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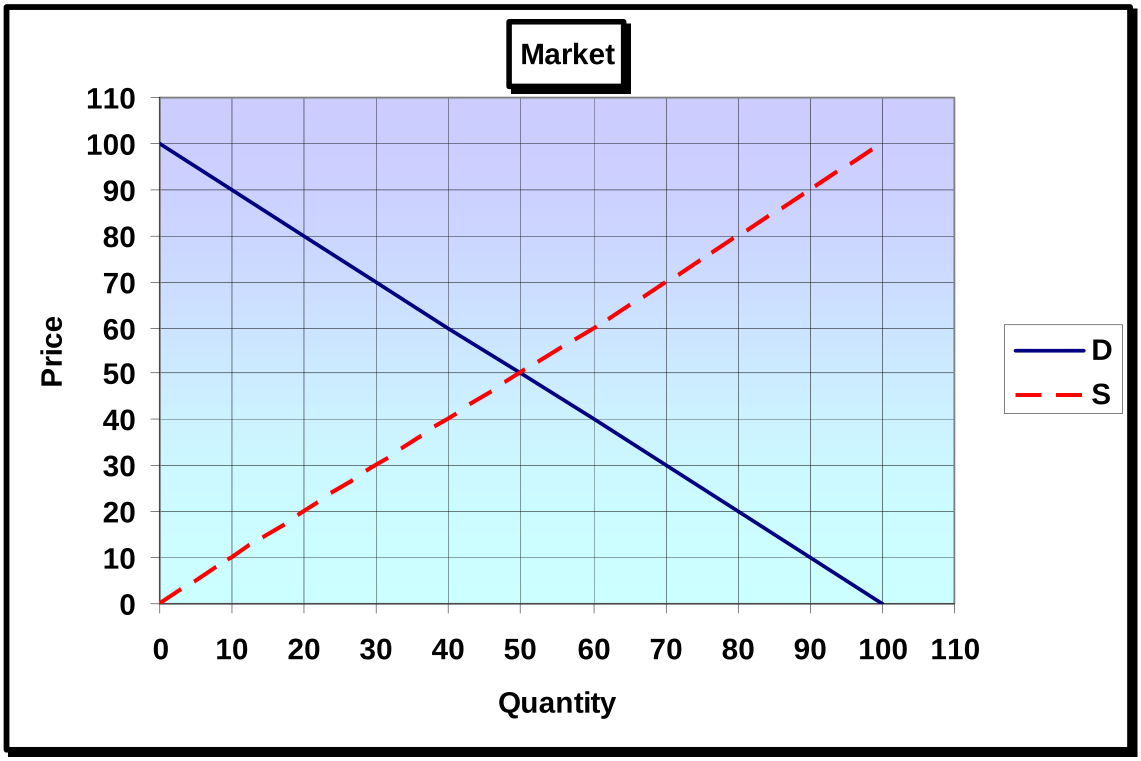
**ECON 302** — **Fall 2011**

## Problem set 1 - Solution

### Preliminaries

1. Suppose that the demand curve for some good is given by $P = 100 - Q$ and the supply curve is given by $P = Q$.
   a. Plot the graphs of the market. Make sure that the curves are plotted to scale.




   b. Solve for the market equilibrium, i.e. find the equilibrium price and quantity and denote them by $P^*, Q^*$.

$$100 - Q = Q$$
$$100 = 2Q$$
$$Q^* = 50,\ P^* = 50$$

2. Suppose that the price of a pair of shoes was \$60 in 2004 and the \$72 in 2005. What is the percentage increase in the price? Show your calculations.

$$\frac{72 - 60}{60} = \frac{12}{60} = \frac{1}{5} = 20\%$$

3. How much is 25% out of 300? Show your calculations.

$$25\% \cdot 300 = \frac{25}{100} \cdot 300 = 25 \cdot 3 = 75$$

4. Prove that $\ln(x^{\alpha} y^{\beta}) = \alpha \ln x + \beta \ln y$. Hint: there are two steps. In each step you need you need to use one of the rules of logarithms from the notes. Write the steps and indicate which rule you used.

Step 1: $\ln(x^{\alpha} y^{\beta}) = \ln(x^{\alpha}) + \ln(y^{\beta})$, using the rule $\ln(x \cdot y) = \ln(x) + \ln(y)$, in words, the ln of a product is the sum of ln's.

