San Francisco State University Michael Bar

Money

1 What is Money?

We define money as the *medium of exchange* in the economy, i.e. a commodity or financial asset that is generally acceptable in exchange for goods and services. Currency which consists of coins and bank notes[1] are a medium of exchange. Checking accounts can also be used as a medium of exchange, since a consumer can write a check in exchange for goods. Travelers checks are another example of money. There are other assets where it is not clear if they should be considered as money, for example saving accounts. A consumer can withdraw from a saving account and pay for goods, but the main purpose saving accounts is to store value not to serve as a medium of exchange.

Are credit cards considered a form of money? The answer is no. If you buy goods from a supermarket using a credit card, the credit card company will pay the shopkeeper today and you will have an obligation to pay the credit card company when your credit card bill comes in. This obligation to the credit card company *does not represent money.* The money part of the transaction between you and the credit card company only comes into play when you pay your bill.

There are several definitions money aggregates:

1. **M1** - Measure of the U.S. money stock that consists of currency held by the public, travelers checks, demand deposits and other checkable deposits including NOW (negotiable order of withdrawal) and ATS (automatic transfer service) account balances and share draft account balances at credit unions.

2. **M2** - Measure of the U.S. money stock that consists of M1, certain overnight repurchase agreements and certain overnight Eurodollars, savings deposits (including money market deposit accounts), time deposits in amounts of less than $100,000 and balances in money market mutual funds (other than those restricted to institutional investors).

3. **M3** - Measure of the U.S. money stock that consists of M2, time deposits of $100,000 or more at all depository institutions, term repurchase agreements in amounts of $100,000 or more, certain term Eurodollars and balances in money market mutual funds restricted to institutional investors.

The most important monetary aggregate is M1 and is often referred to as *the money supply* in the economy.

[1] Take a look at $1 bill, or any other U.S. paper money. It is called "Federal Reserve Note".

2 The Demand for Money

There are many theories that try to explain how the quantity of money that households want to hold is determined. Most of the modern theories are quite complicated and cannot be presented in this class. We will describe two theories here: (1) Quantity Theory of Money[2], and (2) Money in the Utility Function.

2.1 Quantity Theory of Money

According to this theory, households want to hold money in proportion to the dollar value of goods produced in the economy. Let Y_t be the real GDP at time t and let P_t be the price level (GDP deflator). Thus, the nominal GDP is P_tY_t. The demand for money according to this theory is given by

$$M_t^D = k_t \cdot P_tY_t \tag{1}$$

where M_t^D is the demand for money and k_t is the propensity to hold money. Typically, $k_t < 1$ since each dollar can be used more than once every year, so if the households spend P_tY_t during year t, they need to keep only a fraction of their planned spending as money. In 2005 $k_t = 0.11$, which means that in 2005 households held money at the amount of 11% of the GDP. In other words, each dollar circulated 9 times during 2005 ($1/0.11 = 9$). We define the *velocity of money* as the average number of times a piece of money circulates during the year. The velocity is denoted by V_t and defined as

$$V_t = \frac{P_tY_t}{M_t^D}.$$

Thus, equation (1) can be written as

$$M_t^D V_t = P_tY_t \tag{2}$$

Notice that $V_t = 1/k_t$. In equilibrium, the money demand $\left(M_t^D\right)$ is equal to the money supply $\left(M_t^S\right)$, and denoted by M_t and the above can be written as

$$M_tV_t = P_tY_t \tag{3}$$

Equation (3) is called the *quantity equation*. It is important to realize that the quantity equation always holds for every economy in the world. This is simply because we define the velocity to be such that the above equation holds.

The quantity theory is silent about what determines the velocity V_t and real GDP Y_t, but nevertheless this equation is useful for relating money, inflation and real GDP. Divide equation (3) at time $t+1$ by the same equation at time t:

$$\begin{aligned} \frac{M_{t+1}V_{t+1}}{M_tV_t} &= \frac{P_{t+1}Y_{t+1}}{P_tY_t} \\ \left(1+\hat{M}\right)\left(1+\hat{V}\right) &= \left(1+\hat{P}\right)\left(1+\hat{Y}\right) \end{aligned} \tag{4}$$

[2] Sometimes called the Classical Theory.

