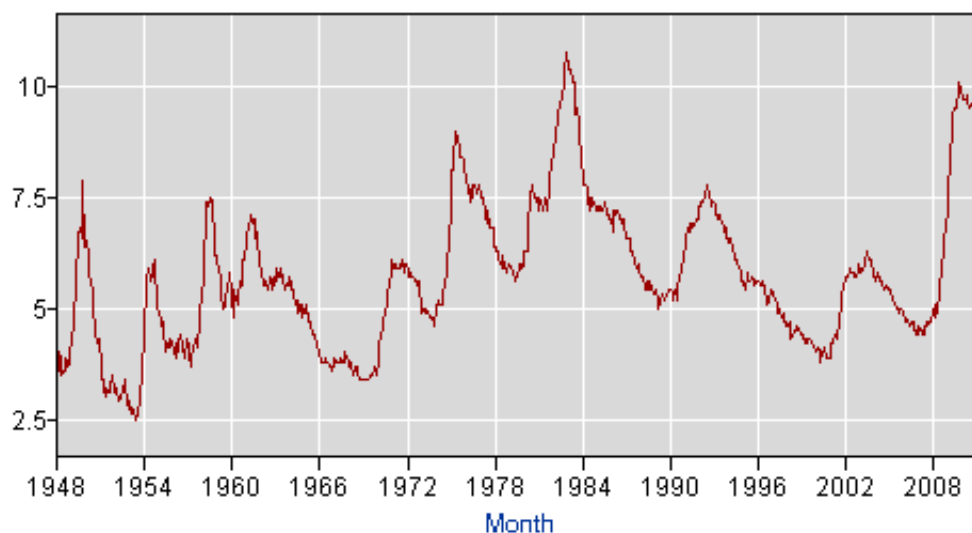


Unemployment¹

In the classical model the labor market is cleared by assumption. This means that all people looking for a job were able to find a job. In reality, there are always some unemployed people in any economy. The next figure shows the unemployment rate in the U.S. since 1948.

Labor Force Statistics from the Current Population Survey

Series Id: LNS14000000
Seasonally Adjusted
Series title: (Seas) Unemployment Rate
Labor force status: Unemployment rate
Type of data: Percent or rate
Age: 16 years and over



Note that for the most part, the unemployment rate was around 5%, and rarely exceeded 10%. Many European countries experience double digits unemployment rate on a regular basis, i.e. on average. Even within the U.S., there are large differences in unemployment rates across states and cities.

This unemployment figure is important, because 1% increase in unemployment rate in the U.S. for example means that about 1.5 million workers lose their jobs. All policymakers agree on the objective to keep unemployment low. High unemployment stands in the way of achieving full productive capacity and increases inequality. In these notes we learn how to measure unemployment rate, and design a theory that explains how it changes over time and across locations.

¹These notes are based on the notes by Oksana Leukhina, at UNC at Chapel Hill, <http://www.unc.edu/~oksana/>

1 Labor Market Definitions

- **Civilian noninstitutional population** - Included are persons 16 years of age and older residing in the 50 States and the District of Columbia who are not inmates of institutions (for example, penal and mental facilities, homes for the aged), and who are not on active duty in the Armed Forces.
- **Unemployed persons** - Persons aged 16 years and older who had no employment during the reference week, were available for work, except for temporary illness, and had made specific efforts to find employment sometime during the 4-week period ending with the reference week. Persons who were waiting to be recalled to a job from which they had been laid off need not have been looking for work to be classified as unemployed.
- **Employed persons** - Persons 16 years and over in the civilian noninstitutional population who, during the reference week, (a) did any work at all (at least 1 hour) as paid employees; worked in their own business, profession, or on their own farm, or worked 15 hours or more as unpaid workers in an enterprise operated by a member of the family; and (b) all those who were not working but who had jobs or businesses from which they were temporarily absent because of vacation, illness, bad weather, child-care problems, maternity or paternity leave, labor-management dispute, job training, or other family or personal reasons, whether or not they were paid for the time off or were seeking other jobs. Each employed person is counted only once, even if he or she holds more than one job. Excluded are persons whose only activity consisted of work around their own house (painting, repairing, or own home housework) or volunteer work for religious, charitable, and other organizations.
- **Labor force** - The labor force includes all persons classified as employed or unemployed.
- **Not in the labor force** - Includes persons aged 16 years and older in the civilian noninstitutional population who are neither employed nor unemployed.

The next diagram illustrates the breakdown of the population into different categories.

$$Population = \underbrace{\overbrace{employed + unemployed}^{\text{labor force}} + not\ in\ labor\ force}_{\text{Civilian Noninstitutional Population of Working Age}} + \left\{ \begin{array}{l} \text{people younger than 16,} \\ \text{in the military,} \\ \text{or institutionalized} \end{array} \right.$$

The next table shows the data for the U.S., January 2006 (in thousands).

