

Problem set 6 - Answer key

Investment

1. (30 points). Consider the model of optimal investment discussed in class.  
a. Write the firm's maximization problem.

$$\left\{ \begin{array}{l} \max_{L_1, L_2, I, K_2} V = A_1 K_1^\theta L_1^{1-\theta} - w_1 L_1 - I + \frac{A_2 K_2^\theta L_2^{1-\theta} + (1-\delta)K_2 - w_2 L_2}{1+r} \\ s.t. \\ K_2 = (1-\delta)K_1 + I \end{array} \right.$$

- b. Explain in words what the firm wants to maximize.

The firm wants to maximize the present value of the stream of dividends:

$$V = \pi_1 + \frac{\pi_2}{1+r}$$

- c. According to this model, what should be stock price of the firm?

The stock price should be the maximized value of  $V$ .

- d. Derive the optimal investment condition and provide economic interpretation of it.

Substituting the constraint into the objective gives

$$\max_{L_1, L_2, I, K_2} V = A_1 K_1^\theta L_1^{1-\theta} - w_1 L_1 - K_2 + (1-\delta)K_1 + \frac{A_2 K_2^\theta L_2^{1-\theta} + (1-\delta)K_2 - w_2 L_2}{1+r}$$

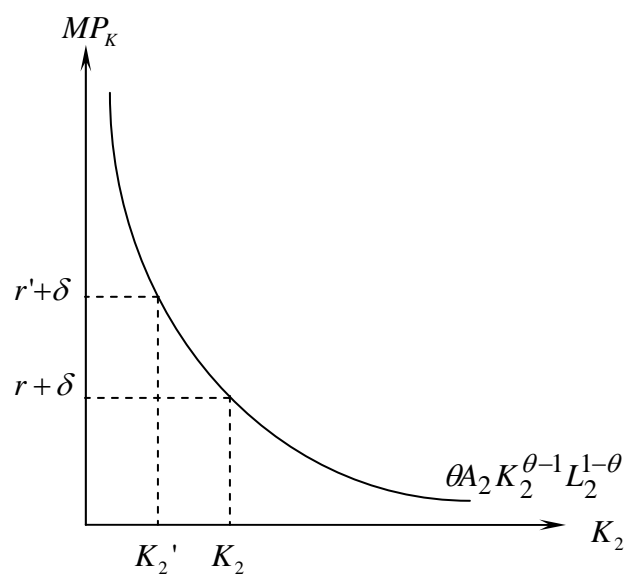
F.O.C. for  $K_2$ :

$$\frac{\partial V}{\partial K_2} = -1 + \frac{\theta A_2 K_2^{\theta-1} L_2^{1-\theta} + 1 - \delta}{1+r} = 0$$

At the optimum, the cost of increasing future capital by 1 unit must be equal to the benefit from that extra unit of capital. The benefit in the next period consists of the marginal product of capital and the non-depreciated value of the extra unit of capital. Dividing the next period's benefit by  $1+r$  gives the present value of the benefit.

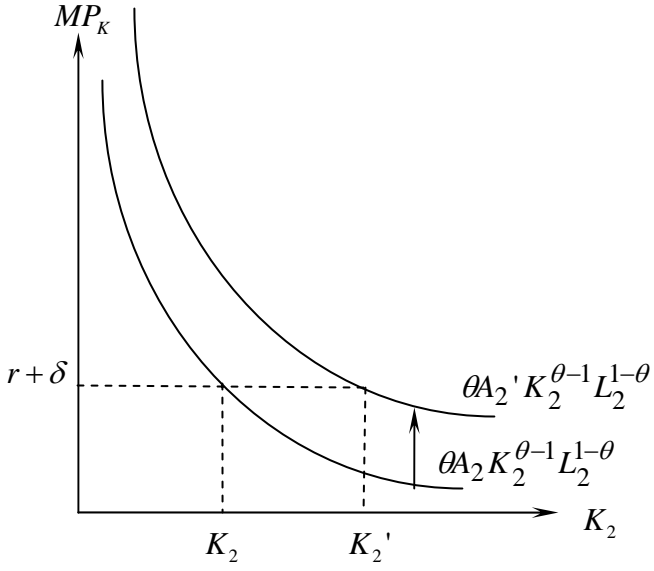
We can rearrange the above to obtain  $\theta A_2 K_2^{\theta-1} L_2^{1-\theta} - \delta = r$ . The left hand side is the net return on investment in physical capital and the right hand side is the net return on investment in the financial market.