where "hat" above a variable denotes its growth rate, for example $\hat{M} = (M_{t+1} - M_t)/M_t$. Taking logs of equation (4) gives

$$\ln\left(1+\hat{M}\right) + \ln\left(1+\hat{V}\right) = \ln\left(1+\hat{P}\right) + \ln\left(1+\hat{Y}\right)$$

For small growth rates the above is approximately

$$\hat{M} + \hat{V} = \hat{P} + \hat{Y} \tag{5}$$

Suppose that velocity is constant, i.e. $\hat{V} = 0$. Then we have

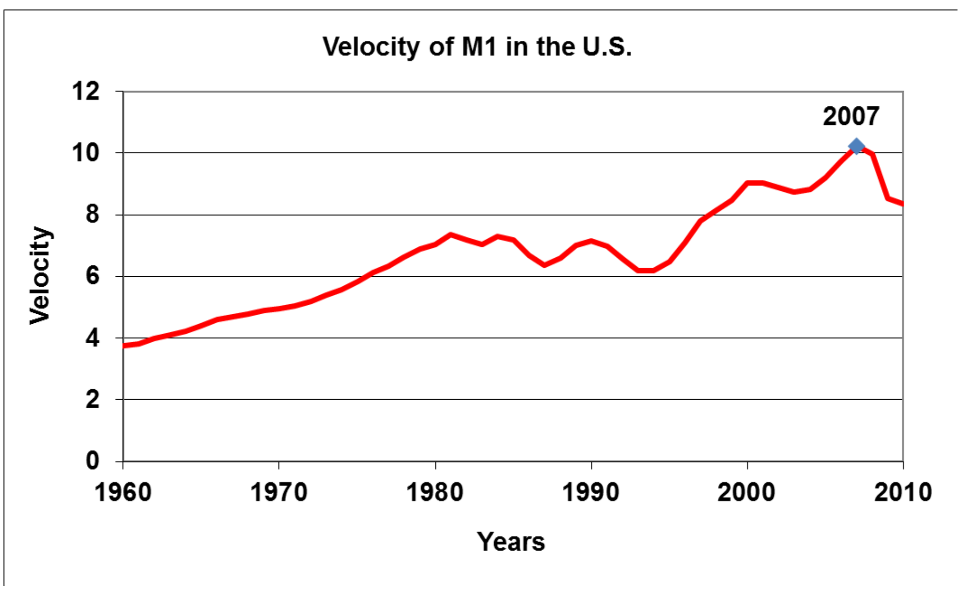
$$\hat{M} = \hat{P} + \hat{Y}$$

Rearranging

$$\hat{P} = \hat{M} - \hat{Y} \tag{6}$$

The growth rate of the price level is inflation $\pi = \hat{P}$. Equation (6) tells us that *if velocity is constant*, then the inflation rate in the economy is approximately equal to the growth rate of money supply minus the growth rate of the real GDP. For example, suppose that during 2005 the money supply increased by 4% and the growth rate of real GDP was 1.5%, then the inflation rate must be 2.5%, if velocity did not change during 2005.

The next figure shows the velocity in the U.S. since 1960.



Notice that velocity has increased during the time period in question by a factor of 2.5 (from 1960 to 2007). During the decade of 1995-2005 the velocity increased from 6 to 9, which is 50% increase. This means that each piece of money circulates more times than it used to in the past. In other words, people economize on money holdings. In the next section we attempt to develop a theory of velocity. Notice that after 2007, we experience a decline in velocity of money, due to recession (from the velocity formula, we see that $V_t = GDP_t/M_t$, and because of the ressesion the numerator experienced slow growth, while the denominator experiences fast growth). In other words, after 2007 we see that people are holding more money, but spending less.