Step 2: $\ln(x^{\alpha}) + \ln(y^{\beta}) = \alpha \ln x + \beta \ln y$, using the rule $\ln(x^{a}) = a \ln x$, "the exponent comes out".

5. Suppose that you deposit certain amount of money in the saving account with interest rate of 2% per year. How long will it take for your money to double? Show your calculations. You are not allowed to use any approximation formulas, such as "the rule of 70".

Let $y_0$ be the initial amount of your deposit. This amount doubling means that after certain time you have $2y_0$. Thus, the equation to be solved is

$$y_0 (1.02)^t = 2y_0$$

Notice that the initial amount cancels out, so the equation becomes $(1.02)^t = 2$, and the only unknown is *t*. This means that the time that takes your saving to double does not depend on how much money you have in the beginning, but only depends on the interest rate (how fast your savings are growing). Taking ln's of both sides

$$t \ln(1.02) = \ln(2)$$

$$t = \frac{\ln 2}{\ln 1.02} \approx 35$$

**Remark:** I said not use the "rule of 70" here, but it is kind of "cool" thing to know. So here I derive it for you. Suppose that the growth rate is *r*, then the doubling time is

$$t = \frac{\ln 2}{\ln(1+r)}$$

Now $\ln(2) = 0.693147 \approx 0.7$ and $\ln(1+r) \approx r$ for small *r*. Thus

$$t \approx \frac{0.7}{r} = \frac{70}{100 \cdot r\%}$$

So the doubling time is approximately 70 divided by the percentage growth rate. For example, if *r=2%* then the doubling time is approximately 70/2=35.

## Introduction

6. Suppose that some variable is growing at constant rate.
   a. Prove that the natural logarithm of that variable is a linear function of time.

If a variable is growing at constant rate $g$, then its value at time $t$ is given by

$$y_t = y_0(1+g)^t$$

Taking logs of that

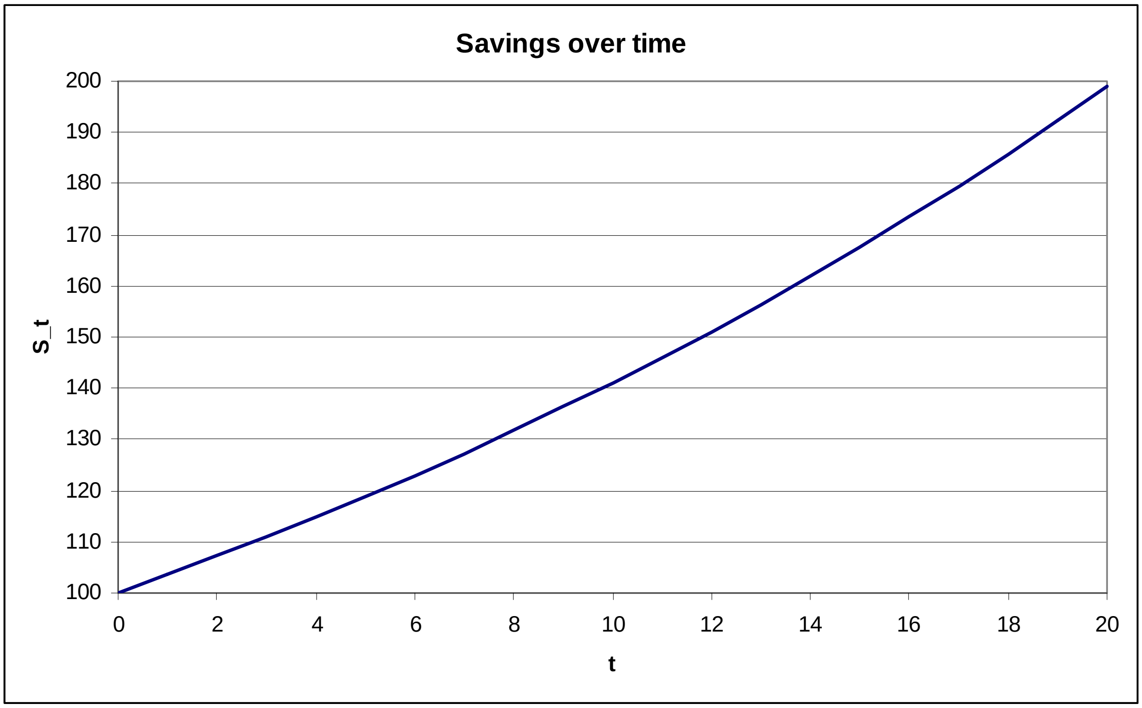
$$\ln y_t = \ln y_0 + t\ln(1+g)$$

Which is a linear function of $t$.