Total Population	298048
Civilian Noninstitutional Population	227553
Labor Force	150114
Employed	143074
Unemployed	7040
Not in the Labor Force	77439

The most important indicators of the labor market are: (1) Unemployment Rate, and (2) Labor Force Participation Rate.

$$\begin{aligned}\text{Unemployment Rate} &= \frac{\#\text{Unemployed}}{\#\text{Labor Force}} \\ \text{Labor Force Participation} &= \frac{\#\text{Labor Force}}{\#\text{Civilian Noninstitutional Population}}\end{aligned}$$

Based on the above data,

$$\begin{aligned}\text{Unemployment Rate} &= \frac{7040}{150114} = 4.7\% \\ \text{Labor Force Participation} &= \frac{150114}{227553} = 66\%\end{aligned}$$

Labor force participation in the U.S. is approximately 70% for men and 60% for women. In the last 50 years, participation rates for women more than doubled while for men, participation rate slightly declined. Some of the prominent hypotheses for why women increasingly entered the labor force include the closing of the gender wage gap, the declining price of home appliances, and the wide spread use of a contraceptive pill. All of these stories have the feature of increasing the return to women of educating themselves and working relative to staying at home. Participation rate is a completely different thing from the unemployment rate. It measures the degree of willingness of people to work for paid wage. Unemployment rate measures the degree of difficulty of finding a job. In these notes, we only focus on the unemployment rate.

Some Determinants of the Unemployment Rate

1. Aggregate economic activity. High levels of output are associated with lower unemployment rates. In other words, unemployment is countercyclical, as the figure 1 shows.

2. Demographic structure of the population. For example, younger workers tend to switch jobs more often, they have less to lose by getting fired, etc. Hence, younger populations, all things equal, tend to have higher unemployment rates. For example, if during the 50's there was a baby boom in the U.S., then 20 years later when the baby boom cohort enters the labor market, we expect the unemployment rate to increase.

3. Sectorial Shifts. For example, a shift away from manufacturing has displaced many workers. Finding a new job for these workers involves acquiring different skills. Hence, societies with a greater degree of restructuring tend to have higher unemployment rates.

4. Government policies. These include unemployment insurance programs as well as welfare, training programs and job matching services for the unemployed. The unemployment insurance (UI) program in the U.S. is run by state governments. Typically, unemployed workers in the U.S. draw benefits for 6 months and the replacement ratio (ratio of UI benefits to the wage the unemployed worker used to receive) is 1/2. Existence of this government policy affects the behavior of both, employed and unemployed.

The unemployment rate in the data exhibits both, fluctuations at business cycle frequencies (determinant #1) as well as longer run trends (determinants #2,3,4).

Figure 16.2 Deviations from Trend in the Unemployment Rate and Percentage Deviations from Trend in Real GDP for 1948–2003

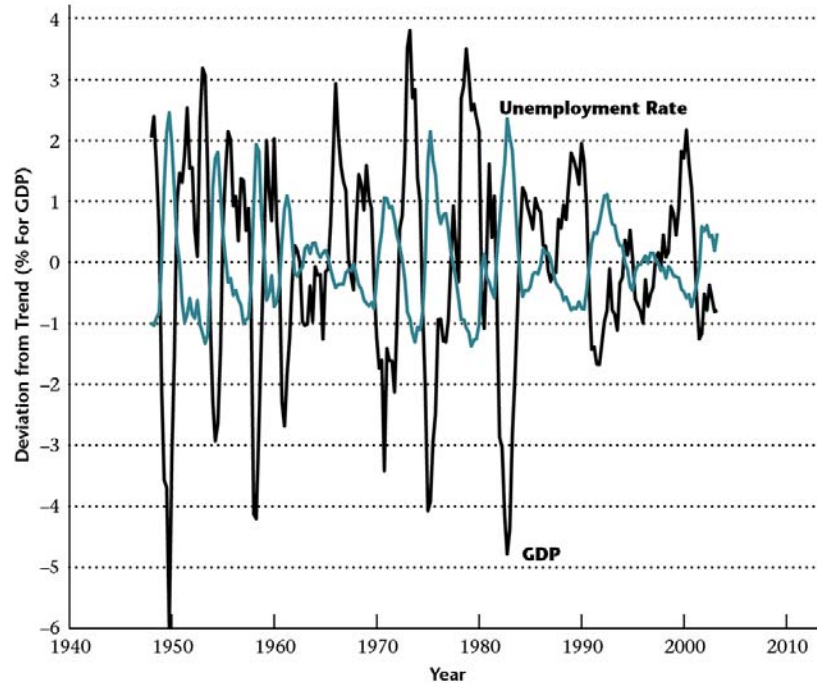


Figure 1: Unemployment rate is countercyclical.

2 The Search Model

This model will provide us with some simple insight into how the unemployment rate is determined. This model will also allow us to study how the unemployment rate can be affected by government policy with respect to unemployment benefits, labor income or unemployment income taxation as well as changes in informational technology.