- e. Illustrate graphically the impact of an increase in real interest rate on the demand for investment.



As real interest rate goes up, the optimal capital in the next period goes down, as shown in the figure above (from  $K_2$  to  $K_2'$ ). As a result, the optimal investment also goes down, since  $I = K_2 - (1 - \delta)K_1$ .

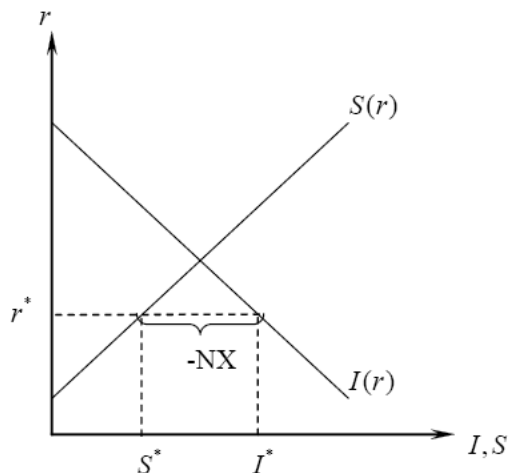
- f. Illustrate graphically the impact of future technological improvement on the demand for investment.



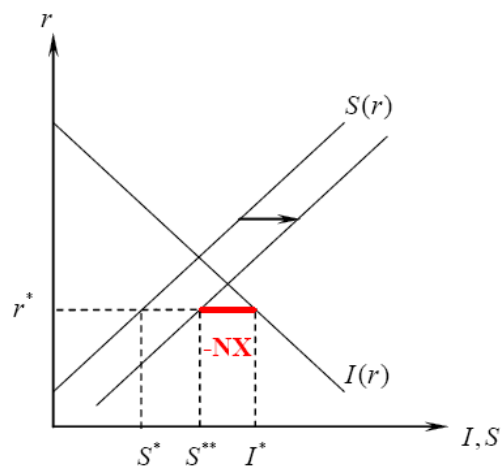
Notice that the marginal product curve shifts upward, so that for any given level of  $K_2$  its marginal product increases. The optimal level of  $K_2$  (and also of investment) will therefore increase.

### Capital Market

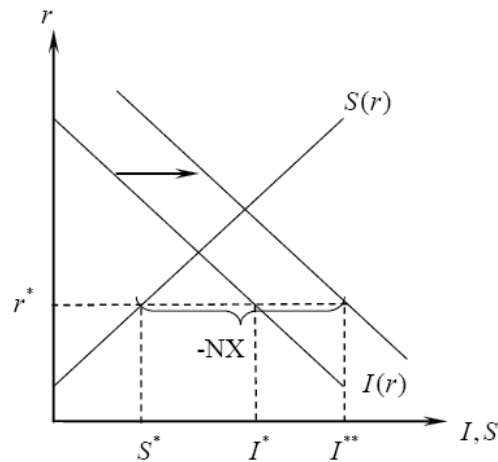
2. (5 points). Draw a fully labeled diagram of the capital market for an open economy with trade deficit.



3. (5 points). Suppose the government decreases its deficit. Illustrate graphically the impact of this event on the capital market. Show what happens to the equilibrium saving, investment, and trade deficit.



4. (5 points). Suppose that future productivity in the economy is expected to increase. Illustrate graphically the impact of this event on the capital market. Show what happens to the equilibrium saving, investment, and trade deficit.



### Money and Banking

5. (10 points). Quantity theory.  
 a. Write the quantity equation of money, both in levels and in growth rates.