2.2 Money in the Utility Function

Consumers derive utility from consumption C and real balances M/P. We can think of the later as liquidity services, or the purchasing power of the nominal money holdings. The opportunity cost of holding money is the nominal interest rate i. The consumer's income is equal to the nominal GDP (PY). The consumer's problem is therefore

$$\max_{C,M} \alpha \ln C + (1-\alpha) \ln \left(\frac{M}{P} \right)$$
$$s.t.$$
$$PC + iM = PY$$

Now divide the budget constraint by the price level

$$\max_{C,M} \alpha \ln C + (1-\alpha) \ln (M) - (1-\alpha) \ln P$$
$$s.t.$$
$$C + i\frac{M}{P} = Y$$

Notice that the last term in the utility function can be dropped because utility is invariant with respect to monotone transformations. Hence, the consumer's problem is

$$\max_{C,M} \alpha \ln C + (1-\alpha) \ln (M)$$
$$s.t.$$
$$C + i\frac{M}{P} = Y$$

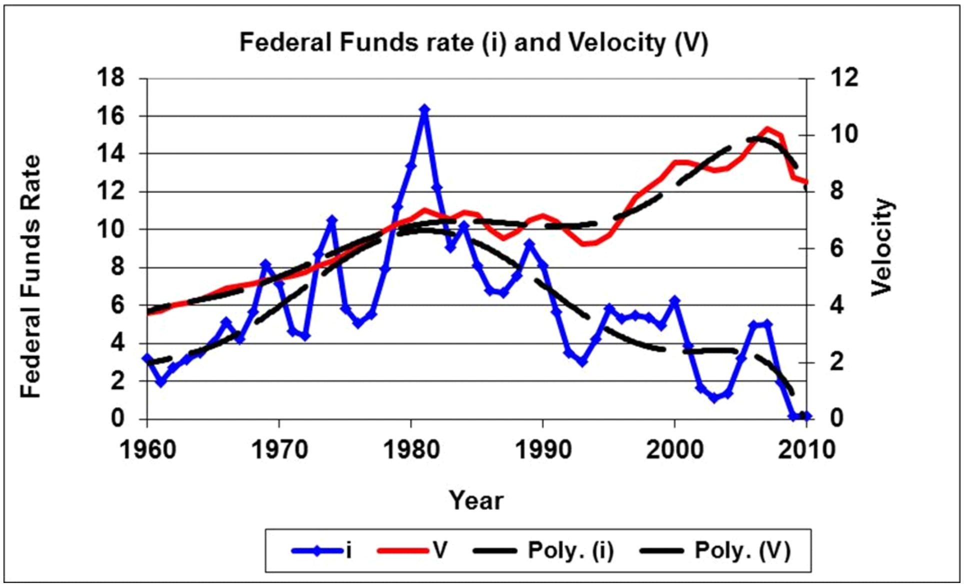
We know that when preferences are of the Cobb-Douglas form, the demand is

$$\begin{aligned} C &= \alpha Y \\ M^D &= \frac{(1-\alpha)}{i} PY \end{aligned}$$

The demand for money that we derived, is increasing GDP and decreasing in nominal interest rate - which represents the cost of holding money. Thus, according to this model the velocity is

$$V = \frac{i}{1-\alpha}$$

This result makes intuitive sense: as interest rate goes up, the opportunity cost of holding money goes up and the households economize on money holdings. As a result, each piece of money is used more times during the year. The next figure shows the graphs of velocity and federal funds rate since 1960.



The dashed lines are polynomial trends (degree 6). As we see from the above figure, until early 80's the trends of velocity and interest rates move in the same direction, as the model predicts. After 1981 however, the trends move in the opposite directions. The only way that our model can reconcile this observation is by increasing α. Recall that α is the weight of consumption in the utility function while $1-\alpha$ is the weight on real balances in the utility function. Decline in $1-\alpha$ means that liquidity services provided by the real balances are not as valuable as before. Our model cannot offer any explanation why this might happen though. One might conjecture that the sharp increase in velocity in the last decade (50% increase) has something to do with innovations in the banking and payment system. These include ATM machines, electronic transfers, etc. Anything that allows faster payments and smaller average holdings of money will increase the velocity. For example, if households receive income every two weeks instead of every month, the average money holding will be lower and velocity higher. Notice that since 2007, velocity and interest rate are moving again in the same direction. This trend can be accounted in the above model by increasing $1-\alpha$, (equivalently a decline in α), which means that people hold more money and spend less.

3 Money Supply

In this section we explain how the Federal Reserve System[3] (FED for short) and the commercial banks create money. We define the Monetary Base (MB) as all the coins and paper money that is created by the FED. The Monetary Base is held partly as reserves (R) of the commercial banks and partly as currency in the hands of public (CU). Thus,

$$MB = R + CU$$

The money supply M consists of currency and checking deposits, which is the $M1$ money aggregate defined above. Thus

$$M = CU + D$$

[3] To learn about the history, the structure and the activities of the Federal Reserve System, visit http://www.federalreserveeducation.org/fed101/index.htm.

