

Problem set 2 - Solution

NIPA

1. (15 points). The next table presents actual data collected by the Bureau of Economic Analysis in 2004. All numbers are in billions of dollars.

Personal Consumption Expenditures	8,214.30
Gross Investment	2300.6
Government Consumption Expenditures	1,843.40
Net Exports	-624
Compensation of Employees	6,693.40
Net Interest	630.9
Rental Income	134.2
Proprietors' income	889.6
Corporate profits	976.6
Indirect business tax net of subsidies	897.4
Consumption of fixed capital	1,435.30
Statistical Discrepancy	76.90

- a. Based on the above data calculate the GDP using the expenditure approach.

$$\begin{aligned}GDP_{EXP} &= C + I + G + NX \\ &= 8,214.30 + 2300.6 + 1,843.40 - 624 = 11,734.30\end{aligned}$$

- b. Calculate the National Income.

$$\begin{aligned}NI &= W + \text{int}_{net} + \text{rent} + \pi_p + \pi_B \\ &= 6,693.40 + 630.9 + 134.2 + 889.6 + 976.6 = 9,324.70\end{aligned}$$

- c. Calculate the GDP using the Income approach. (Self check: I provided the number for the statistical discrepancy so you can check that your calculations are correct. The statistical discrepancy is the difference between the GDP calculated by expenditure approach and the GDP calculated by the income approach).

$$\begin{aligned}GDP_{INC} &= NI + Dep + Ti \\ &= 9,324.70 + 1,435.30 + 897.4 = 11,657.40\end{aligned}$$

2. (20 points). The next table provides data on prices and output in some artificial economy for the years 2000 – 2005. The goods are labeled 1 and 2. See the illustration posted on the course web page for help. For all the questions it is recommended to use Excel.

Year	P ₁	Q ₁	P ₂	Q ₂
Year	1	100	10	2
2000	2	101	11	3
2001	3	102	12	4
2002	4	103	13	5
2003	5	104	14	6
2004	6	105	15	7

- a. Construct the series of GDP, real GDP (Q), Implicit Price Deflator (P), growth rate of real GDP (g_Q), growth rate of P (g_P) using the base year approach, when the base year is 2000. In other words, complete the table below.

Year	P ₁	Q ₁	P ₂	Q ₂	GDP	Q	P	g _Q	g _P
2000	1	100	10	2	120	120	1.000	-	-
2001	2	101	11	3	235	131	1.794	1.092	1.794
2002	3	102	12	4	354	142	2.493	1.084	1.390
2003	4	103	13	5	477	153	3.118	1.077	1.251
2004	5	104	14	6	604	164	3.683	1.072	1.181
2005	6	105	15	7	735	175	4.200	1.067	1.140

- b. Repeat part a when the base year is 2005.

Year	P ₁	Q ₁	P ₂	Q ₂	GDP	Q	P	g _Q	g _P
2000	1	100	10	2	120	630	0.190	-	-
2001	2	101	11	3	235	651	0.361	1.033	1.895
2002	3	102	12	4	354	672	0.527	1.032	1.459
2003	4	103	13	5	477	693	0.688	1.031	1.307
2004	5	104	14	6	604	714	0.846	1.030	1.229
2005	6	105	15	7	735	735	1.000	1.029	1.182

- c. Decompose the growth in nominal GDP between 2004 and 2005 into the growth of real GDP and Inflation when the base year is 2000 and when the base year is 2005.

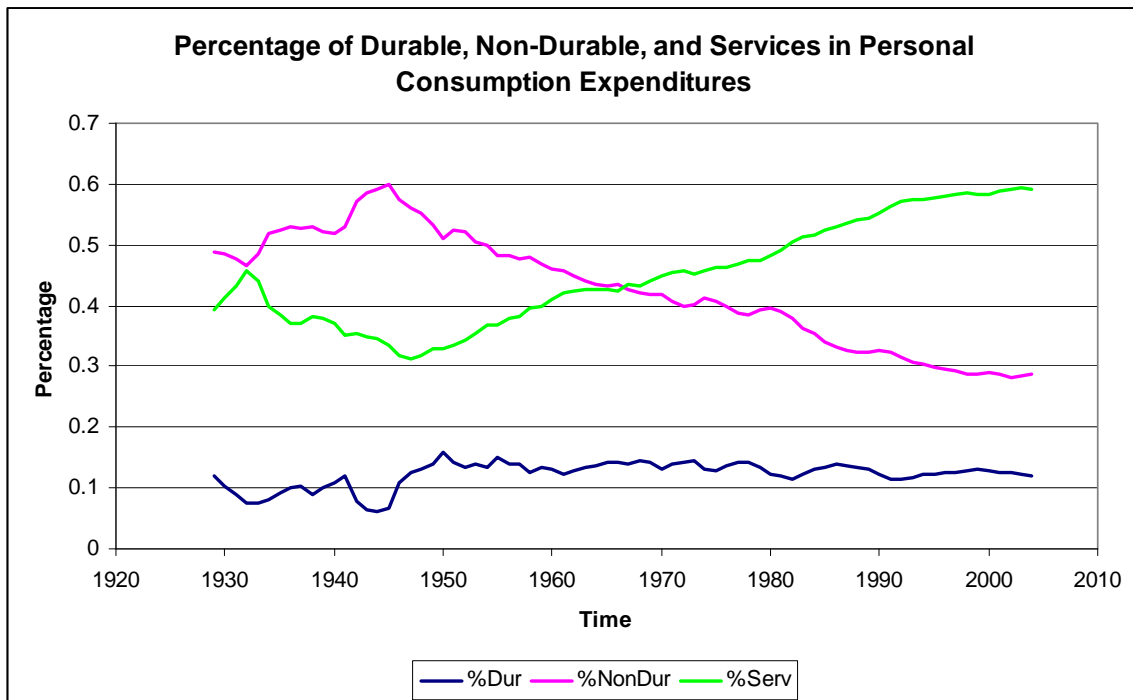
$$\begin{aligned}
 & \underline{\text{Base} = 2000} \\
 & \frac{GDP_{2005}}{GDP_{2004}} = \frac{Q_{2005}}{Q_{2004}} \cdot \frac{P_{2005}}{P_{2004}} \\
 & \frac{735}{604} = \frac{175}{164} \cdot \frac{4.20}{3.68} \\
 & 1.217 = 1.067 \cdot 1.140
 \end{aligned}$$

