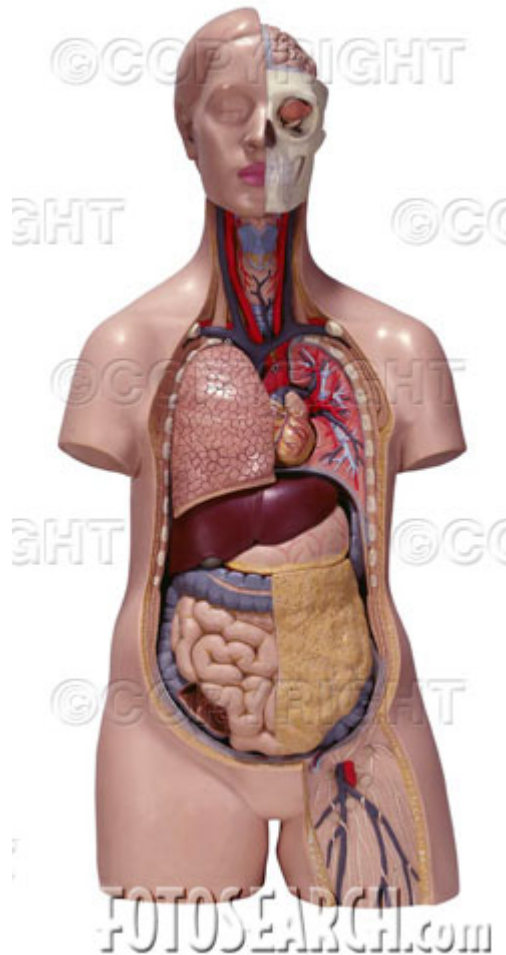


Problem set 3

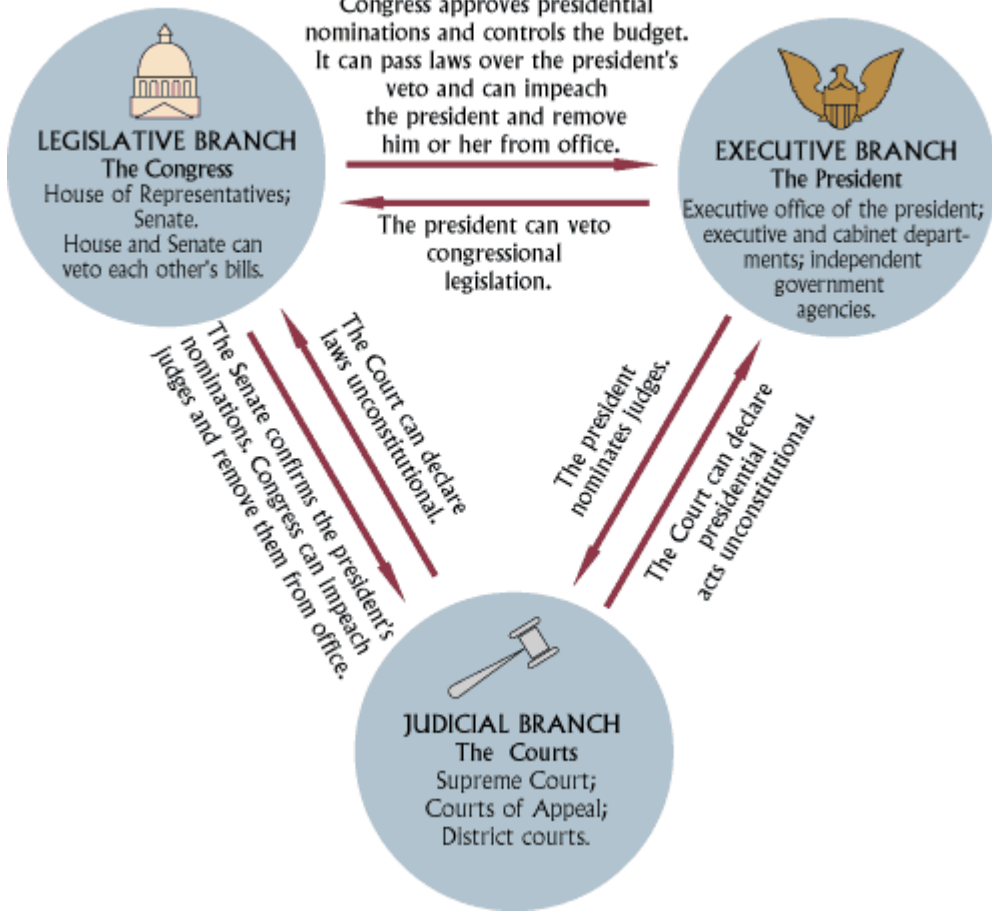
Philosophy of Models

1. (15 points). Based on our discussion of models in economics and science, answer the following questions.
 - a. Give an example of a model not from economics.

Example 1: a model of human body.