2.1 Preferences

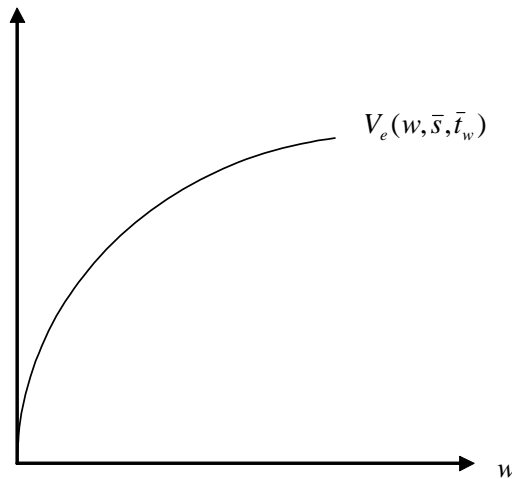
There are many jobs with different real wages w . The only characteristics of a job that people care about is the wage that it pays. People are either employed or unemployed. This means that everybody is in the labor force. Let U represent the fraction of people that are unemployed. Then fraction $1 - U$ of people are employed. A fraction s of all the employed will be separated from their jobs at any given period. We call s the separation rate and assume that it is fixed and the same for all jobs. The separation rate is exogenously given parameter and can be thought of as the probability that any employed worker will lose his job. A fraction p of all unemployed people get a job offer. Again, p is just a given parameter; people have no control over it. We can think of p as the probability that any unemployed

worker will get a job offer.

Let $V_e(w, s, t_w)$ denote the utility of being employed at wage w , with separation rate s and taxes on labor income t_w . We assume that V_e is increasing in w , but at a decreasing rate (which means that V_e is concave in w). Also assume that V_e is decreasing in separation rate s and in taxes on labor income t_w . Thus $V_e \left(\begin{matrix} w, \\ + \\ s, \\ - \\ t_w \\ - \end{matrix} \right)$. For example, V_e could be of the following form

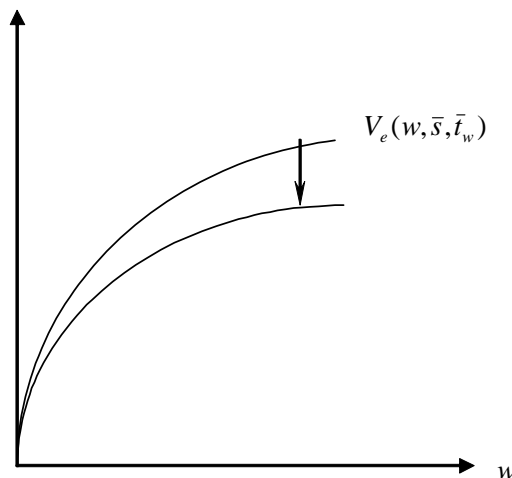
$$V_e(w, s, t_w) = (1 - s) \sqrt{w(1 - t_w)}$$

If we plot V_e as a function of w (keeping s and t_w fixed), the graph would look like the following



Utility of Employed, as a Function of w

The notation \bar{s} and \bar{t}_w means that the above graph was plotted for some fixed values of s and t_w . Changes in these values will shift the entire curve. In particular, an increase in either s or t_w will shift the entire curve down, as shown in the next picture



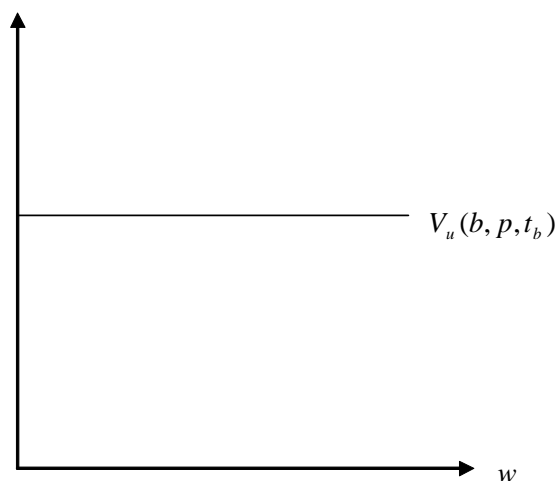
Shift in the Utility as $s \uparrow$ or $t_w \uparrow$.

In what follows, we will use the notation $V_e(w)$ to denote the utility of employed person for given and fixed values of s and t_w .