The commercial banks are required to keep certain minimum percentage of deposits (D) as liquid reserves of cash and currency. This fraction is called the **required reserve ratio** and denoted by rd, where

$$rd = \frac{R}{D}$$

Suppose that consumers want to hold certain amount of cash, as a fraction of their deposits. Let this fraction (currency/deposits ratio) be cd, that is

$$cd = \frac{CU}{D}$$

Under these assumptions, M, D, R, and CU are all proportional to the monetary base.

$$\frac{M}{MB} = \frac{D + CU}{R + CU} = \frac{1 + cd}{rd + cd}$$

The last equality is obtained by dividing the numerator and the denominator by D. Similarly,

$$\begin{aligned} \frac{D}{MB} &= \frac{D}{R + CU} = \frac{1}{rd + cd} \\ \frac{R}{MB} &= \frac{R}{R + CU} = \frac{rd}{rd + cd} \\ \frac{CU}{MB} &= \frac{CU}{R + CU} = \frac{cd}{rd + cd} \end{aligned}$$

Thus

$$\begin{aligned} \Delta M &= \left(\frac{1 + cd}{rd + cd}\right) \Delta MB \\ \Delta D &= \left(\frac{1}{rd + cd}\right) \Delta MB \\ \Delta R &= \left(\frac{rd}{rd + cd}\right) \Delta MB \\ \Delta CU &= \left(\frac{cd}{rd + cd}\right) \Delta MB \end{aligned}$$

The magnitude $mm = \left(\frac{1+cd}{rd+cd}\right)$ is called the money multiplier, and it gives us the change in the money supply that results from \$1 change in the monetary base.

Now we are ready to illustrate how the FED, together with the commercial banks, affects the money supply. The FED has several policy instruments, the most popular of which is open market operations. An open market operation is purchasing and selling government bonds. Every purchase of bonds by the FED increases the monetary base (injects money into the economy). Because of the partial reserve requirements, the commercial banks can lend some of the money received from the FED and thereby generating additional money. The FED can also alter the reserve requirement ratio, but it rarely does so. We will demonstrate the two modes of operation in the next example.

3.1 Example of Money Creation

Suppose the currency/deposit ratio that the public wants is 20% and the reserve requirement ratio is 10%. The following is a consolidated balance sheet of the commercial banks.

Balance sheet of the commercial banks

Assets	Liabilities
$R = 10$	$D = 100$
$B_G = 25$	
$L = 65$	
100	100

where R is the reserves, B_G government bonds, L is loans, and D is deposits. Notice that the balance sheet must always be balanced. Also observe that the banks conform to the reserve requirement and indeed $R/D = 0.1$.

1. Find the monetary base in this economy.

$$\begin{aligned} MB &= CU + R \\ CU &= cd \cdot D = 0.2 \cdot 100 = 20 \\ R &= 10 \end{aligned}$$

   Thus

$$MB = 20 + 10 = 30$$

2. Find the money supply in this economy

$$M = CU + D = 20 + 100 = 120$$

3. Find the money multiplier in this economy.

$$mm = \left(\frac{1 + cd}{rd + cd}\right) = \left(\frac{1 + 0.2}{0.1 + 0.2}\right) = 4$$

4. Now suppose that the FED performs an open market operation and buys government bonds at the amount of 5. Find the new monetary base, the money supply and describe the new balance sheet of the commercial banks.

$$\begin{aligned} MB &= 30 + 5 = 35 \\ \Delta M &= \left(\frac{1 + cd}{rd + cd}\right) \Delta MB = \left(\frac{1 + 0.2}{0.1 + 0.2}\right) \cdot 5 = 20 \\ \Delta D &= \left(\frac{1}{rd + cd}\right) \Delta MB = \left(\frac{1}{0.1 + 0.2}\right) \cdot 5 = 16\frac{2}{3} \\ \Delta R &= \left(\frac{rd}{rd + cd}\right) \Delta MB = \left(\frac{0.1}{0.1 + 0.2}\right) \cdot 5 = 1\frac{2}{3} \\ \Delta CU &= \left(\frac{cd}{rd + cd}\right) \Delta MB = \left(\frac{0.2}{0.1 + 0.2}\right) \cdot 5 = 3\frac{1}{3} \end{aligned}$$

