Macroeconomics, ECON 302
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<u>Midterm1, sec 2</u>

Thursday, February 26

1 hour and 15 minutes

Name: ____Answer Key________________________

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (10 points). Suppose that GDP in the U.S. is 3 times as large as that of China. Also suppose that the U.S. GDP grows at constant rate of 2% per year, while Chinese GDP grows at 7% per year. How many years would it take for China to catch up with the U.S. in terms of GDP? (**Instructions:** Since you don't have calculators, leave the unknown variable *t* as expression that could be calculated with a calculator).

$$GDP_{US}(1+0.02)^t = GDP_{CHN}(1+0.07)^t$$
$$3 \cdot GDP_{CHN}(1+0.02)^t = GDP_{CHN}(1+0.07)^t$$
$$3 \cdot (1+0.02)^t = (1+0.07)^t$$
$$\ln(3) + t\ln(1.02) = t\ln(1.07)$$
$$t = \frac{\ln(3)}{\ln(1.07) - \ln(1.02)}$$

2. (10 points). If a variable grows at constant rate, then the natural log of the variable is
   a. Increasing function of time
   b. Decreasing function of time
   c. Linear function of time
   d. Quadratic function of time
   e. Impossible to tell without more information

3. (15 points). The next table provides data on prices and output in some artificial economy for the years 2000 – 2002. The goods are labeled 1 and 2, so that P_1, P_2, Q_1, Q_2 are prices and quantities of the two goods respectively.

Year	P_1	Q_1	P_2	Q_2
2000	2	50	30	5
2001	3	57	30	19

Calculate the inflation rate between the years 2000 and 2001, using 2001 as the base year.

$P_{2001} = 1$ (base year).

$$P_{2000} = \frac{GDP_{2000}}{RGDP_{2000}} = \frac{2 \cdot 50 + 30 \cdot 5}{3 \cdot 50 + 30 \cdot 5} = \frac{250}{300} = \frac{5}{6}$$

Inflation: $\pi_{2000-2001} = \frac{P_{2001}}{P_{2000}} - 1 = \frac{6}{5} - 1 = \frac{1}{5} = 20\%$

4. (30 points). Suppose that consumer's utility function is $u(x, y)$. The prices of goods X and Y are p_x, p_y and his income is I.
   a. Write the consumer's problem.

$$\max_{x,y} u(x, y)$$
$$s.t.$$
$$p_x x + p_y y = I$$

   b. Using the Lagrange method, derive the **mathematical condition** for optimality of consumption bundle.

$$L = u(x, y) - \lambda[p_x x + p_y y - I]$$
$$(1).\ L_x = u_x(x, y) - \lambda p_x = 0$$
$$(2).\ L_y = u_y(x, y) - \lambda p_y = 0$$

Dividing (1) by (2) gives

$$\frac{U_x(x, y)}{U_y(x, y)} = \frac{p_x}{P_y}$$

   c. Give a verbal interpretation of the above condition.

Interpretation 1: The left hand side is the slope (in absolute value) of the indifference curve and the right hand side is the slope (in absolute value) of the budget constraint. Thus, at the optimum, the indifference curve is tangent to the budget constraint.

Interpretation 2: The above condition can be written as

$$\frac{U_x(x, y)}{p_x} = \frac{U_y(x, y)}{P_y}$$

The left hand side is the utility generated by extra dollar spent on X and the right hand side is the utility from extra dollar spent on Y. The optimal allocation of income between the two goods requires that those should be the same.

d. Suppose that the utility function is $u(x, y) = x \cdot y$. Write the consumer's demand for *X* and *Y*.

$$x = 0.5 \cdot \frac{I}{p_x}$$
$$y = 0.5 \cdot \frac{I}{p_y}$$

e. Suppose that the consumer's income is $I = \$100$, and the prices are $p_x = \$10, \quad p_y = \$25$. Can the consumer in section d afford the consumption bundle $(x, y) = (10,0)$? Prove your answer.

The budget constraint is: $10x + 25y = 100$.
The consumption bundle $(x, y) = (10,0)$ satisfies the budget constraint:
$10 \cdot 10 + 25 \cdot 0 = 100$

f. Does the bundle $(x, y) = (10,0)$ satisfy the condition for optimality you found in section b. Prove your answer.

No, MRS is not equal to the price ratio at the above bundle.

$$\frac{U_x(x, y)}{U_y(x, y)} = \frac{p_x}{P_y}$$
$$\frac{y}{x} = \frac{10}{25}$$
$$\frac{0}{10} \neq \frac{10}{25}$$

5. (20 points). Suppose that a firm operates a technology given by the following production function $F(K,L) = 100K^{0.3}L^{0.7}$.
   a. Prove that this technology exhibits constant returns to scale.

$$F(\lambda K, \lambda L) = 100(\lambda K)^{0.3}(\lambda L)^{0.7} = 100\lambda^{0.3}K^{0.3}\lambda^{0.7}L^{0.7} = \lambda 100K^{0.3}L^{0.7} = \lambda F(K,L)$$

   b. Derive the marginal products of capital and labor.

$$MP_K = \frac{\partial F(K,L)}{\partial K} = 0.3 \cdot 100K^{0.3-1}L^{0.7}$$
$$MP_L = \frac{\partial F(K,L)}{\partial L} = 0.7 \cdot 100K^{0.3}L^{0.7-1}$$

   c. Prove that in perfect competition (if being a price taker), the above firm pays 30% of it's output to capital and 70% of it's output to labor.

A price taking firm pays the inputs their marginal product,

$$r = MP_K$$
$$w = MP_L$$

The payment to capital: $r \cdot K = 0.3 \cdot 100K^{0.3-1}L^{0.7} \cdot K = 0.3 \cdot 100K^{0.3}L^{0.7} = 0.3Y$

The payment to labor: $w \cdot L = 0.7 \cdot 100K^{0.3}L^{0.7-1} \cdot L = 0.7 \cdot 100K^{0.3}L^{0.7} = 0.7Y$