Macroeconomics, ECON 302 **San Francisco State University**
Michael Bar **Fall 2011**

Midterm Exam 1

Tuesday, September 27

1 hour and 15 minutes

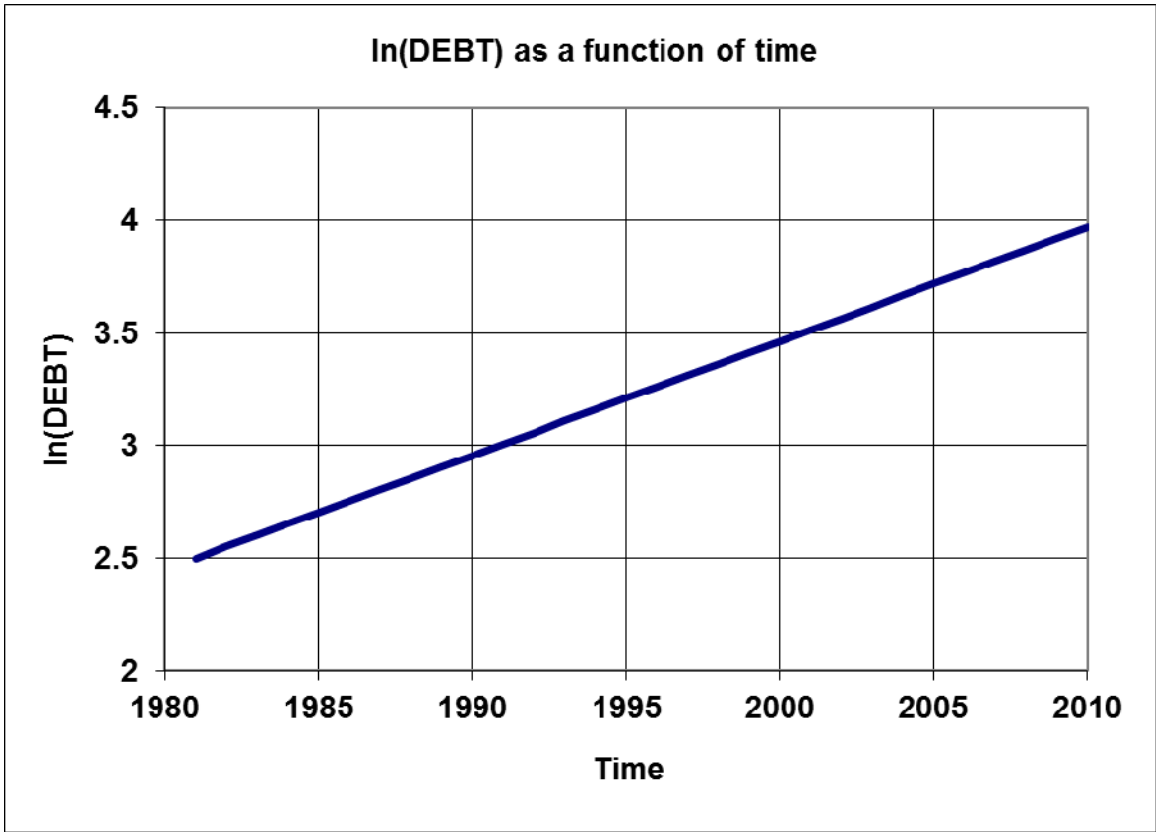
Name: ____Answer Key_____

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (15 points). The next graph shows the ln of the U.S. national debt over the 30 years period 1981 – 2010.



a. Based on the above graph (circle the correct answer):
   i. The ln of debt growing at constant rate
   ii. The debt is growing at increasing rate
   iii. The debt is growing at constant rate
   iv. The debt is growing at decreasing rate
b. Provide a brief (one sentence) explanation for your chosen answer in the previous section.

The $\ln(DEBT)$ is a linear function of time.

c. Based on the above graph, what is the approximate average growth rate of the debt over the given period? Show your calculations

The slope of $\ln(DEBT)$ is approximately equal to the growth rate of the debt:

$$g(DEBT) \approx \frac{4-2.5}{30} = \frac{1.5}{30} = 0.05 = 5\% \text{ per year.}$$

2. (15 points). Suppose that GDP in the U.S. is twice a large as that of China. Also suppose that the U.S. GDP is not growing at all, while Chinese GDP grows at 7% per year.
    a. How many years would it take China to catch up with the U.S. in terms of GDP? (**Instructions:** simplify your equations up to the point when you must use a calculator).