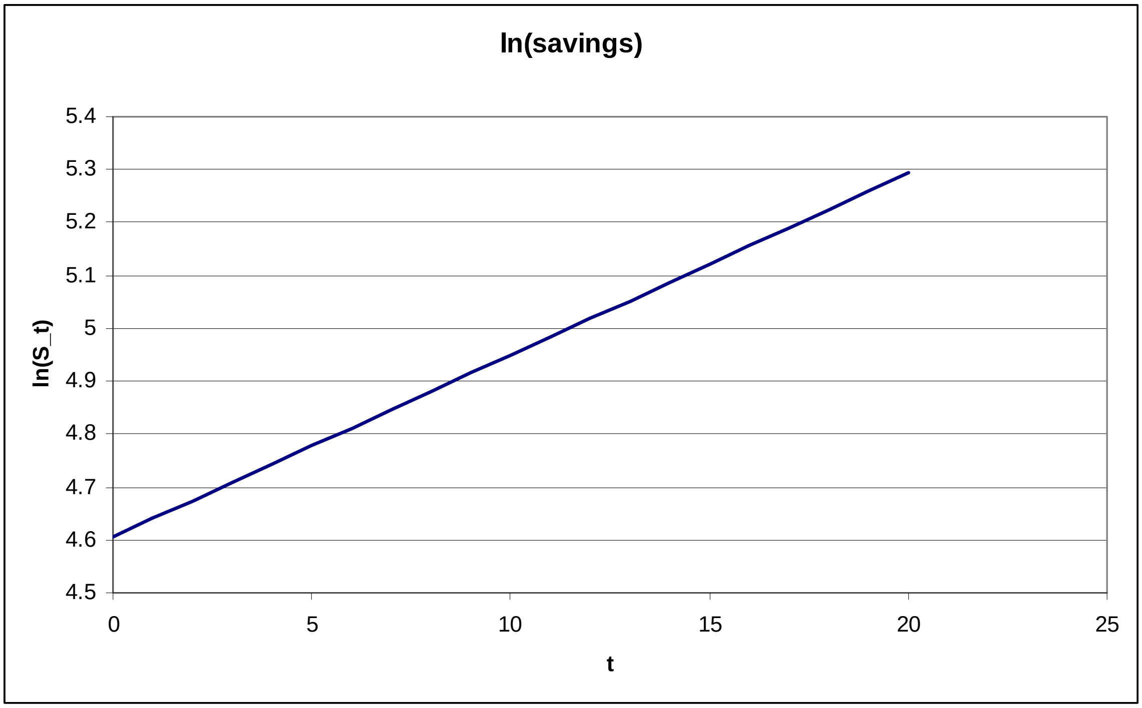
   b. Find the intercept and slope of the linear function in part a.

The intercept is $\ln(y_0)$ and the slope is $\ln(1+g)$.

7. This question illustrates that when a variable grows at constant rate, then the graph of the ln of the variable is a linear function of time, with slope that is approximately equal to the growth rate of the original variable (when that growth rate is small). Suppose that you put 100$ in a savings account at annual interest rate of 3.5%. Let $S_t$ be the amount of savings at time $t$, where $t = 0, 1, \ldots, 20$.
   a. Using Excel, plot the graph that shows the amount of savings that you have in each of the years $t = 0, 1, \ldots, 20$. That is, plot $S_t$ against $t$.