Let $V_u(b, p, t_b)$ denote the utility of being unemployed, where b is the real unemployment benefit, p is the probability of getting a job offer, and t_b is the tax on income from unemployment. We assume that $V_u \left(\underset{+}{b}, \underset{+}{p}, \underset{-}{t_b} \right)$, which means that V_u is increasing in the unemployment benefits b , increasing in the chances of getting an offer p , and decreasing in the taxes on unemployment benefit. For example, the function V_u could be of the following form

$$V_u(b, p, t_b) = p\sqrt{b(1 - t_b)}$$

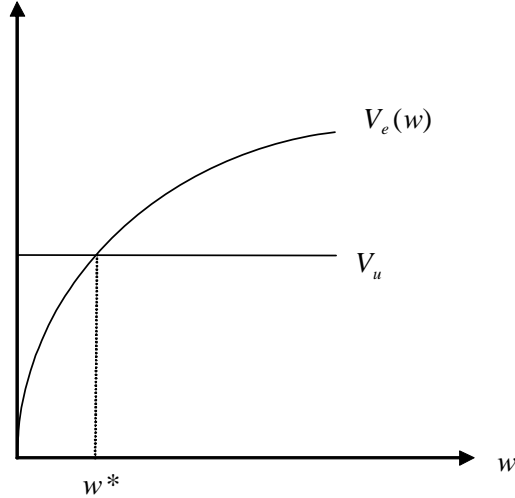
Plotting the graph of V_u against the real wage (for given values of b, p, t_b) looks like a horizontal curve since V_u does not depend on w , as shown in the next picture



In what follows, we will use the notation V_u as a shorthand for the utility of unemployed for given and fixed values of b, p and t_b .

2.2 Job Offer Acceptance

When an unemployed worker receives a job offer w he accepts it if $V_e(w) \geq V_u$. The minimum wage offer which an unemployed worker accepts is an offer w^* such that $V_e(w^*) = V_u$. We refer to this w^* as the *reservation wage*. For all job offers $w \geq w^*$, $V_e(w) \geq V_u$ and therefore the unemployed will accept those job offers. For all job offers $w < w^*$, $V_e(w) < V_u$ and therefore the unemployed will reject those job offers. The next graph illustrates the job offer acceptance decision.



The reservation wage w^* is crucial for determining the unemployment rate. Suppose that there are 1000 job offers, and 700 of them are $\geq w^*$. Then we know that 70% of those receiving job offers will accept them and become employed.

Examples

1. Suppose that $V_e(w, s, t) = (1 - s) \sqrt{w(1 - t_w)}$, and $V_u(b, p, t_b) = 3p\sqrt{b(1 - t_b)}$. Find the reservation wage w^* .

Solution: the reservation wage solves

$$(1 - s) \sqrt{w(1 - t_w)} = 3p\sqrt{b(1 - t_b)}$$

Thus

$$w^* = \left(\frac{3p}{1 - s} \right)^2 \frac{b(1 - t_b)}{(1 - t_w)}$$

2. Explain how does w^* depend on the exogenous parameters b, p, s, t_w, t_b and give some intuition for your results.

Solution: The reservation wage, w^* , is increasing in b, p, s and t_w . This makes intuitive sense. Higher unemployment benefit b means that the unemployed are more comfortable with being unemployed and it will take higher wage to induce them to accept the job offer. Higher p means that greater fraction of unemployed receive job offers, so the chances of finding a higher paid job (everything else equal) is higher and therefore w^* is higher. Higher s means greater separation rate, or greater risk of losing the job. Thus, the unemployed will demand higher wage to compensate for that risk. Finally, higher tax on labor t_w lowers the net of tax real wage and hence it takes higher before tax wage to induce the unemployed to accept a job.

Also observe that w^* is decreasing in the tax on unemployment benefit t_b , which is also intuitive; higher tax on unemployment benefit lowers the net of tax unemployment benefit and hence lowers the utility from being unemployed. Thus, it will take lower wage to induce the unemployed to accept the job.

3. In the above example, suppose that all types of income are taxed at the same rate t , how does the reservation rate w^* depend on t ?

Solution: It doesn't, t cancels out

$$w^* = \left(\frac{3p}{1-s} \right)^2 \frac{b(1-t)}{(1-t)} = \left(\frac{3p}{1-s} \right)^2 b$$

4. Suppose that in the above example we have $b = 5$, $p = 0.6$, $s = 0.1$, $t_w = t_b = 0.3$. Find the reservation wage w^* .

Solution:

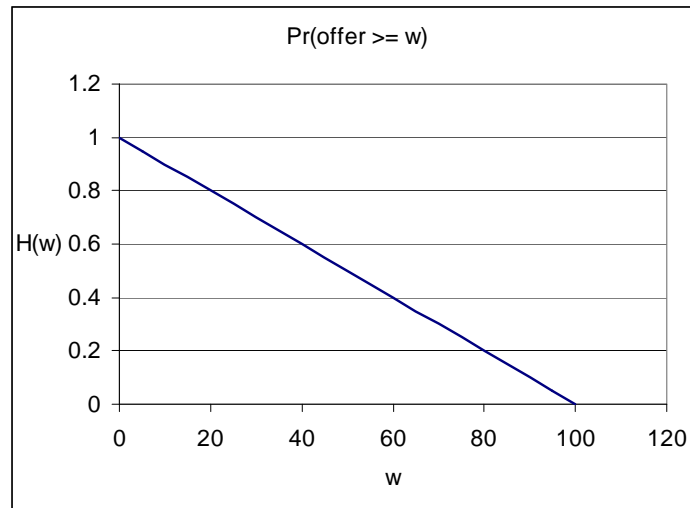
$$\begin{aligned} w^* &= \left(\frac{3p}{1-s} \right)^2 \frac{b(1-t_b)}{(1-t_w)} \\ &= \left(\frac{3 \cdot 0.6}{1-0.1} \right)^2 \left[\frac{5 \cdot (1-0.3)}{(1-0.3)} \right] \\ &= \left(\frac{1.8}{0.9} \right)^2 5 = 20 \end{aligned}$$