$$GDP_{US}(1+0)^t = GDP_{CHN}(1+0.07)^t$$
$$2GDP_{CHN} = GDP_{CHN}(1+0.07)^t$$
$$2 = (1+0.07)^t$$
$$\ln(2) = t\ln(1.07)$$
$$t = \frac{\ln(2)}{\ln(1.07)}$$

    b. Using the "rule of 70", give approximate answer to the previous question.

$$t \approx \frac{70}{7} = 10 \ \textit{years}$$

    c. Using the "rule of 70" again, how many years approximately would it take China to catch up with the U.S., if the U.S. GDP grows at 3.5% per year?

$$t \approx \frac{70}{(7-3.5)} = 20 \ \textit{years}$$

Now China is catching up with the U.S. at 3.5% rate.

3. (10 points). The next table provides data on prices and output in some artificial economy for the years 2000 – 2001. The goods are labeled 1 and 2, so that P_1, P_2, Q_1, Q_2 are prices and quantities of the two goods respectively.

Year	P_1	Q_1	P_2	Q_2
2000	2	50	20	10
2001	3	50	25	10

a. Calculate the growth rate of real GDP between the years 2000 and 2001, using 2000 as the base year.

$$RGDP_{2000} = 2 \cdot 50 + 20 \cdot 10 = 300$$
$$RGDP_{2001} = 2 \cdot 50 + 20 \cdot 10 = 300$$
$$\frac{RGDP_{2001}}{RGDP_{2000}} = \frac{300}{300} = 1$$

The net growth rate is zero:

$$g(RGDP) = \frac{RGDP_{2001}}{RGDP_{2000}} - 1 = 0$$

Notice that this result is not surprising, since there is no growth in output of the two goods.

b. Calculate the inflation rate between the years 2000 and 2001, using 2000 as the base year.

Nominal GDP in 2001: $GDP_{2001} = 3 \cdot 50 + 25 \cdot 10 = 400$

Price level in 2001: $P_{2001} = \frac{GDP_{2001}}{RGDP_{2001}} = \frac{400}{300} = \frac{4}{3}$

Price level in 2000: $P_{2000} = 1$ (price level in the base year is always 1)

Inflation between 2000 and 2001: $\pi_{2000-2001} = \frac{P_{2001}}{P_{2000}} - 1 = \frac{4}{3} - 1 = \frac{1}{3} = 33.33\%$

4. (10 points). The next table presents data from the National Income and Product accounts of some country.

Personal Consumption Expenditures	6,000
Gross Investment	2,000
Government Consumption Expenditures	1,500
Exports	500
Imports	1000
Compensation of Employees	5,000
Net Interest	1,000
Rental Income	500
Proprietors' income	500
Corporate profits	600
Indirect business tax net of subsidies	500
Consumption of fixed capital	1000
Statistical Discrepancy	-100

a. Based on the above data calculate the GDP using the expenditure approach.

$$GDP_{EX} = C + I + G + X - IM = 6{,}000 + 2{,}000 + 1{,}500 - 500 = 9{,}000$$

b. Calculate the GDP using the Income approach. (Self check: I provided the number for the statistical discrepancy so you can check that your calculations are correct. The statistical discrepancy is the difference between the GDP calculated by expenditure approach and the GDP calculated by the income approach).

$$GDP_{INC} = W + Int + \mathrm{Re}nt + \pi_P + \pi_B + Ti + Dep$$
$$= 5{,}000 + 1{,}000 + 500 + 500 + 600 + 500 + 1{,}000 = 9{,}100$$

5. (35 points). Suppose that consumer derives utility from two goods, X and Y, and has a utility function $u(x, y)$. The prices of goods are P_x, P_y and his income is I.

   a. Write the consumer's optimization problem.

$$\max_{x,y} u(x, y)$$
$$s.t.$$
$$p_x x + p_y y = I$$

   b. Write the Lagrange function associated with the consumer's optimization problem.

$$L = u(x, y) - \lambda\left[p_x x + p_y y - I\right]$$

   c. Derive the mathematical condition for optimal consumption bundle, assuming that indifference curves are convex.

The first order conditions for the above Lagrange function:

$$L_x = U_x(x, y) - \lambda p_x = 0 \quad \Rightarrow \qquad U_x(x, y) = \lambda p_x$$
$$L_y = U_y(x, y) - \lambda p_y = 0 \quad \Rightarrow \qquad U_y(x, y) = \lambda p_y$$

Dividing the two conditions:

$$\frac{U_x(x, y)}{U_y(x, y)} = \frac{p_x}{p_y}$$

   d. Give a verbal interpretation of the above condition.

