San Francisco State University **Michael Bar**
ECON 302 **Fall 2008**

Final Exam

Wednesday, December 17

2 hours, 30 minutes

Name: ___________________________________

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (30 points). Consider the Classical model studied in class, and briefly described as follows. The consumer derives utility from consumption C and leisure l according to $U(C,l) = \alpha \ln C + (1-\alpha)\ln l$. He is endowed with h hours which he can allocate between leisure and work L_S . The real wage is w. The consumer owns a firm and receives dividend income (profit) π . The firm produces output Y using technology $Y = AK^{\theta} L_D^{1-\theta}$, where A is productivity parameter (TFP), K is the capital owned by the firm, and L_D is labor employed by the firm. The government taxes labor income at the rate of t_w and dividend income at the rate of t_π **.**

   a. (5 points). Write the consumer's problem.

Consumer's problem

$$\max_{C,l} \alpha \ln C + (1-\alpha)\ln l$$

$$s.t.$$

$$C = w(h-l)(1-t_w) + \pi(1-t_\pi)$$

   b. (5 points). Write the consumer's demand for consumption.

For finding the demand it is convenient to rewrite the budget constraint as follows:

$$C + w(1-t_w)l = wh(1-t_w) + \pi(1-t_\pi)$$

Demand for consumption: $C = \alpha\left[wh(1-t_w) + \pi(1-t_\pi)\right]$

   c. (5 points). Write the consumer's demand for leisure and his labor supply.

Demand for leisure: $l = \dfrac{(1-\alpha)\left[wh(1-t_w) + \pi(1-t_\pi)\right]}{w(1-t_w)} = (1-\alpha)\left(h + \dfrac{\pi}{w}\dfrac{(1-t_\pi)}{(1-t_w)}\right)$

Labor supply: $L_S = h - l = h - (1-\alpha)\left(h + \dfrac{\pi}{w}\dfrac{(1-t_\pi)}{(1-t_w)}\right)$

d. (5 points). In this model, changes in productivity do not affect the equilibrium labor. True/False, circle the correct answer and provide a brief mathematical proof.

Plugging the profit and wage from the firm's problem into the labor supply, we get

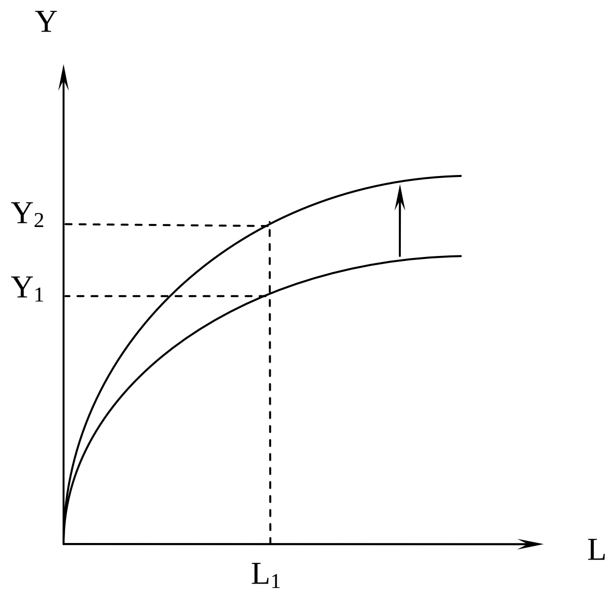
$$L = h - (1-\alpha)\left(h + \frac{\theta A K^{\theta} L^{1-\theta}}{(1-\theta) A K^{\theta} L^{-\theta}} \frac{(1-t_{\pi})}{(1-t_{w})} \right)$$

$$= h - (1-\alpha)\left(h + \left(\frac{\theta}{1-\theta} \right) L \frac{(1-t_{\pi})}{(1-t_{w})} \right)$$

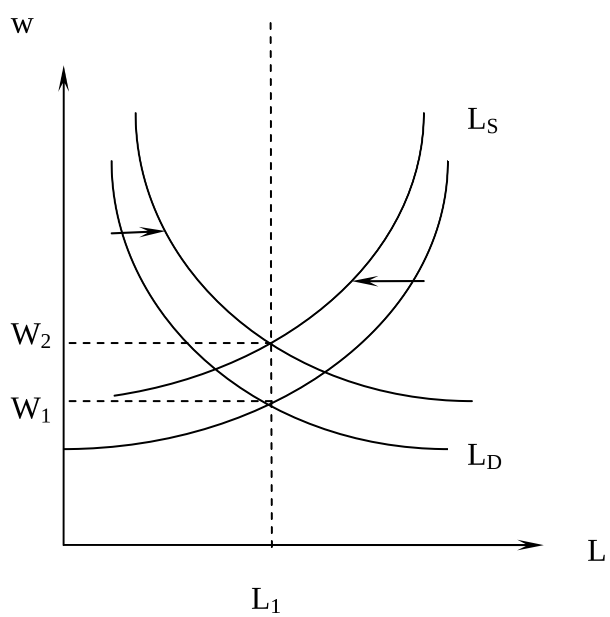
Thus, the productivity (A) cancels out and labor supply is independent of taxes.

e. (10 points). Using fully labeled graphs of the production function and labor market, illustrate the effect of productivity growth ($A\uparrow$) on equilibrium output (Y^*), equilibrium real wage (w^*) and equilibrium employment (L^*).