2.3 Distribution of wage offers

We need one more piece of information in order to find the unemployment rate, namely the distribution of wage offers. We assume that the distribution of job offers is given by a function $H(w)$ which gives the probability that an offer is at least w . For example, suppose that

$$H(w) = 1 - \frac{1}{100}w$$

The next figure is the plot of this function.



Important! $H(w)$ does not give the probability of receiving an offer of at least w . What it does tell us is that *if* some unemployed person received an offer, then $H(w)$ is the probability that that offer is at least w .

Examples

1. Suppose that the distribution of job offers is as given above, and suppose that I am unemployed who received an offer. What is the probability that this offer is above 20?

Solution:

$$H(20) = 1 - \frac{1}{100}20 = 0.8$$

2. Suppose that I am an unemployed person and a fraction $p = 0.6$ receive job offers. What is the probability that I get an offer of at least 20?

Solution:

$$p \cdot H(20) = 0.6 \cdot 0.8 = 0.48$$

3. Explain the difference between part 1 and 2.

Solution: In part 1 it was already given that I received an offer, so $H(w)$ tells us what is the probability that an offer that was received is at least w . In part 2 it is not known whether I will receive an offer or not. In fact, there is only 60% chance that I will. Thus, the chances that I will get an offer of at least 20 are 0.6 times what they are in part 1.

2.4 Equilibrium

Now we have all the information we need in order to compute the law of motion of unemployment rate:

$$U_{t+1} = U_t + s(1 - U_t) - pH(w^*)U_t$$

The unemployment rate in the next period U_{t+1} is equal to the sum of 3 elements. The first is the current unemployment rate U_t .

The term $s(1 - U_t)$ is the addition to the unemployment rate due to separation of currently employed from their jobs. Suppose that 90% of the labor force are currently employed ($1 - U_t = 0.9$) and the separation rate is 0.1, which means that 10% of the currently employed will lose their jobs. Thus the term $s(1 - U_t) = 0.1 \cdot 0.9 = 0.09$, i.e., the unemployment rate next period will increase by 0.9% due to some of the currently employed losing their jobs.

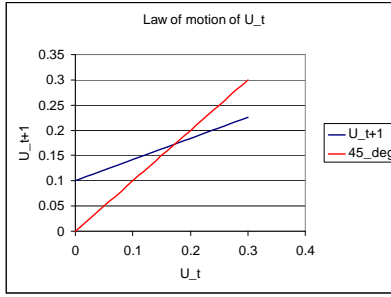
The last term on the right hand side represents the decline in unemployment rate due to some of the currently unemployed finding jobs. Suppose that currently there is 10% unemployment rate. Suppose that $p = 0.6$, which means that 60% of the currently unemployed will receive a job offer. What fraction of them will accept the offer? It is given by $H(w^*)$, which is the probability that an offer exceeds the reservation wage (remember that unemployed people accept an offer if it is at least as high as their reservation wage). Suppose that in our example $w^* = 20$, so $H(w^*) = 0.8$. This means that 80% of the unemployed *who received and offer* will accept it. Thus, $pH(w^*)U_t = 0.6 \cdot 0.8 \cdot 0.1 = 0.048\%$, which is the decline in the unemployment rate due to unemployed finding jobs. In this numerical example we see that the unemployment rate will increase from period t to period $t + 1$ because the addition to unemployment rate from employed who lose their jobs is greater than the decline in unemployment rate from unemployed who find jobs.

Long-run equilibrium

Rearranging the law of motion of unemployment rate gives

$$\begin{aligned} U_{t+1} &= U_t + s(1 - U_t) - pH(w^*)U_t \\ U_{t+1} &= U_t + s - sU_t - pH(w^*)U_t \\ U_{t+1} &= U_t[1 - s - pH(w^*)] + s \end{aligned}$$

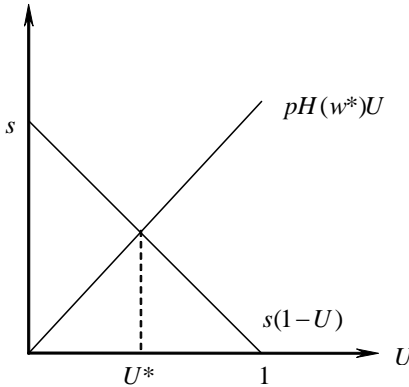
If $1 - s - pH(w^*) < 1$ then the law of motion has a steady state, as shown in the next figure