In levels:

$$MV = PY$$

In growth rates:

$$\hat{M} + \hat{V} = \hat{P} + \hat{Y}$$

- b. Suppose that velocity is fixed, the growth rate of real GDP is 1.5% and the FED wants to achieve inflation of 2%. What is the required growth of the money supply?

$$\begin{aligned} \hat{M} + \hat{V} &= \hat{P} + \hat{Y} \\ ? + 0 &= 2\% + 1.5\% \\ \Rightarrow \hat{M} &= 3.5\% \end{aligned}$$

6. (15 points). Suppose that the public wants to hold currency/deposit ratio of  $cd = 0.1$ , and the required reserve/deposit ratio is  $rd = 0.2$ . The initial consolidated balance sheet of commercial banks is as follows:

Assets	Liabilities
$R = 20$	$D = 100$
$B_G = 25$	
$L = 55$	
100	100

- a. Find the monetary base, the money supply and the money multiplier in this economy.

$$CU = cd \cdot D = 0.1 \cdot 100 = 10$$

$$MB = R + CU = 20 + 10 = 30$$

$$M = CU + D = 10 + 100 = 110$$

$$mm = \left( \frac{1 + cd}{rd + cd} \right) = \left( \frac{1 + 0.1}{0.2 + 0.1} \right) = 3 \frac{2}{3}$$

- b. Now suppose that the FED performs an open market operation and buys government bonds from the commercial banks at the amount of 5. Find the new monetary base, the money supply and show the new balance sheet of the commercial banks.

$$MB = 30 + 5 = 35$$

$$M = \left( \frac{1 + cd}{rd + cd} \right) \cdot MB = \left( \frac{1 + 0.1}{0.2 + 0.1} \right) \cdot 35 = 128 \frac{1}{3}$$

$$D = \left( \frac{1}{rd + cd} \right) \cdot MB = \left( \frac{1}{0.2 + 0.1} \right) \cdot 35 = 116 \frac{2}{3}$$

$$R = \left( \frac{rd}{rd + cd} \right) \cdot MB = \left( \frac{0.2}{0.2 + 0.1} \right) \cdot 35 = 23 \frac{1}{3}$$

$$CU = \left( \frac{cd}{rd + cd} \right) \cdot MB = \left( \frac{0.1}{0.2 + 0.1} \right) \cdot 35 = 11 \frac{2}{3}$$

Assets	Liabilities
$R = 23 \frac{1}{3}$	$D = 116 \frac{2}{3}$
$B_G = 20$	
$L = 73 \frac{1}{3}$	
$116 \frac{2}{3}$	$116 \frac{2}{3}$

- c. Suppose that instead of the open market operation, the FED reduces the required reserve ratio to 10%. Find the new monetary base, money multiplier, the money supply and present the new balance sheet of the commercial banks.

$$MB = 30$$

$$mm = \frac{1 + cd}{rd + cd} = \frac{1 + 0.1}{0.1 + 0.1} = 5.5$$

$$M = \left( \frac{1 + cd}{rd + cd} \right) \cdot MB = \left( \frac{1 + 0.1}{0.1 + 0.1} \right) \cdot 30 = 165$$

$$D = \left( \frac{1}{rd + cd} \right) \cdot MB = \left( \frac{1}{0.1 + 0.1} \right) \cdot 30 = 150$$

$$R = \left( \frac{rd}{rd + cd} \right) \cdot MB = \left( \frac{0.1}{0.1 + 0.1} \right) \cdot 30 = 15$$

$$CU = \left( \frac{cd}{rd + cd} \right) \cdot MB = \left( \frac{0.1}{0.1 + 0.1} \right) \cdot 30 = 15$$

Assets	Liabilities
$R = 15$	$D = 150$
$B_G = 25$	
$L = 110$	
150	150

7. (15 points). Answer the following questions about the structure of the Federal Reserve System. Two sources of information about the Federal Reserve System are available on the course web page: (1) the FED 101 (or the FED's educational website) by clicking on "About the FED", and (2) a short movie about the FED.
- This group has broad control over the growth of the nation's money supply (circle one answer):
    - Advisory Councils
    - Federal Open Market Committee
    - Board of Governors
    - Member Banks
  - How many districts make up the Federal Reserve System (circle one answer)?
    - 3
    - 12
    - 25
    - 52
  - Which of the following is NOT one of the 3 primary responsibilities of the Federal Reserve System (circle one answer)?
    - Printing money and minting coins
    - Conducting the U.S. monetary policy
    - Banking supervision
    - Providing financial services to the commercial banks (such as check clearing).