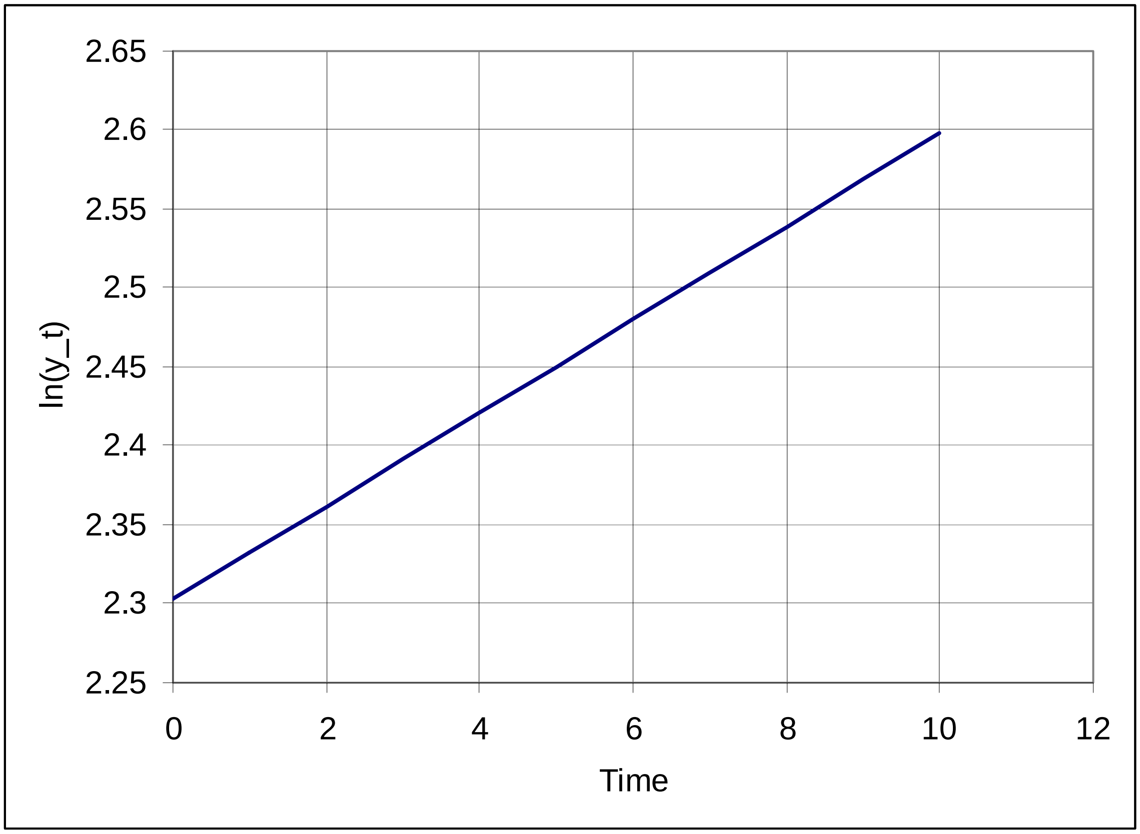
b. Using Excel, plot the graph of $\ln(S_t)$ against *t*.




c. What can you say about the shape of the graph in part b? What is the slope of the graph in part b?

The $\ln(S_t)$ is a linear function of time. The slope of the line is $\ln(1.035) \cong 0.0344$, which is approximately the (net) growth rate of the original variable $S_t$.

8. Based on the following graph, what is the approximate growth rate of the variable $y_t$?




a. 2.3%
b. 10%
c. 3%
d. 6%

To see why, recall that the slope of the $\ln(y_t)$ is approximately equal to the growth rate of $y_t$. The slope of the above line is $\Delta Y / \Delta X = (2.6 - 2.3)/10 = 0.3/10 = 0.03 = 3\%$.

9. Based on the graphs from the "Introduction" slides that displays the real GDP per capita and the ln of that variable in different countries (ARG = Argentina, CHN = China, KOR = South Korea, TCD = Chad, USA = USA).
   a. Are all the above countries experiencing sustained growth in real GDP per capita?

No. Argentina had negative growth in real GDP per capita during the 80's. Chad experienced negative growth most of the time.

   b. What can you say about the growth rate of real GDP per capita in the USA and Argentina prior to 1975? Explain how you reached your conclusion.

About the same. By looking at the ln(real GDP per capita) of both countries, we observe that before the mid seventies, the two trends were approximately parallel and straight lines. This means that the slopes of the trends were the same, and thus the growth rate was about the same.

10. Based on the introduction, what was the highest unemployment rate in the U.S. since WWII? What was the lowest?

The highest is about 11% and the lowest is about 2.5%