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ECON 302 **Fall 2011**

Problem set 5
Consumption-Saving Decision and Ricardian Equivalence

1. (10 points). Saving and Investment equation.
   a. Derive the saving and investment equation.

The formula for GDP using the expenditure approach is

$$GDP = C + I + G + NX$$

Subtract the total taxes and add the transfer payments to both sides, and rearranging

$$GDP - T + TR = C + I + G - T + TR + NX$$

$$YD = C + I - S_G + NX$$

$$YD - C + S_G = I + NX$$

<u>The Saving and Investment Equation:</u>

$$\boxed{S_P + S_G = I + NX}$$

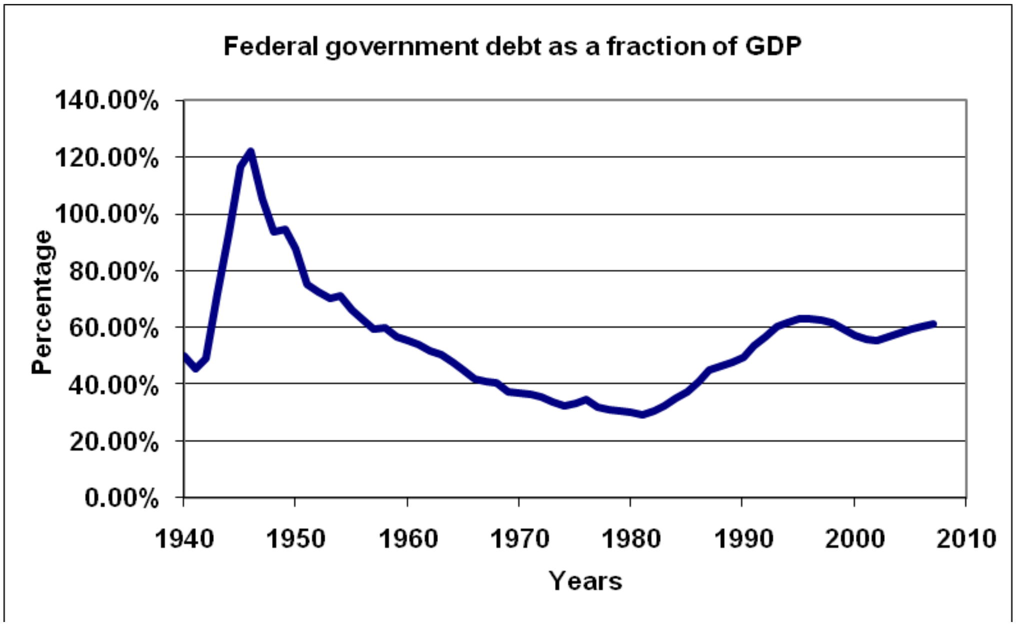
   b. Suppose that in some economy the private saving is 15, the domestic investment is 20, and the government runs a deficit of 3. What must be the trade deficit in that country?

Using the saving and investment equation:

$$\underbrace{S_P}_{15} + \underbrace{S_G}_{-3} = \underbrace{I}_{20} + \underbrace{NX}_{?}$$

Thus $NX = -8$, and the trade deficit is therefore 8.

2. (35 points). In this question you need to use Excel and data for HW4 posted on the course web page. The questions that we address here are how big is the government debt, and what is the cost of the debt, i.e. how much interest does government pay because of the debt.
   a. Plot the graph of the government debt as a fraction of GDP over the period 1940 - 2005.



b. Why does it make sense to look at the debt as a fraction of GDP, and not at the debt itself?

What matters is the size of the debt relative to the entire income of the country – GDP. Just like with our personal debt, in order to get an idea how large the debt is, we need to compare it to our earning. The GDP is the total income of the economy, so it makes sense to look at the debt as a fraction of GDP. This also allows us to compare the debt situation of different countries. Suppose I tell you that the debt in the U.S. is 8 trillion dollars, while in some other country it is 25 billion. These numbers are totally uninformative and do not allow us to conclude which country has more serious debt problem.

c. How big is the debt as a fraction of GDP in 2006?

The debt in 2006 was 60% of the GDP.

d. Is this the biggest debt that we had since 1940? When did the U.S. have the largest debt and what was the fraction of debt to GDP then?

That debt today is not the highest ever. After WWII (1946) the U.S. debt was more than 120% of GDP.

e. Plot the graph of the interest payments as a fraction of government income over the period 1940 - 2006.

f. Why does it make sense to look at the government interest payments as a percentage of its income?