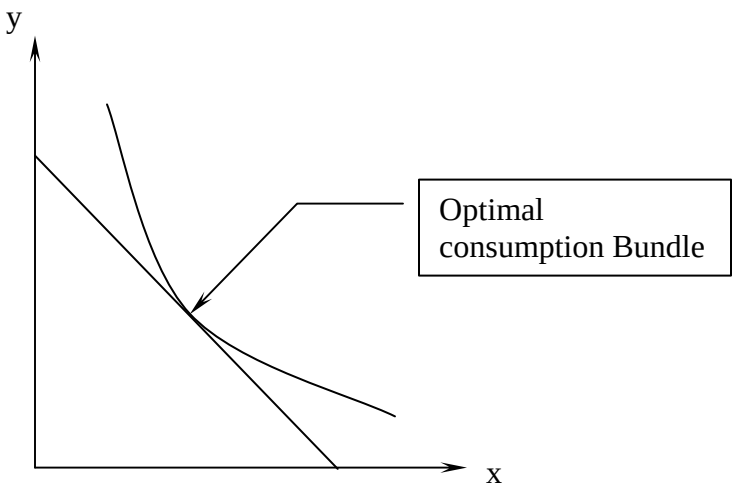
Interpretation 1: The left hand side is the slope (in absolute value) of the indifference curve and the right hand side is the slope (in absolute value) of the budget constraint. Thus, at the optimum, the indifference curve is tangent to the budget constraint.

Interpretation 2: The above condition can be written as

$$\frac{U_x(x, y)}{p_x} = \frac{U_y(x, y)}{P_y}$$

The left hand side is the utility generated by extra dollar spent on X and the right hand side is the utility from extra dollar spent on Y. The optimal allocation of income between the two goods requires that those should be the same.

e. Illustrate the condition you derived in part c with a graph of the budget constraint and indifference curve.



f. Suppose that the utility function is $u(x,y) = x^{0.5}y^{0.5}$. Write the consumer's demand functions.

$$x^* = 0.5\frac{I}{p_x}$$
$$y^* = 0.5\frac{I}{p_y}$$

g. Suppose that the utility function is $v(x,y) = 40xy$. This consumer spends 40 percent of his income on good X and 60 percent of his income on Y, regardless of the prices of these goods, and regardless of his income. True/(False), circle the correct answer and provide a brief explanation.

This utility function is obtained by applying a **monotone increasing transformation** to $u(x,y) = x^{0.5}y^{0.5}$, which we know leads to the demand in the last section and implies spending 50% of income on each good. The transformation is: $v(x,y) = 40 \cdot u(x,y)^2$. Thus the utility functions $u(x,y) = x^{0.5}y^{0.5}$ and $v(x,y) = 40xy$ yield the same demand functions and imply spending equal fractions of income (50%) on each good.

6. (15 points). Suppose that a firm operates a technology given by the following production function $F(K,L) = 100K^{0.3}L^{0.7}$.
    a. Prove that this technology exhibits constant returns to scale.

$$F(\lambda K, \lambda L) = 100(\lambda K)^{0.3}(\lambda L)^{0.7} = 100\lambda^{0.3}K^{0.3}\lambda^{0.7}L^{0.7} = \lambda 100K^{0.3}L^{0.7} = \lambda F(K,L)$$

    b. Derive the marginal products of capital and labor.

$$MP_K = \frac{\partial F(K,L)}{\partial K} = 0.3 \cdot 100K^{0.3-1}L^{0.7}$$
$$MP_L = \frac{\partial F(K,L)}{\partial L} = 0.7 \cdot 100K^{0.3}L^{0.7-1}$$

    c. Prove that in perfect competition (if being a price taker), the above firm pays 30% of it's output to capital and 70% of it's output to labor.

A price-taking firm pays the inputs their marginal product,

$$r = MP_K$$
$$w = MP_L$$

The payment to capital: $r \cdot K = 0.3 \cdot 100K^{0.3-1}L^{0.7} \cdot K = 0.3 \cdot 100K^{0.3}L^{0.7} = 0.3Y$

The payment to labor: $w \cdot L = 0.7 \cdot 100K^{0.3}L^{0.7-1} \cdot L = 0.7 \cdot 100K^{0.3}L^{0.7} = 0.7Y$