Example 2: Model of the U.S. government.



- b. Define the following concepts:
 - i. Exogenous variable – determined outside of the model.
 - ii. Endogenous variable – determined within the model.
- c. List all the exogenous and endogenous variables of the Classical Model.

Exogenous: t, A, K (some might write h , which is o.k.).

Endogenous variables: w, L, l, c, G, Y, π

Classical Model

2. (50 points). Consider the Classical model discussed in class.

- a. Write the consumer's problem.

Consumer's problem:

$$\begin{cases} \max_{C,l} \alpha \ln C + (1 - \alpha) \ln l \\ s.t. C = [w(h - l) + \pi](1 - t) \end{cases}$$

In words, the consumer's problem is to maximize utility subject to his budget constraint, by choosing the optimal consumption and leisure.

- b. Write the consumer's demand for consumption and leisure, and its labor supply. You don't need to derive anything.

If you rewrite the budget constraint as $C + w(1-t)l = (wh + \pi)(1-t)$, you can immediately see that the demand is:

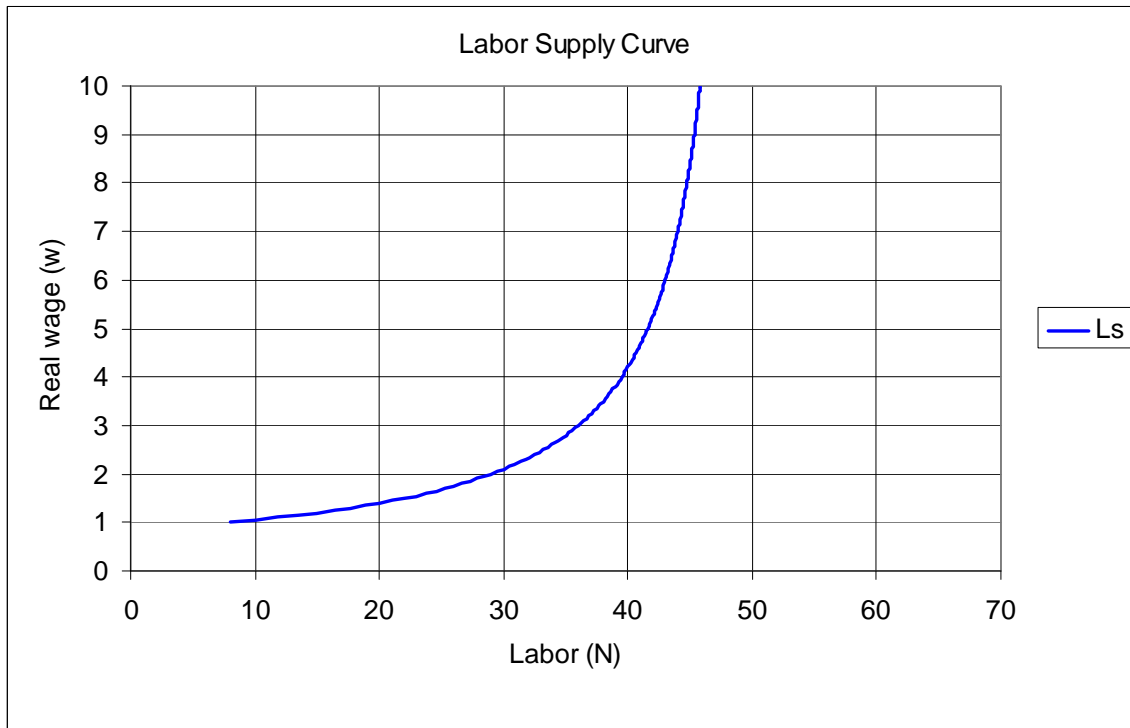
$$C = \alpha(wh + \pi)(1-t)$$

$$l = \frac{(1-\alpha)(wh + \pi)(1-t)}{w(1-t)} = (1-\alpha)\left(h + \frac{\pi}{w}\right)$$

Labor supply:

$$L_s = h - l = h - (1-\alpha)\left(h + \frac{\pi}{w}\right)$$

c. Draw the labor supply curve.



d. What happens to the labor supply curve when the real wage goes up?