What matters is the interest payments relative to the government's income. The dollar value of any of the government expenditures is totally uninformative. If we want to learn anything about government spending, we must look at that spending as a fraction of government income. For example, spending on national defense, education, interest payments, all need to be presented as a fraction of government income.

g. What fraction of the government income is paid as interest on debt in 2006? What is the dollar amount that the government paid as interest on its debt in 2006?

The government spent 377.1 billion dollar, which is 9.5% of its income, on paying interest on its debt in 2006.

Discussion: Does U.S. government have large debt? Debt above 60% of GDP is considered by most economists as quite large, but not catastrophic. The interest payments represent the burden of the debt, and as we have seen this burden is quite large (almost 10% of government's income). At the micro level, it is not unheard of that people with annual income of $70,000 take mortgage of $700,000, so their debt is 1000% of their annual income. Compared to this, 60% does not seem that large of a debt. Most economists however, when looking at these numbers would advice the U.S. government to act more responsibly in the future and reduce the government deficit. One way to reduce the deficit is to cut spending. **Can the government reduce it's debt by raising taxes?** It is not obvious that higher taxes will help to decrease the deficit. If higher taxes will discourage people from working, then higher taxes will lead to lower *tax base*, which is the total income being taxed. So it is not obvious at all that higher taxes will increase the total tax revenues.

3. (50 points). Consider the two-period model of consumption and saving.
   a. Write the consumer's problem of utility maximization subject to the budget constraints in two periods.

$$\begin{cases} \max\limits_{c_1, c_2, s} U(c_1, c_2) \\ s.t. \\ BC_1: \quad c_1 + s = y_1 - t_1 \\ BC_2: \quad c_2 = y_2 - t_2 + (1+r)s \end{cases}$$

   b. Derive the lifetime budget constraint from the budget constraints in each period. Show your derivations.

Substitute *s* from the second period budget constraint into the first period's budget constraint. It is easy to do when you divide both sides of BC_2 by $1+r$ to get

$$BC_2: \quad \frac{c_2}{1+r} = \frac{y_2}{1+r} - \frac{t_2}{1+r} + s$$

Now add the two budget constraints and get the **lifetime budget constraint:**

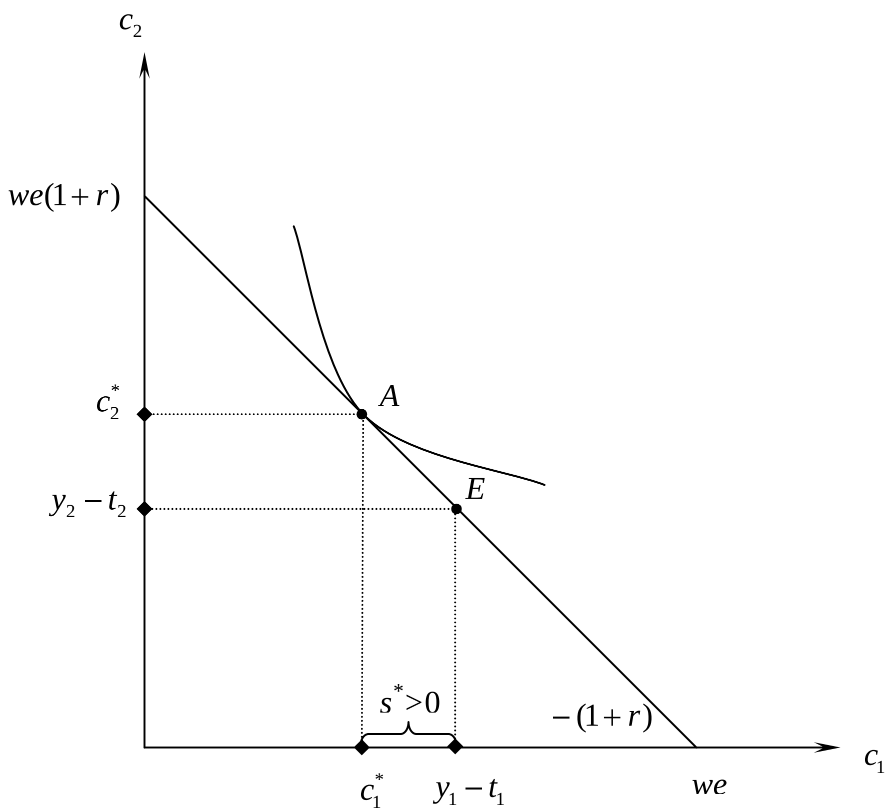
$$\boxed{\underbrace{c_1 + \frac{c_2}{1+r}}_{PV\ of\ lifetime\ consumption} = \underbrace{y_1 - t_1 + \frac{y_2 - t_2}{1+r}}_{we\ =\ lifetime\ wealth}}$$

   c. Give the economic interpretation of the left hand side and the right hand side of the lifetime budget constraint.