8. (15 points). Answer the following questions based on the Bernanke’s speech on monetary policy (“Implementing Monetary Policy”) posted on the course web page.

a. Can the FED directly control interest rates on mortgages?

No. The FED has more or less full control over the Federal Funds Rate, but other interest rates are affected only indirectly by the FED’s actions.

b. How can the FED indirectly control the long-term interest rates in the economy?

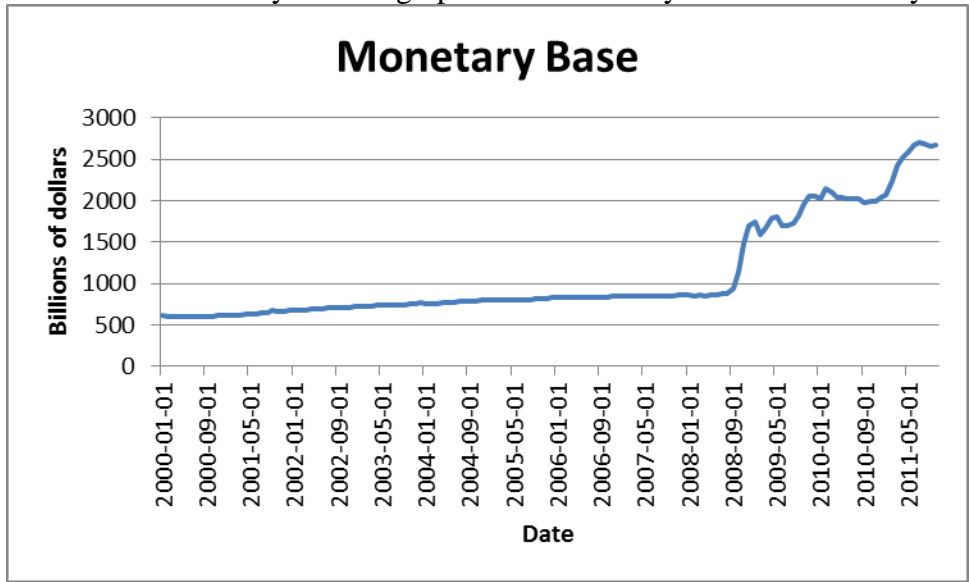
The FED can influence public’s expectations about future monetary policies through announcements. For these announcements to be perceived as credible, the FED needs to establish a “track record” of previous policies and clarify to the public how the FED responds to different economic situations. This is called “transparency” of monetary policy. For more about the importance of transparency, read the interview with Bernanke posted on the course web page.

c. How can the FOMC learn about the impact of their announcements on the public’s expectations about future interest rate? Be explicit.

The FOMC can learn about public’s expectations from the prices of futures contracts on the funds rate.

9. (30 points). **Excel required.** Use the data for HW6 posted on the course webpage.

a. Present a fully labeled graph of the monetary base since January 2000.