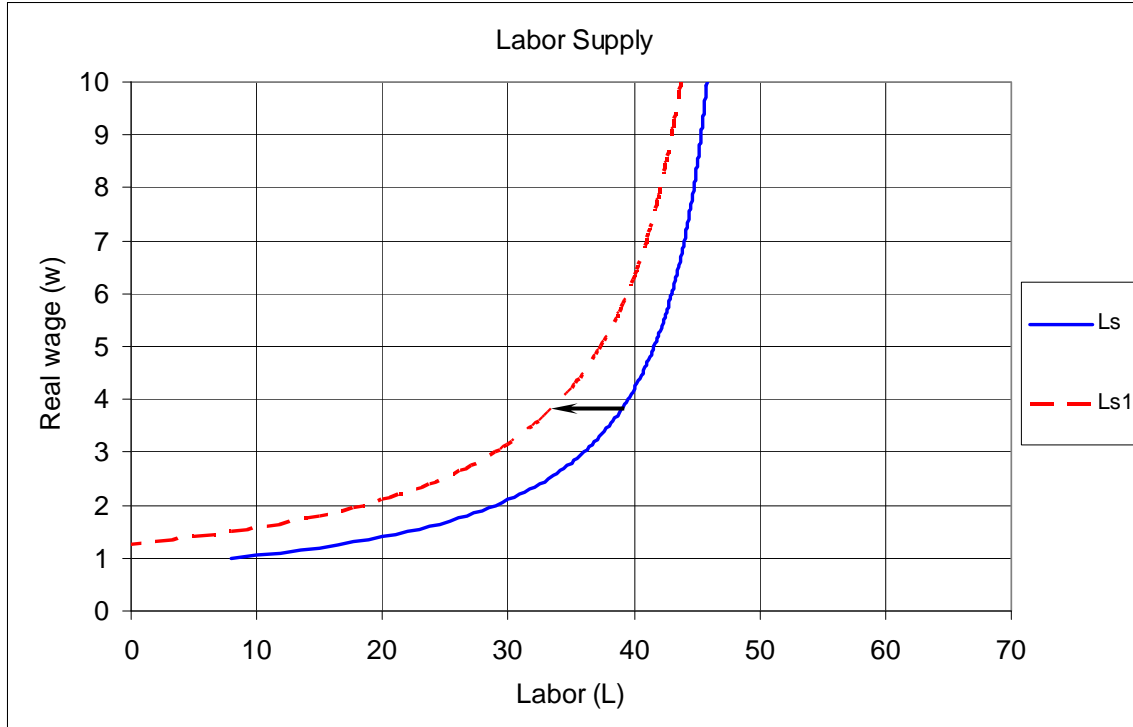
Changes in w are reflected in movement **along the same supply curve**. The supply curve tells us how much labor the consumer wants to supply **at any given wage**. The supply curve does not shift when the price (in this case w) changes. In the above picture, if the real wage goes up from 2 to 4, the consumer will increase his working hours from 30 to 40, which is a movement **along the same supply curve**.

e. What happens to the labor supply curve when the profit goes up? Illustrate your answer by drawing the labor supply curve before and after the increase in profit.

The supply curve will shift to the left (decrease in labor supply). From the labor supply equation

$$L_S = h - (1 - \alpha) \left(h + \frac{\pi}{w} \right)$$

we see that if $\pi \uparrow$, then leisure will go down and labor supply **for any given wage** will decrease. In the next picture we see that the consumer works less at any given wage (shift to the left of the labor supply curve).



f. Provide economic intuition for the previous part.

As the non-labor income increases ($\pi \uparrow$), the consumer does not need to work as much. For example, scholarships are non-labor income, and are given to students so they don't need to work too much and they can study more.

g. Write the firm's problem. Derive the firm's demand for labor.

$$\text{Firm's problem: } \max_{L_D} \pi = AK^\theta L_D^{1-\theta} - wL_D$$

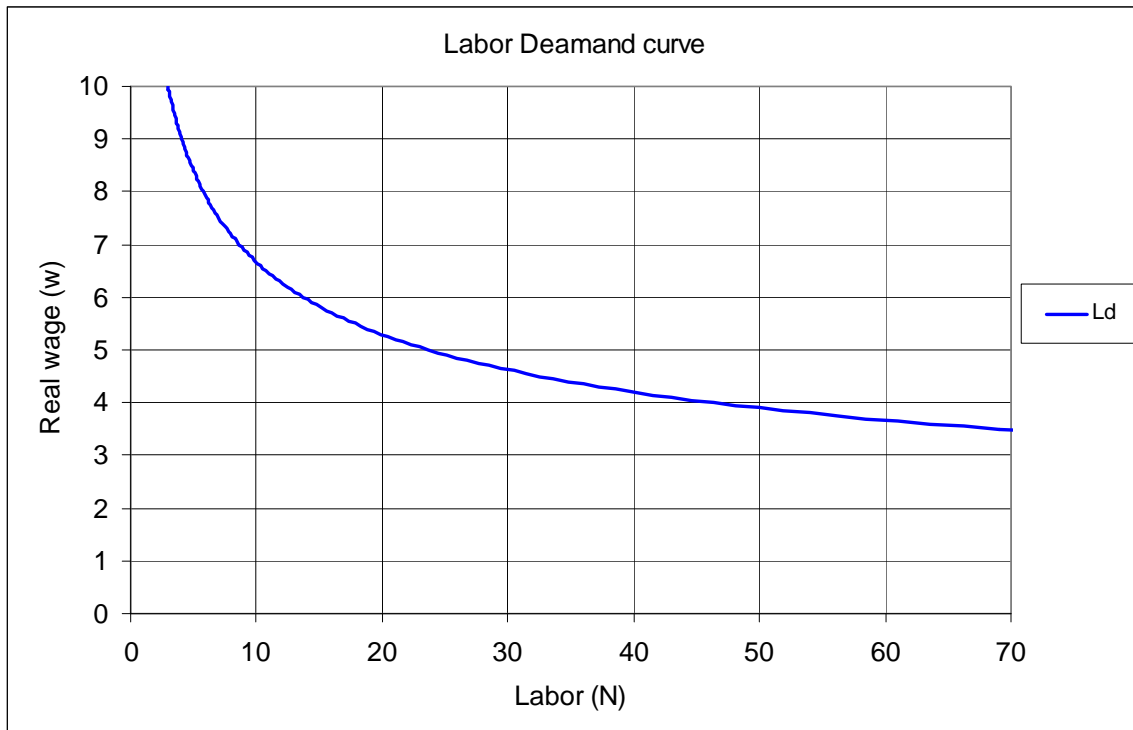
In words, the firm maximizes profit by choosing the optimal amount of labor to employ. Solving the firm's problem (deriving the demand):