The left hand side is the present value of lifetime consumption, and the right hand side is the present value of lifetime net of taxes income, which we call the lifetime wealth (*we*).

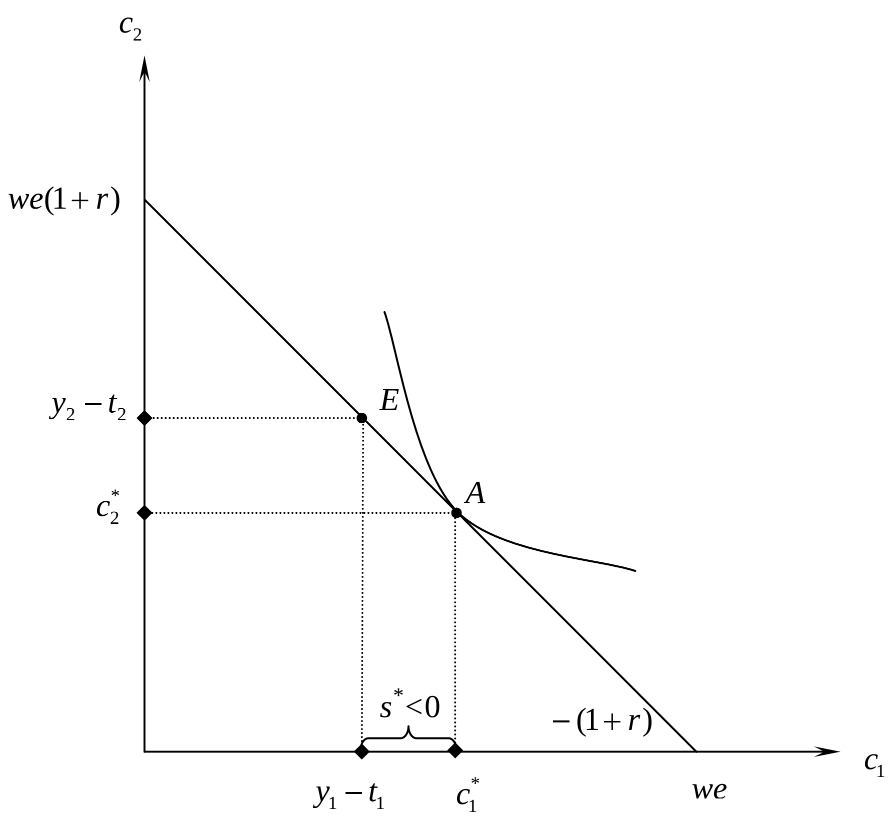
d. Draw a fully labeled graph of the lifetime budget constraint with a tangent indifference curve indicating the optimal choice for a lender, and label the saving as well. In order to receive all the points your graph should be clear and the lines are carefully drawn with a ruler.

The next figure shows the optimal choice for a consumer who is a lender. The optimal choice (optimal consumption bundle) is at point A.



e. Draw a fully labeled graph of the lifetime budget constraint with a tangent indifference curve indicating the optimal choice for a borrower, and label the saving as well. In order to receive all the points your graph should be clear and the lines are carefully drawn with a ruler.

The next figure shows the optimal choice for a consumer who is a borrower. The optimal choice (optimal consumption bundle) is at point A.



f. An increase in current income y_1 will increase the current consumption c_1 by the same amount (i.e. $\Delta c_1 = \Delta y_1$). True/$\boxed{\text{false}}$, prove your answer.

We see from the lifetime budget constraint that an increase in y_1 will shift the budget constraint to the right. Given that both goods (current consumption and future consumption) are normal, the consumer will increase the consumption in both periods. In order to increase the consumption in the second period the consumer must increase his saving. Thus, an increase in the current income will increase the current consumption by less than the change in the current income. We call this result **consumption smoothing**.

To summarize: $\boxed{y_1 \uparrow \;\Rightarrow\; c_1^* \uparrow, s \uparrow, c_2^* \uparrow,\ \Delta c_1 < \Delta y_1}$

g. Consider the government budget constraint and suppose that the real interest rate is 6%.