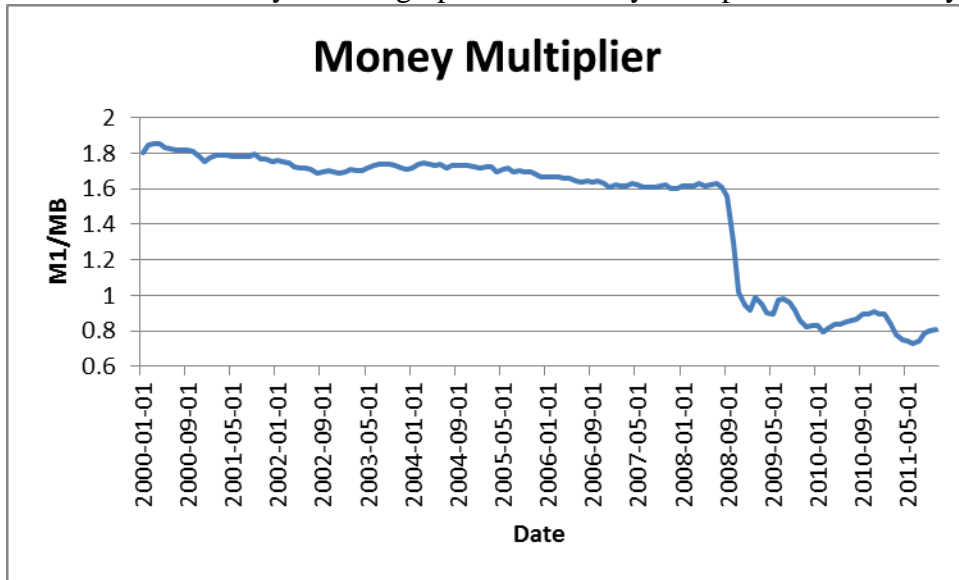


b. Based on the graph of monetary base, we can clearly see that the FED is trying to increase the money supply in the economy since 2008.

True /  False, circle the correct answer and briefly explain.

$M = MB \cdot mm$ , thus if the money multiplier is unchanged, an increase in monetary base will lead to an increase in the money supply.

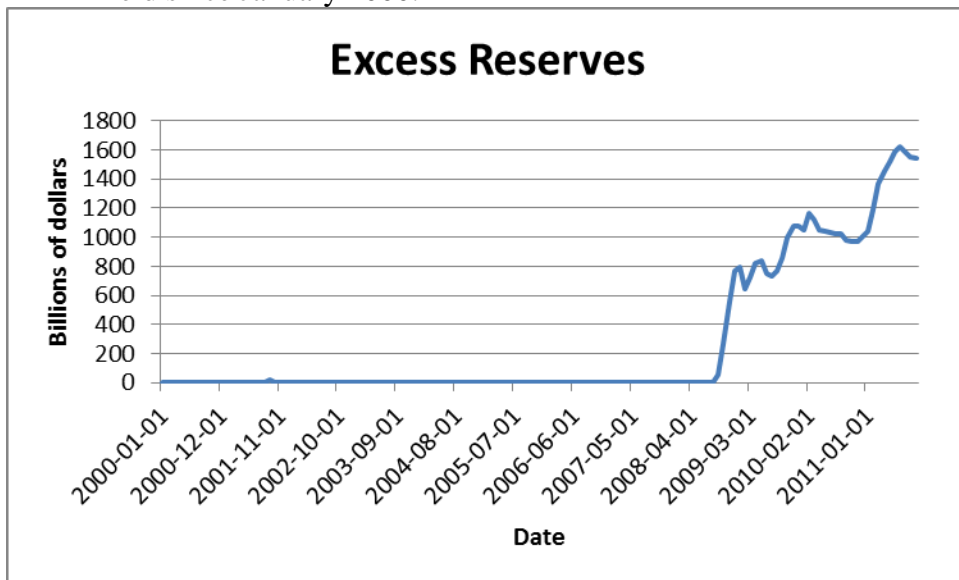
c. Present a fully labeled graph of the money multiplier since January 2000.



d. Based on the above graph, we can conclude that the money supply did not increase as much as the monetary base since 2008. True False, circle the correct answer and briefly explain.

The money multiplier  $mm = M1/MB$  dropped sharply, which means that the growth of money supply (the numerator) was much less than the growth of the monetary base (the denominator).

e. Present a fully labeled graph of excess reserves that commercial banks held since January 2000.



f. Based on the above graph, we see that commercial banks always keep the minimum required reserves. False True, circle the correct answer and explain briefly.

This is true throughout the 2000's, but recently (since 2008) we see that banks keep much more reserves than what is required. This means that the banks are acting cautiously now about giving loans.