$$\frac{\partial \pi}{\partial L_D} = (1 - \theta)AK^\theta L_D^{-\theta} - w = 0$$

$$(1 - \theta)AK^\theta L_D^{-\theta} = w$$

$$L_D = \left(\frac{(1 - \theta)AK^\theta}{w} \right)^{1/\theta}$$

h. Draw the graph of the labor demand curve.



i. What happens to the labor demand curve when the real wage goes up?

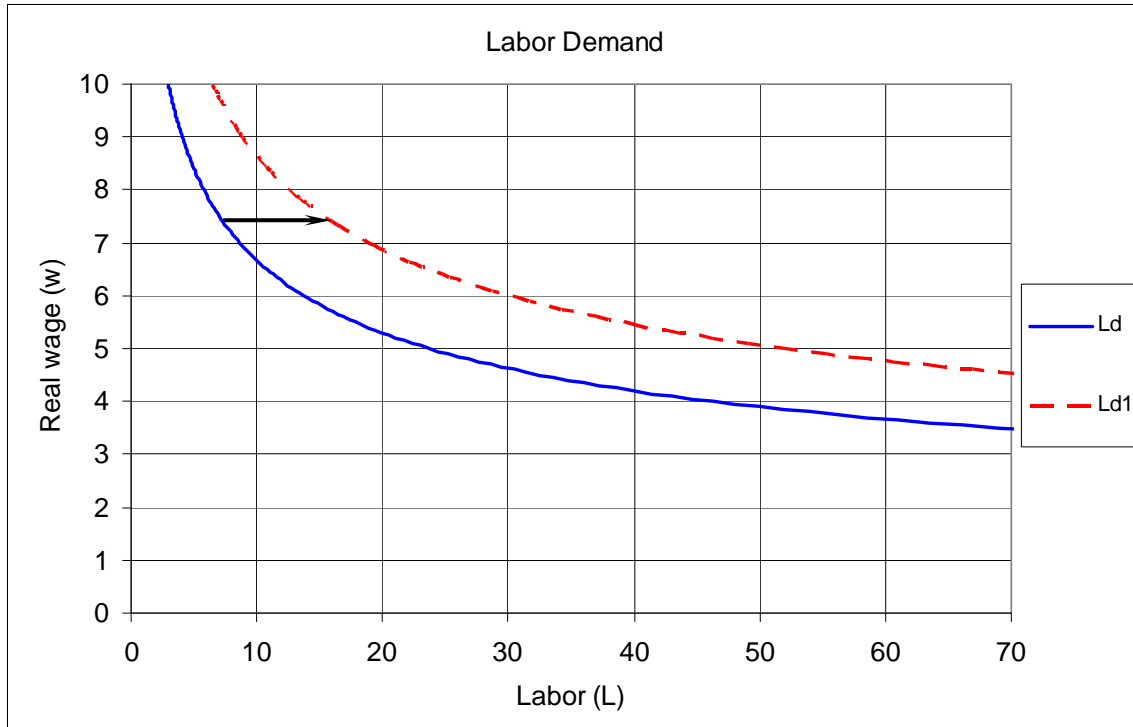
Changes in w are reflected in movement **along the same demand curve**. The demand curve tells us how much labor does the firm want to employ **at any given wage**. The demand curve does not shift when the price (in this case w) changes. In the above picture, if the wage goes up from 4 to 5, the firm will change its labor demanded from about 45 to about 24, which is a movement **along the same demand curve**.

j. What happens to the labor demand curve when the productivity parameter A goes up? Illustrate your answer by drawing the labor demand curve before and after the increase in productivity.