$$\begin{aligned} & \text{Base} = 2005 \\ \frac{GDP_{2005}}{GDP_{2004}} &= \frac{Q_{2005}}{Q_{2004}} \cdot \frac{P_{2005}}{P_{2004}} \\ \frac{735}{604} &= \frac{735}{714} \cdot \frac{1}{0.846} \\ 1.217 &= 1.029 \cdot 1.182 \end{aligned}$$

- d. What did you learn from this exercise (Does the choice of the base year matter the resulting growth in real GDP and inflation)?

When using the base year method, the choice of the base year affects the decomposition of growth in nominal GDP into real GDP and inflation. In this example, when the base year was chosen to be 2000, we got much bigger growth in real GDP and smaller inflation than if the base year was chosen to be 2005.

3. (20 points). Using the data on Personal Consumption Expenditures posted next to “HW1” on the course web page, calculate the consumption of durables, consumption of nondurables, and consumption of services as **percentages** of total consumption and plot these time series. Comment on the changes that have taken place over time in the **percentage** of services, durables and nondurables out of total consumption.

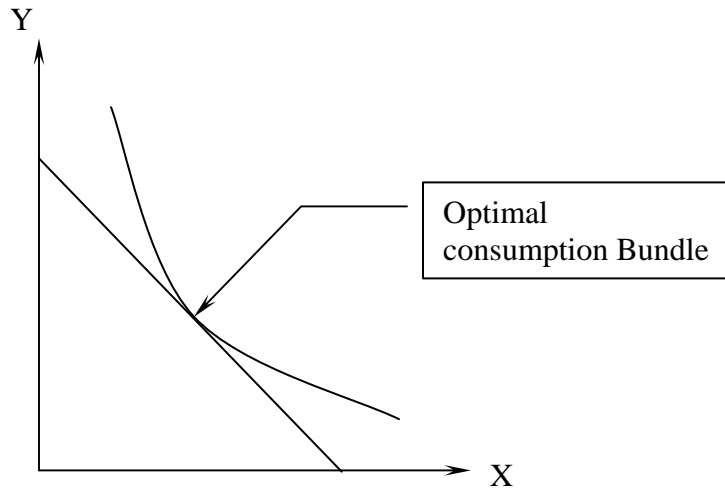


During 1932 – 1947, there was a decline in the fraction of services. After 1947 there is a steady increase in the fraction of Services. Any thoughts about what happened? This is an

interesting topic for final project and there is plenty of data on detailed components of consumption (prices and quantities) on the BEA site.

Micro Foundations

4. (10 points). Consumer choice model.
- Consider a model in which the consumer chooses optimal consumption of (X, Y) . Draw a diagram of a typical budget constraint and indifference curves, and show the optimal consumption bundle.



- Write the mathematical condition for optimality of consumption bundle and give a verbal interpretation of the condition.

The condition for optimal consumption bundle:

$$\frac{U_x(x, y)}{U_y(x, y)} = \frac{p_x}{P_y}$$

Interpretation 1: The left hand side is the slope (in absolute value) of the indifference curve and the right hand side is the slope (in absolute value) of the budget constraint. Thus, at the optimum indifference curve and the budget constraint must be tangent.

Interpretation 2: The above condition can be written as

$$\frac{U_x(x, y)}{p_x} = \frac{U_y(x, y)}{P_y}$$

The left hand side is the utility generated by extra dollar spent on X and the right hand side is the utility from extra dollar spent on Y. The optimal allocation of income between the two goods requires that those should be the same.

5. (35 points). Consumer choice model with Cobb-Douglas preferences.
- Suppose that consumer's utility is given by $U(x, y) = x^\alpha y^{1-\alpha}$, $0 < \alpha < 1$. The prices of goods X and Y are P_x, P_y and his income is I . Use the Lagrange method to derive this consumer's demand for the goods X and Y .