Thus

$$
\begin{aligned}
M &= 120 + 20 = 140 \\
D &= 100 + 16\frac{2}{3} = 116\frac{2}{3} \\
R &= 10 + 1\frac{2}{3} = 11\frac{2}{3} \\
CU &= 20 + 3\frac{1}{3} = 23\frac{1}{3}
\end{aligned}
$$

Balance sheet of the commercial banks

Assets	Liabilities
$R = 11\frac{2}{3}$	$D = 116\frac{2}{3}$
$B_G = 20$	
$L = 85$	
$116\frac{2}{3}$	$116\frac{2}{3}$

The loans are simply set to balance the balance sheet. Notice that when the FED increased the monetary base by 5, the money supply increased by 20. This illustrates the fact that the FED does not have direct control over the money supply, but rather the commercial banks together with the FED create the money supply.

5. Suppose that instead of the open market operation, the FED sets the required reserve ratio to 5%. Find the new monetary base, money multiplier, the money supply and describe the new balance sheet of the commercial banks.

The monetary base does not change, because the FED did not buy or sell any asset.

$$
\begin{aligned}
MB &= 30 \\
mm &= \frac{1+cd}{rd+cd} = \frac{1+0.2}{0.05+0.2} = 4.8 \\
M &= \left(\frac{1+cd}{rd+cd}\right) MB = \left(\frac{1+0.2}{0.05+0.2}\right) \cdot 30 = 144 \\
D &= \left(\frac{1}{rd+cd}\right) MB = \left(\frac{1}{0.05+0.2}\right) \cdot 30 = 120 \\
R &= \left(\frac{rd}{rd+cd}\right) MB = \left(\frac{0.05}{0.05+0.2}\right) \cdot 30 = 6 \\
CU &= \left(\frac{cd}{rd+cd}\right) MB = \left(\frac{0.2}{0.05+0.2}\right) \cdot 30 = 24
\end{aligned}
$$

Balance sheet of the commercial banks

Assets	Liabilities
$R = 6$	$D = 120$
$B_G = 25$	
$L = 89$	
120	120

Just to check that we did not make an error, lets verify that the consumers hold currency/deposits at the right ratio, and also that the banks hold reserves/deposits at the right ratio:

$$\frac{CU}{D} = \frac{24}{120} = 0.2$$
$$\frac{R}{D} = \frac{6}{120} = 0.05$$

As was mentioned before, the FED does not use the second type of policy (changing the required reserve ratio) frequently. One of the goals of the FED is to maintain a stable banking system and if the required reserve ratio changes, the banks have to adjust their loans and deposits in a complicated way.

4 Illustration of the Money Multiplier

In this section we show in detail how an open market operation affects the balance sheet of the commercial banks and the money supply. The following steps illustrate the working of the money multiplier when the FED buys bonds at the amount of x from the commercial banks.

	$CU = 20$
0.	$CU = 20$
3.	$+cd\frac{x}{1+cd}$
4.	$+cd\frac{(1-rd)x}{(1+cd)^2}$
5.	$+cd\frac{(1-rd)^2 x}{(1+cd)^3}$
$\vdots$	$\vdots$

Balance Sheet of Banks

	Assets		Liabilities
0.	$R = 10$	0.	$D = 100$
1.	$+x$	3.	$+\frac{x}{1+cd}$
2.	$-x$	4.	$+\frac{(1-rd)x}{(1+cd)^2}$
3.	$+rd\frac{x}{1+cd}$	5.	$+\frac{(1-rd)^2 x}{(1+cd)^3}$
4.	$+rd\frac{(1-rd)x}{(1+cd)^2}$	$\vdots$	$\vdots$
5.	$+rd\frac{(1-rd)^2 x}{(1+cd)^3}$		
$\vdots$	$\vdots$		
0.	$B_G = 25$		
1.	$-x$		
0.	$L = 65$		
2.	$+x$		
3.	$+\frac{(1-rd)x}{1+cd}$		
4.	$+\frac{(1-rd)^2 x}{(1+cd)^2}$		
5.	$+\frac{(1-rd)^3 x}{(1+cd)^3}$		
$\vdots$	$\vdots$		