The demand curve will shift to the right (increase in demand). As can be seen from the labor demand equation,

$$L_D = \left(\frac{(1-\theta)AK^\theta}{w} \right)^{1/\theta},$$

when A goes up, then **for any give wage** the firm wants to hire more labor. In the next picture we see that for any given wage the firm wants to hire more labor.



3. (40 points). Consider the classical model discussed in class. The parameters of the model are as follows: $A = 10, K = 10, \theta = 1/3, \alpha = 0.5, h = 100, t = 0.2$.
- a. Solve for the equilibrium. That is, find the values of all the endogenous variables: w, ℓ, L, C, G, Y, π . You are required to write the equations that you used. You can use the Excel file accompanying the notes, in order to verify your answers.

$$L^* = \frac{(1-\theta)\alpha h}{1-\alpha\theta} = \frac{(1-1/3) \cdot 0.5 \cdot 100}{1-0.5 \cdot 1/3} = 40$$

$$l^* = h - L^* = 100 - 40 = 60$$

$$w^* = (1-\theta)AK^\theta L^{*-\theta} = (1-1/3) \cdot 10 \cdot 10^{1/3} \cdot 40^{-1/3} = 4.200$$

$$Y^* = AK^\theta L^{*1-\theta} = 10 \cdot 10^{1/3} 40^{2/3} = 251.984$$

$$\pi = \theta Y^* = 83.995$$

$$C^* = (1-t)Y^* = 201.587$$

$$G = tY^* = 50.397$$

- b. Suppose that there was a technological progress, so that $A = 15$. Solve for the new equilibrium.

Using the same equations as above, with $A = 15$, gives the following results:

	Old	New
w	4.200	6.300
l	60	60
L	40	40
C	201.587	302.381
G	50.397	75.595
Y	251.984	377.976
π	83.995	125.992

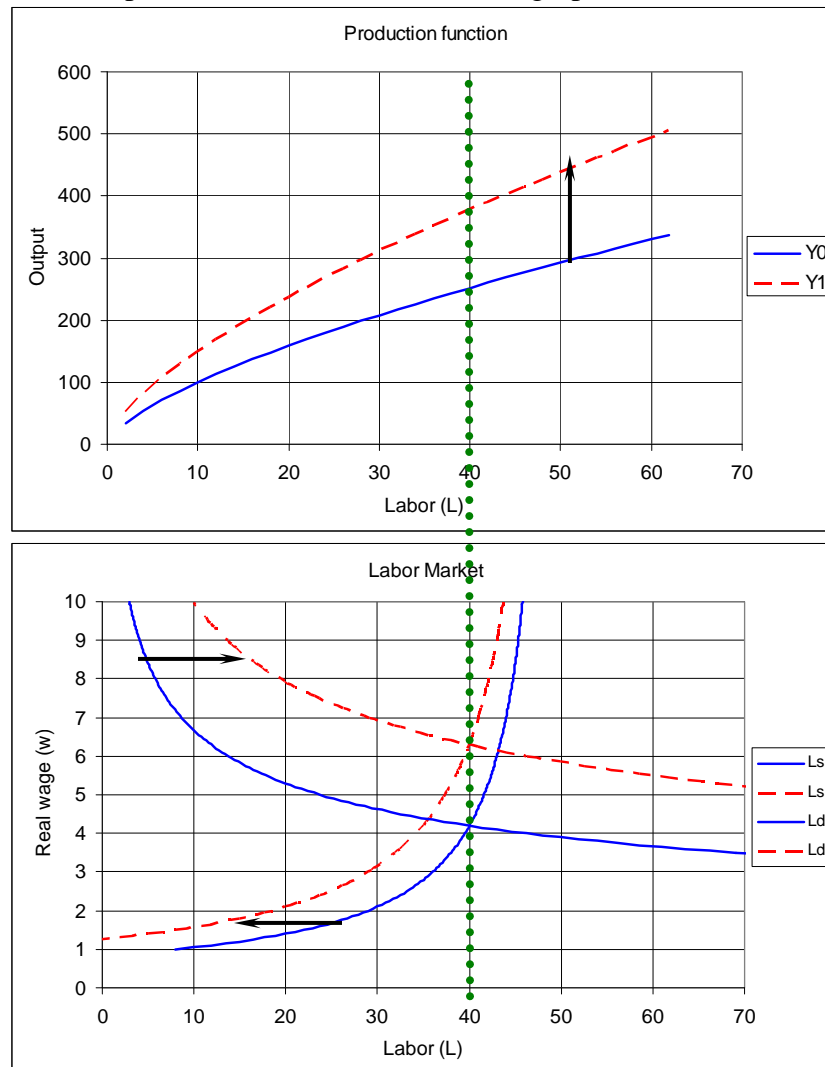
- c. What happened to the real wage, employment, consumption, and output as a result of the technological progress?

Real wage, consumption and output increased, but employment did not change. Observe that equilibrium employment is given by

$$L^* = \frac{(1-\theta)\alpha h}{1-\alpha\theta}$$

As you can see, it does not depend on the level of technology.