The Lagrangian is:

$$L = x^\alpha y^{1-\alpha} - \lambda[p_x x + p_y y - I]$$

Step 1: First order conditions:

$$\begin{aligned} L_x = \alpha x^{\alpha-1} y^{1-\alpha} - \lambda p_x &= 0 & (1) \quad \alpha x^{\alpha-1} y^{1-\alpha} &= \lambda p_x \\ L_y = (1-\alpha)x^\alpha y^{-\alpha} - \lambda p_y &= 0 & (2) \quad (1-\alpha)x^\alpha y^{-\alpha} &= \lambda p_y \end{aligned}$$

Step 2: Dividing (1) by (2) gives

$$\begin{aligned} \frac{\alpha x^{\alpha-1} y^{1-\alpha}}{(1-\alpha)x^\alpha y^{-\alpha}} &= \frac{\lambda p_x}{\lambda p_y} \\ \frac{\alpha}{1-\alpha} \frac{y}{x} &= \frac{p_x}{p_y} \end{aligned}$$

Step 3: Solving for y :

$$y = \frac{1-\alpha}{\alpha} \frac{p_x}{p_y} x$$

Step 4: Plugging into the budget constraint:

$$\begin{aligned} p_x x + p_y \left(\frac{1-\alpha}{\alpha} \frac{p_x}{p_y} x \right) &= I \\ p_x x \left(1 + \frac{1-\alpha}{\alpha} \right) &= I \\ p_x x \left(\frac{1}{\alpha} \right) &= I \end{aligned}$$

Which gives the demand for X :

$$\boxed{x = \frac{\alpha I}{p_x}}$$

And from step 3, the demand for Y :

$$\boxed{y = \frac{(1-\alpha)I}{p_y}}$$

- Based on your answers in a, what happens to the demand for X and Y if income goes up?

The demand for both goods goes up as income goes up. This means that both goods are **normal**.

- c. Based on your answer in a, what happens to the demand for X and Y if P_x goes up?

The quantity demanded of X will decrease while the demand for Y will not change.

- d. Based on your answer in a, what happens to the demand for X and Y if P_y goes up?

The demand for X will not change while the quantity demanded of Y will decrease.

- e. Based on your answer in a, what happens to the fraction of income spent on X , Y if the Income or prices change?

The fraction of income spent on each good is fixed and independent of income and prices. We see from the demand equations that the consumer will always spend a fraction α of his income on X , and the remaining fraction $1 - \alpha$ on Y .

- f. Repeat part a for $U(x, y) = \alpha \ln x + (1 - \alpha) \ln y$, $0 < \alpha < 1$.

The Lagrangian is:

$$L = \alpha \ln x + (1 - \alpha) \ln y - \lambda [p_x x + p_y y - I]$$

Step 1: First order conditions:

$$L_x = \frac{\alpha}{x} - \lambda p_x = 0 \quad (1) \quad \frac{\alpha}{x} = \lambda p_x$$

$$L_y = \frac{(1 - \alpha)}{y} - \lambda p_y = 0 \quad (2) \quad \frac{(1 - \alpha)}{y} = \lambda p_y$$

Step 2: Dividing (1) by (2) gives

$$\frac{\alpha}{1 - \alpha} \frac{y}{x} = \frac{p_x}{p_y}$$

This is the same first order condition for optimal bundle as the one obtained at the end of step 2 of section a. The rest of the steps are thus identical to part a.

Step 3: Solving for y :

$$y = \frac{1 - \alpha}{\alpha} \frac{p_x}{p_y} x$$

Step 4: Plugging into the budget constraint:

$$p_x x + p_y \left(\frac{1 - \alpha}{\alpha} \frac{p_x}{p_y} x \right) = I$$

$$p_x x \left(1 + \frac{1 - \alpha}{\alpha} \right) = I$$

$$p_x x \left(\frac{1}{\alpha} \right) = I$$

Which gives the demand for X:

$$x = \frac{\alpha I}{p_x}$$

And from step 3, the demand for Y:

$$y = \frac{(1 - \alpha)I}{p_y}$$

- g. Explain why did you get the same demand in parts a and f even though the utility functions are different.

The utility function in part f is a logarithmic transformation of the utility in part a.

In part a: $U(x, y) = x^\alpha y^{1-\alpha}$

In part f: $V(x, y) = \ln(U(x, y)) = \ln(c^\alpha y^{1-\alpha}) = \alpha \ln x + (1 - \alpha) \ln y$

We have shown in class that applying a monotone increasing transformation to a utility function creates a **different** utility function, but the new one represents the **same preferences**. In other words, suppose that consumer A has utility $U(x, y) = x^\alpha y^{1-\alpha}$ and consumer B has utility $V(x, y) = \alpha \ln x + (1 - \alpha) \ln y$, then they have the same preferences and thus will make the same choices and will have the same demand.

Formally, we say that utility functions are invariant with respect to monotone increasing transformations.