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

If the government gives a tax cut of 30 in the first period (i.e. $\Delta T_1 = -30$), find the necessary change in the second period's taxes ($\Delta T_2 = ?$) that would keep the present value of taxes unchanged.

$$T_1 - 30 + \frac{T_2 + \Delta_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

$$-30 + \frac{\Delta_2}{1+r} = 0$$

$$\Delta_2 = 30(1+r) = 30 \cdot 1.06 = 31.8$$

h. State the Ricardian equivalence theorem.

If the present value of government spending remains unchanged, then changes in the taxes do not affect the households' optimal consumption choice (c_1^*, c_2^*).

4. (10 points). Suppose that instead of a lump-sum taxes, the taxes are proportional to income ($0 < t_1, t_2 < 1$), so that the budget constraints are now

$$BC_1: \quad c_1 + s = y_1(1-t_1)$$
$$BC_2: \quad c_2 = y_2(1-t_2) + (1+r)s$$

Prove that the Ricardian Equivalence theorem still holds.

The consumer's lifetime BC is: $c_1 + \frac{c_2}{1+r} = y_1 + \frac{y_2}{1+r} - \left[t_1 y_1 + \frac{t_2 y_2}{1+r} \right]$. From the government lifetime BC we see that $G_1 + \frac{G_2}{1+r} = N\left(t_1 y_1 + \frac{t_2 y_2}{1+r} \right)$, thus, the lifetime tax liability of the consumers is still fixed when the present value of government spending is fixed.

5. (15 points). Consider the two-period model of consumption and saving. Suppose that the consumer's utility is $U(c_1, c_2) = \ln(c_1) + \beta \ln(c_2)$.

a. Write the consumer's problem

$$\begin{cases} \max\limits_{c_1, c_2, s} \ln(c_1) + \beta \ln(c_2) \\ s.t. \\ BC_1: \quad c_1 + s = y_1 - t_1 \\ BC_2: \quad c_2 = y_2 - t_2 + (1+r)s \end{cases}$$

b. Write the consumer's demand for consumption in both periods and his supply of saving.

Writing the consumer's problem with the lifetime budget constraint helps for this section.

$$\begin{cases} \max\limits_{c_1, c_2} \ln(c_1) + \beta \ln(c_2) \\ s.t. \\ c_1 + \dfrac{c_2}{1+r} = y_1 - t_1 + \dfrac{y_2 - t_2}{1+r} \end{cases}$$

Now we can see that since the preferences are of the Cobb-Douglas form, the consumer will spend a fixed fraction of his lifetime income on c_1 and c_2:

$$c_1^* = \left(\frac{1}{1+\beta}\right)\left(y_1 - t_1 + \frac{y_2 - t_2}{1+r}\right)$$

$$c_2^* = \left(\frac{\beta}{1+\beta}\right)\left(y_1 - t_1 + \frac{y_2 - t_2}{1+r}\right)(1+r) = \left(\frac{\beta}{1+\beta}\right)\left((y_1 - t_1)(1+r) + y_2 - t_2\right)$$

The saving, from the first budget constraint

$$s^* = y_1 - t_1 - \underbrace{\left(\frac{1}{1+\beta}\right)\left(y_1 - t_1 + \frac{y_2 - t_2}{1+r}\right)}_{c_1^*}$$

c. Prove that saving is increasing in real interest rate.

Notice that c_1^* is decreasing in *r* (see section b), and $s^* = y_1 - t_1 - c_1^*$. Thus, s^* is increasing in *r*. Another way to show that s^* is increasing in *r* is to take the derivative

$$\frac{\partial s^*}{\partial r} = \left(\frac{1}{1+\beta}\right)\frac{y_2 - t_2}{(1+r)^2} > 0$$

(Obviously, the taxes are no greater than income, so the term $y_2 - t_2 > 0$, and the above derivative is always positive).

d. Suppose that income in the first period increases by \$100. Find the change in the first period's consumption.

$$c_1^* = \left(\frac{1}{1+\beta}\right)\left(y_1 + 100 - t_1 + \frac{y_2 - t_2}{1+r}\right) = \left(\frac{1}{1+\beta}\right)\left(y_1 - t_1 + \frac{y_2 - t_2}{1+r}\right) + \left(\frac{1}{1+\beta}\right)\cdot 100$$

Thus, the change in c_1^* is

$$\boxed{\Delta c_1^* = \left(\frac{1}{1+\beta}\right)\cdot 100}$$