- d. Illustrate graphically the impact of the technological improvement on the labor market and the production function. You can use the Excel file that accompanies the notes to create those graphs.



- e. Explain why the labor supply curve shifted to the left in part d.

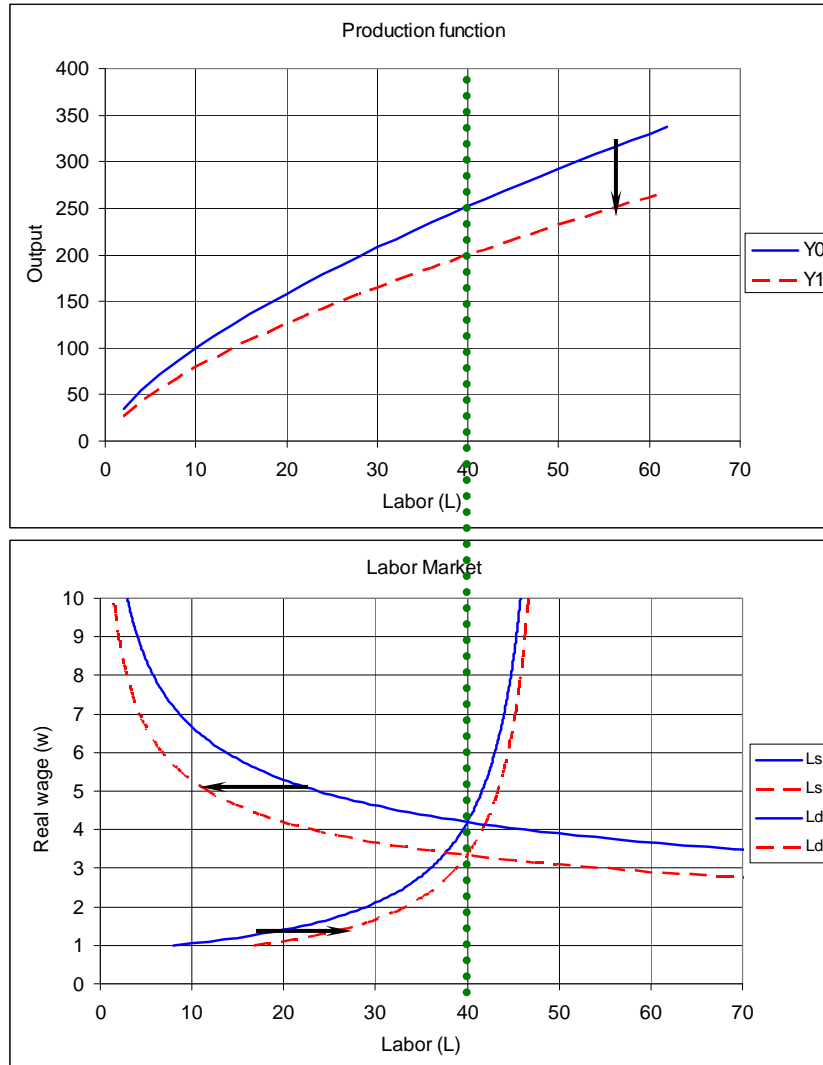
Because equilibrium profit increased. From the labor supply equation

$$L_s = h - (1 - \alpha) \left(h + \frac{\pi}{w} \right)$$

we see that if $\pi \uparrow$, then leisure will go up and labor supply **for any given wage** will decrease. Economic intuition is as follows. As the non-labor income increases ($\pi \uparrow$), the consumer does not need to work as much. For example, scholarships are non-labor income, and are given to students so they don't need to work too much and they can study more.

Remark. If you answered “because the real wage increased”, this means that the concept of supply curve is not clear to you.

- f. Suppose that **instead of the technological progress**, a hurricane destroyed half of the capital stock in the economy. Demonstrate with the diagrams of the labor market and production functions, the impact of the hurricane on the real wage, employment and output.



Real wage, and output decreased, but employment did not change.

- g. Suppose that instead of the above changes, the government increases the tax rate to 50%. What will happen to all the endogenous variables $(w, \ell, L, C, G, Y, \pi)$ as a result of that policy?

Only personal consumption expenditure and government consumption expenditure will change. Remember that in this model, the government takes a fraction t of the total output and that leaves a fraction $(1 - t)$ to the consumer.

$$\begin{aligned} C^* &= (1-t)Y^* \\ G &= tY^* \end{aligned}$$

- h. Answer the three basic questions that we asked in the introduction to the topic of business cycles: (1) what causes business cycles? (2) Can the government smooth them? (3) Should it?

- (1) What causes business cycles? Mostly, shocks to productivity parameter A .
 (2) Can the government smooth them? No.
 (3) Should it? No, because it can't.

4. (15 points). Consider the classical model discussed in class, with a slight change: the labor income is taxed at the rate t_w and the profit is taxed at rate t_π .