We can see that starting from any unemployment rate, the economy will converge to a steady state U^* such that $U_t = U_{t+1} = U^*$ for all t . To find the steady state we solve

$$\begin{aligned} U &= U + s(1 - U) - pH(w^*)U \\ s(1 - U) &= pH(w^*)U \\ U^* &= \frac{s}{pH(w^*) + s} \end{aligned}$$

Once we find the reservation wage w^* , we can plot $pH(w^*)U$ as a function of U , it is just a linear function of U with slope $pH(w^*)$. The intersection of this line with $s(1 - U)$ (also a linear function of U with slope $-s$ and intercept s) determines the long-run (or steady state) equilibrium level of U^* .



In our numerical example

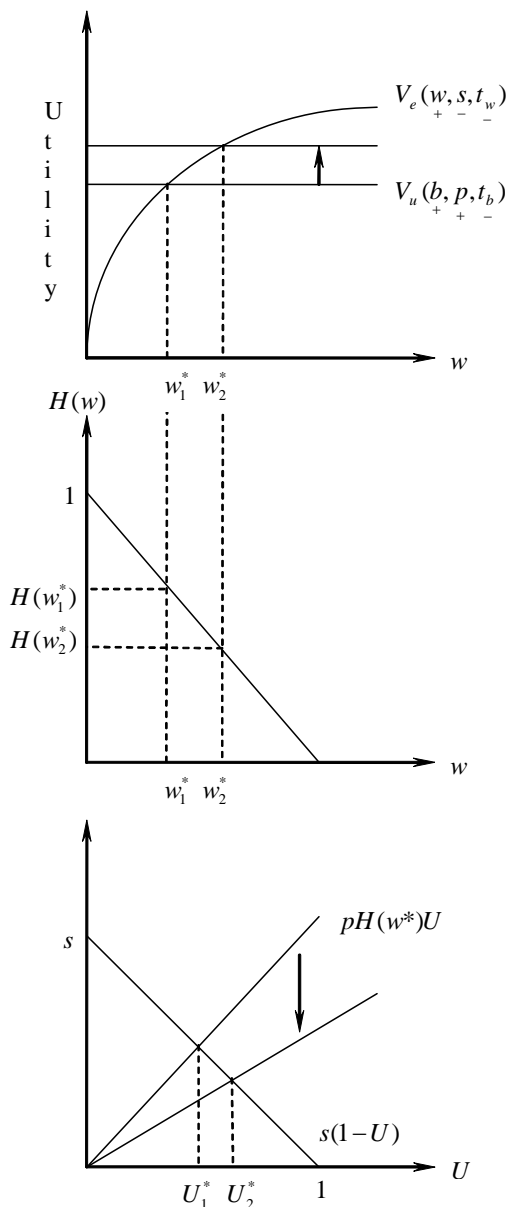
$$U^* = \frac{s}{pH(w^*) + s} = \frac{0.1}{0.6 \cdot 0.8 + 0.1} = 0.172413793$$

Intuitively, the term $s(1 - U)$ represents the "flow in" to the unemployment as a result of employed people separating from their jobs, while the term $pH(w^*)U$ represents the "flow out" of the unemployment as a result of unemployed accepting job offers.

2.5 Experiments with the Search Model

2.5.1 An increase in unemployment insurance benefit ($b \uparrow$)

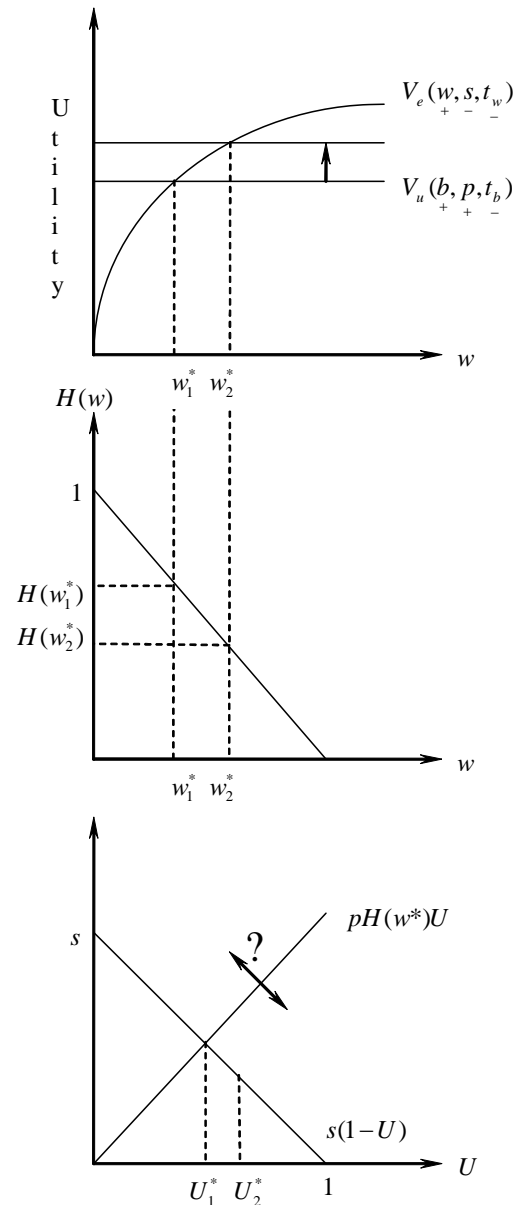
As $b \uparrow$, the value of being unemployed shifts up, hence, the reservation wage increases (the unemployed get more picky about job offers). As a result, fewer of those who are offered jobs (the same job offers are made) accept their offer, i.e., $H(w^*) \downarrow$. Hence, $U^* = \frac{s}{pH(w^*)+s}$ goes up. Notice that in this experiment, s and p remained the same while $H(w^*)$ went down. As the denominator became smaller, the fraction became larger. Intuitively, the flow out of the unemployed is reduced since fewer job offers are accepted. The figures below illustrate all of the steps we mentioned.