Each step is numbered. Step "0" is the initial balance sheet. It is important to make sure that the balance sheet is balanced after each step, i.e. assets are equal to liabilities. In step 1 the FED buys bonds at the amount of x from the commercial banks. As a result, the bonds decreased by x and the reserves increased by x. Now after step 1, the commercial banks have too much reserves and they can lend the extra reserves to households. As a result of step 2 the reserves decreased by x while loans increased by x. In step 3, the loans at the amount of x are distributed between currency and deposits such that $\Delta CU/\Delta D = cd$ and $\Delta CU + \Delta D = x$. Also, the commercial banks keep a fraction rd of the new deposits in reserves and lend the rest (a fraction $1 - rd$ of the new deposits). Observe that all the changes in step 3 in the balance sheet keep it balanced. The same is repeated in step 4, that is the extra loans are distributed between CU and D according to the currency/deposits ratio, and a fraction rd of the new deposits is kept in reserves while the rest is used for extra loans. This process continues indefinitely. In order to follow the above process more easily, color each step in a different color and make sure that the impact of each step on the balance sheet keeps it balanced.

Suppose that the above steps continue forever. We can use the rule of summation of an infinite geometric series

$$\sum_{t=0}^{\infty} q^t = \frac{1}{1-q} \qquad (0 < q < 1)$$

to get the following results:

$$\begin{aligned}
\Delta D &= \frac{x}{1+cd}\sum_{t=0}^{\infty}\left(\frac{1-rd}{1+cd}\right)^t = \frac{x}{1+cd}\left(\frac{1}{1-\frac{1-rd}{1+cd}}\right) \\
&= \frac{x}{1+cd}\left(\frac{1}{\frac{1+cd-1+rd}{1+cd}}\right) = \left(\frac{1}{rd+cd}\right)x \\
\Delta CU &= cd\cdot\Delta D = \left(\frac{cd}{rd+cd}\right)x \\
\Delta M &= \Delta CU + \Delta D = \left(\frac{1+cd}{rd+cd}\right)x \\
\Delta R &= rd\cdot\Delta D = \left(\frac{rd}{rd+cd}\right)x
\end{aligned}$$

Thus, we derived all the multipliers in section 3 as the limit when $t \to \infty$ of the sequence of loans and deposits generated by the open market operation.

We can also compute all the magnitudes above after T steps. This is more realistic because in the real world the money circulates only limited number of times per year. In particular, the money velocity in 2005 is 9. Thus, we would like to compute the summations of the first 9 steps. This can be done using the rule summation of a finite geometric series

$$\sum_{t=0}^{T} q^t = \frac{1-q^{T+1}}{1-q}$$

This gives

$$\begin{aligned}
\Delta D &= \frac{x}{1+cd}\sum_{t=0}^{T}\left(\frac{1-rd}{1+cd}\right)^t = \frac{x}{1+cd}\left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^{T+1}}{1-\frac{1-rd}{1+cd}}\right) \\
&= \frac{x}{1+cd}\left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^{T+1}}{\frac{1+cd-1+rd}{1+cd}}\right) = \left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^{T+1}}{rd+cd}\right)x \\
\Delta CU &= cd\cdot\Delta D = cd\cdot\left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^{T+1}}{rd+cd}\right)x \\
\Delta M &= \Delta CU + \Delta D = (1+cd)\cdot\left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^{T+1}}{rd+cd}\right)x \\
\Delta R &= rd\cdot\Delta D = rd\cdot\left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^{T+1}}{rd+cd}\right)x
\end{aligned}$$

For example, if the FED buys bonds at the amount of 5, then after 9 rounds of loans and deposits we have

$$
\begin{aligned}
\Delta D &= \left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^9}{rd+cd}\right)\cdot 5 \\
\Delta CU &= cd\cdot\left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^9}{rd+cd}\right)\cdot 5 \\
\Delta M &= (1+cd)\cdot\left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^9}{rd+cd}\right)\cdot 5 \\
\Delta R &= rd\cdot\left(\frac{1-\left(\frac{1-rd}{1+cd}\right)^9}{rd+cd}\right)\cdot 5
\end{aligned}
$$

This completes the illustration of money creation by the FED and the commercial banks.