- a. Write the consumer's problem with the above taxes.

The original consumer's problem is:

$$\begin{cases} \max_{C,l} \alpha \ln C + (1 - \alpha) \ln l \\ \text{s.t. } C = w(h - l)(1 - t) + \pi(1 - t) \end{cases}$$

Notice that labor income $w(h - l)$ and non-labor income π are taxed at the same rate of t . Now, when labor income is taxed at the rate t_w and the profit is taxed at rate t_π , the problem becomes

$$\begin{cases} \max_{C,l} \alpha \ln C + (1 - \alpha) \ln l \\ \text{s.t. } C = w(h - l)(1 - t_w) + \pi(1 - t_\pi) \end{cases}$$

Notice that the only difference is the subscripts w and π .

- b. Solve for the equilibrium employment level as a function of the two tax rates.

It helps to write the budget constraint as $C + w(1 - t_w)l = wh(1 - t_w) + \pi(1 - t_\pi)$. The demand for leisure is

$$l = (1 - \alpha) \frac{(wh(1 - t_w) + \pi(1 - t_\pi))}{w(1 - t_w)}$$

$$l = (1 - \alpha) \left(h + \frac{\pi(1 - t_\pi)}{w(1 - t_w)} \right)$$

Recall that previously, when the tax rate on all income was the same, the term $(1 - t_w)/(1 - t_\pi) = (1 - t)/(1 - t)$ canceled out.

The labor supply is

$$L_S = h - (1 - \alpha) \left(h + \frac{\pi(1 - t_\pi)}{w(1 - t_w)} \right)$$

Now repeat the same steps as in the notes, to find equilibrium employment.
 In equilibrium

$$L = h - (1 - \alpha) \left(h + \frac{\theta AK^\theta L^{1-\theta}}{(1-\theta)AK^\theta L^{-\theta}} \left(\frac{1-t_\pi}{1-t_w} \right) \right)$$

$$L = h - (1 - \alpha) \left(h + \frac{\theta}{(1-\theta)} L \left(\frac{1-t_\pi}{1-t_w} \right) \right)$$

$$L = \alpha h - \frac{(1-\alpha)\theta}{(1-\theta)} L \left(\frac{1-t_\pi}{1-t_w} \right)$$

$$\alpha h = L + \frac{(1-\alpha)\theta}{(1-\theta)} L \left(\frac{1-t_\pi}{1-t_w} \right)$$

$$\alpha h = L \left[1 + \frac{(1-\alpha)\theta}{(1-\theta)} \left(\frac{1-t_\pi}{1-t_w} \right) \right]$$

Equilibrium employment:

$$L^* = \frac{\alpha h}{\left[1 + \frac{(1-\alpha)\theta}{(1-\theta)} \left(\frac{1-t_\pi}{1-t_w} \right) \right]}$$

- c. Explain how the government can increase the equilibrium employment and output when it can change the two tax rates.

Notice that equilibrium labor is increasing in t_π and decreasing in t_w . By changing the ratio

$$\left(\frac{1-t_\pi}{1-t_w} \right)$$

The government can affect equilibrium employment. The economic intuition goes like this. The demand curve for labor is not affected by taxes. So we only need to look at the effects of taxes on the labor supply curve:

$$L_S = h - (1 - \alpha) \left(h + \frac{\pi}{w} \left(\frac{1-t_\pi}{1-t_w} \right) \right)$$

Notice that we can denote

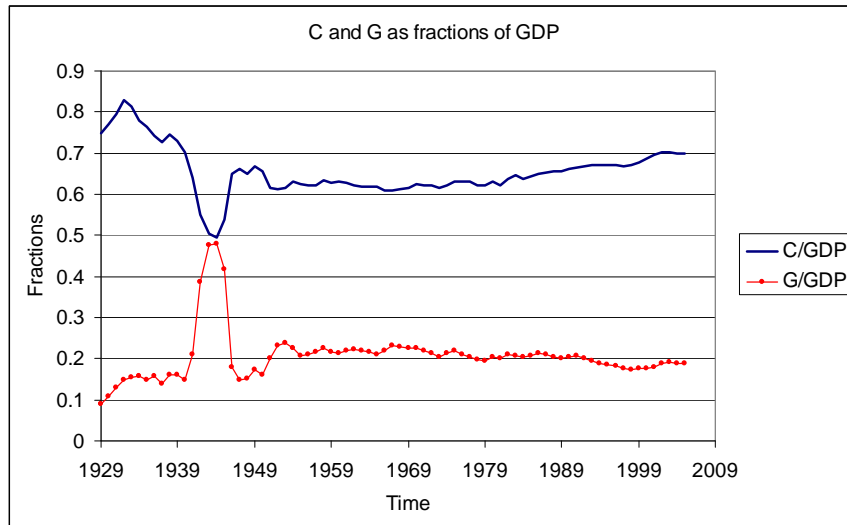
$$\tilde{\pi} = \pi \left(\frac{1-t_\pi}{1-t_w} \right)$$

And the labor supply curve becomes

$$L_S = h - (1 - \alpha) \left(h + \frac{\tilde{\pi}}{w} \right)$$

By changing the taxes such that the net of taxes non-labor income decreases ($\tilde{\pi} \downarrow$), say by increasing t_π or decreasing t_w , will induce the consumer to supply more labor.