2.5.2 An increase in probability of getting a job offer ($p \uparrow$)

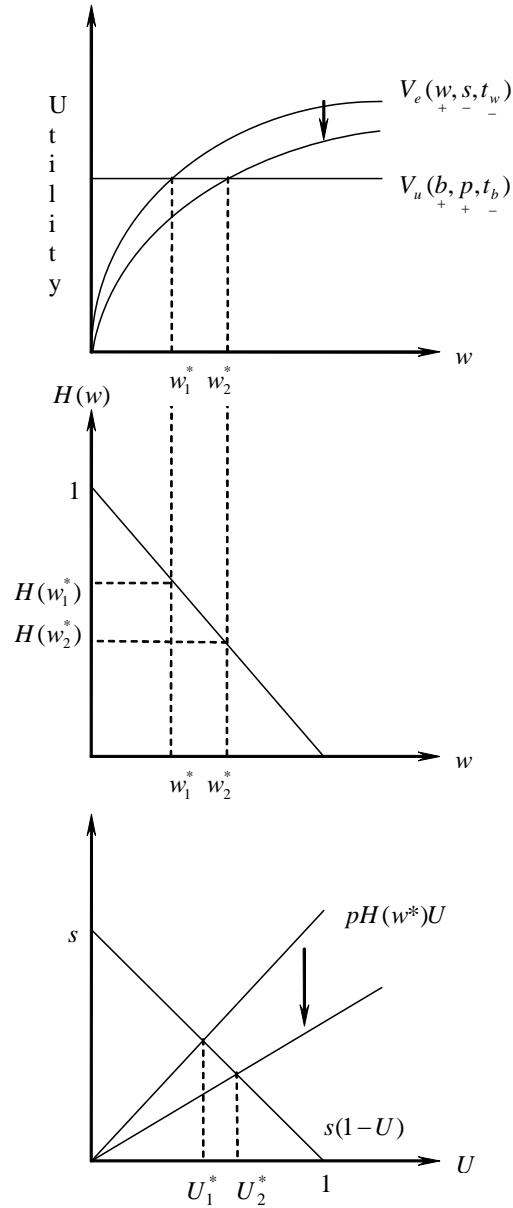
This increase can be a result of an improvement in information technology that facilitates a job search, or government policy that is successful at increasing the chances of the unemployed of finding a job, such as retraining programs.

As p goes up, the value of being unemployed increases, driving the reservation wage up (again the unemployed become more picky). As a result, a lower fraction of unemployed with job offers actually accept their job, that is, $H(w^*) \downarrow$. Let's consider the equilibrium unemployment rate $U^* = \frac{s}{pH(w^*)+s}$. What happens to this fraction? s remained unchanged. p went up but $H(w^*)$ went down. It is unclear whether the product $pH(w^*)$ increased or decreased. Thus,