5. (20 points). **Excel required.** For all the sections use the data provided in the file “Data for HW3”.
- Create two new variables: C/GDP and G/GDP, which are the fractions (or percentages) of personal consumption and government consumption in the GDP. Do not report this data. Plot on the same graph the two time series. Use XY scatter, and choose lines, not columns.

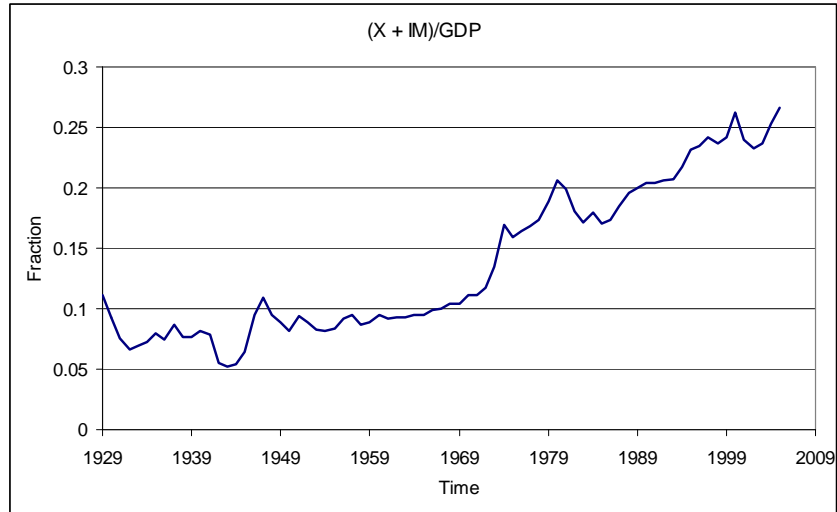


- The classical model predicts that when government expands, this crowds out personal consumption ($G \uparrow \Rightarrow C \downarrow$). Is this prediction consistent with the evidence in your graph of part a? Explain.

Yes. We do observe that the large increase in the share of the government in GDP during WWII was partly at expense of the personal consumption. After WWII we see a slow decline in the government’s share of GDP and a slow increase in the consumption share.

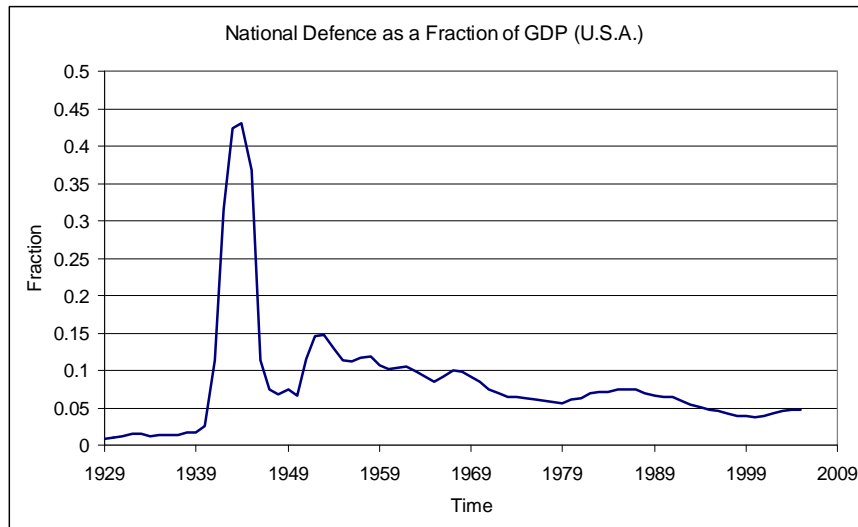
- A measure of openness of an economy is $(X+IM)/GDP$, which is the sum of exports and imports as a fraction of GDP. Plot a graph of the openness in the U.S. since 1929, and based on the graph comment on whether the U.S. economy is more open now than before.

In the graph below we see that the U.S. economy became more open.



- d. Guess what is the fraction of national defense spending in U.S. GDP? Now that you have guessed, plot the graph of the National Defense spending as a fraction of GDP (ND/GDP). Did it go up or down in the last 50 years? Is it bigger or smaller than what you guessed?

My guess is 7%, which is about half of what it is in Israel. It turns out that it is even lower than this, as can be seen in the next graph.



Observe that in the last 50 years there is a slow decline in the fraction of national defense in the GDP. During WWII it was more than 40%.