2.5.3 An increase in labor income taxes ($t_w \uparrow$)

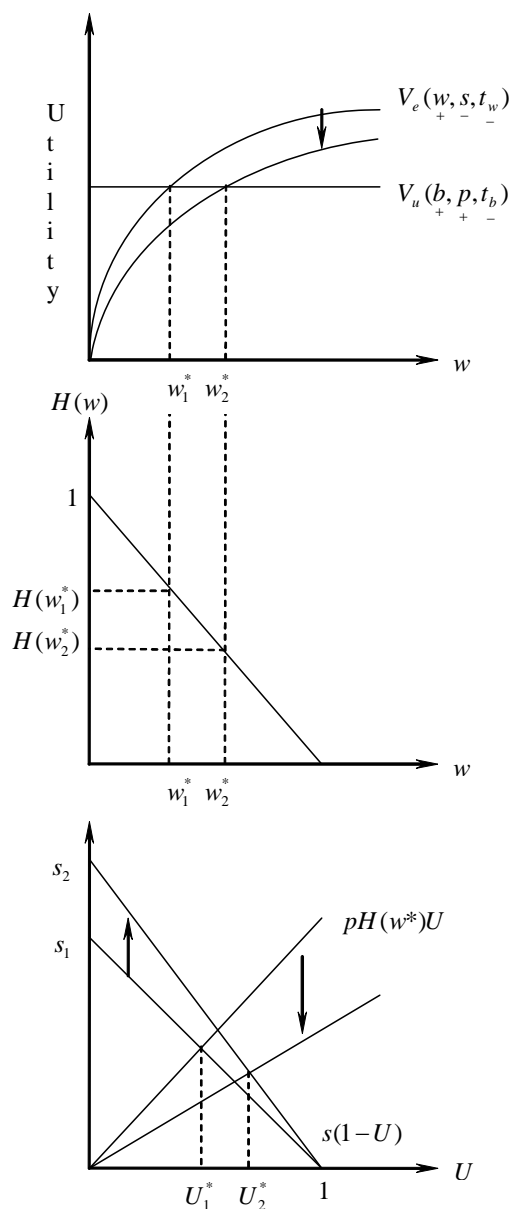
An increase in labor income taxes leads to a fall in the value of being employed for any given wage. As $V_e(w)$ shifts down, the reservation wage goes up and fewer of those unemployed with job offers actually accept their offer, that is, $H(w^*)$ goes down. So, the equilibrium unemployment rate $U^* = \frac{s}{pH(w^*)+s}$ increases. Thus,



2.5.4 An increase in separation rate ($s \uparrow$)

This increase can be a result of moving from a command economy (operated by government) to a market economy where the jobs are less secured and there is a higher risk of being separated from the job (since people are employed based on their skill and not based on their connections to the ruling party).

An increase in the separation rate decreases the utility being employed, so the value of being unemployed goes down, and the reservation wage goes up $w^* \uparrow$. As a result, the probability of getting offers that are accepted goes down $H(w^*) \downarrow$. Thus, the flow out of unemployment is reduced ($pH(w^*)$ goes down). At the same time the flow in to the unemployment goes up $s(1-U) \uparrow$. Thus,



Numerical Examples

Suppose that $V_e(w, s, t) = (1 - s) \sqrt{w(1 - t_w)}$, and $V_u(b, p, t_b) = 3p\sqrt{b(1 - t_b)}$, $b = 5$, $p = 0.6$, $s = 0.1$, $t_w = t_b = 0.3$, $H(w) = 1 - \frac{1}{100}w$.

1. Find the steady state unemployment rate.

Solution:

Step 1: find the reservation wage w^*

$$(1 - s) \sqrt{w(1 - t_w)} = 3p\sqrt{b(1 - t_b)}$$

Thus

$$\begin{aligned} w^* &= \left(\frac{3p}{1 - s} \right)^2 \frac{b(1 - t_b)}{(1 - t_w)} \\ &= \left(\frac{3 \cdot 0.6}{1 - 0.1} \right)^2 \left[\frac{5 \cdot (1 - 0.3)}{(1 - 0.3)} \right] \\ &= \left(\frac{1.8}{0.9} \right)^2 5 = 20 \end{aligned}$$

Step 2: Find the fraction of offers that are at least w^* (finding $H(w^*)$)

$$H(20) = 1 - 0.01 \cdot 20 = 0.8$$

Step 3: Find the steady state unemployment rate (U^*)

$$\begin{aligned} s(1 - U) &= pH(w^*)U \\ U^* &= \frac{s}{pH(w^*) + s} = \frac{0.1}{0.6 \cdot 0.8 + 0.1} = 0.172413793 \end{aligned}$$

2. Suppose that $p = 1$, so that everybody receives an offer. Find the steady state unemployment rate.

Solution:

Step 1: find the reservation wage w^*

$$(1 - s) \sqrt{w(1 - t_w)} = 3p\sqrt{b(1 - t_b)}$$

Thus

$$\begin{aligned} w^* &= \left(\frac{3p}{1 - s} \right)^2 \frac{b(1 - t_b)}{(1 - t_w)} \\ &= \left(\frac{3 \cdot 1}{1 - 0.1} \right)^2 \left[\frac{5 \cdot (1 - 0.3)}{(1 - 0.3)} \right] \\ &= \left(\frac{3}{0.9} \right)^2 5 = 55.55555556 \end{aligned}$$

Step 2: Find the fraction of offers that are at least w^* (finding $H(w^*)$)

$$H(55.55555556) = 1 - 0.01 \cdot 55.55555556 = 0.444444444$$

Step 3: Find the steady state unemployment rate (U^*)

$$\begin{aligned} s(1 - U) &= pH(w^*)U \\ U^* &= \frac{s}{pH(w^*) + s} = \frac{0.1}{0.6 \cdot 0.4444444444 + 0.1} = 0.183673469 \end{aligned}$$

So that the unemployment rate increased.