Production function



Labor market



2. (15 points). Consider the two-period model of consumption and saving discussed in the class. There are *N* identical consumers that live for two periods (1 and 2) and derive utility from consumption c_1 and c_2 in the two periods: $U(c_1, c_2)$. Consumers receive income y_1 and y_2 in the two periods and pay a lump sum tax t_1 and t_2 to the government. The consumers decide how much to consume in each period and how much to save in the first period. We denote the saving in the first period by *s*. Consumers can borrow and lend at real interest rate *r*, which is assumed exogenously given. Thus the budget constraints in the two periods are

$$BC_1: \quad c_1 + s = y_1 - t_1$$

$$BC_2: \quad c_2 = y_2 - t_2 + (1+r)s$$

The government collects tax revenues $T_1 = N \cdot t_1$ and $T_2 = N \cdot t_2$, and spends G_1 and G_2 in the two periods. The government can borrow and lend at real interest rate *r* with the constraint that the present value of spending = present value of taxes

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

a. (5 points). Suppose that the real interest rate is $r = 7\%$ and the government gives a tax cut of 200 in the first period. Find the necessary change in the second period's taxes that would keep the present value of taxes unchanged. Show your calculations.

$$T_1 - 200 + \frac{T_2 + \Delta_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

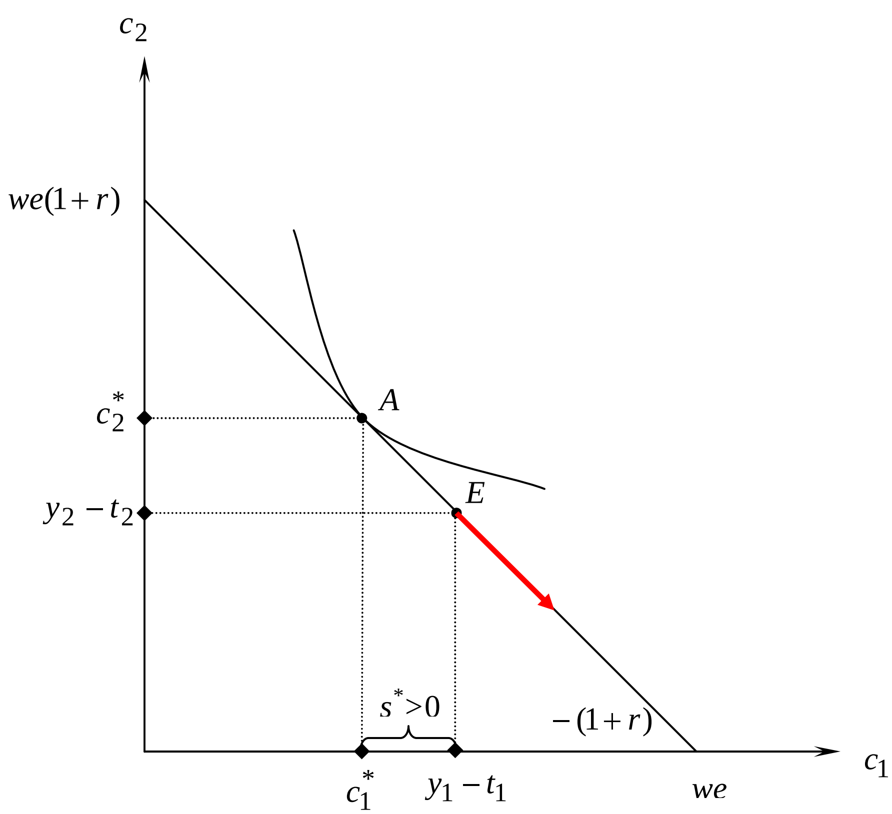
$$-200 + \frac{\Delta_2}{1+r} = 0$$

$$\Delta_2 = 200(1+r) = 200 \cdot 1.07 = 214$$

b. (5 points). State the Ricardian Equivalence Theorem.

Theorem (Ricardian equivalence):
If the present value of government spending remains unchanged, then changes in the taxes do not affect the households' optimal consumption choice (c_1^*, c_2^*).

c. (5 points). Using a graph, explain why the Ricardian Equivalence Theorem holds in this model.



In this model, if the present value of spending does not change, so does the present value of taxes on each consumer (since tax burden is shared equally between all consumers). For example, a tax cut in the 1st period will require a tax increase in the 2nd period, in a way that the endowment point moves down along the budget constraint (as shown in the above graph). This change in endowment will not alter the budget constraint, and therefore the optimal consumption in each period is unchanged (point A in the above graph).

3. (15 points). Consider the model of optimal investment, briefly described as follows. A firm can produce output in two periods according to

$$Y_1 = A_1 K_1^{\theta} L_1^{1-\theta}$$

$$Y_2 = A_2 K_2^{\theta} L_2^{1-\theta}$$

where A_1, A_2 are productivity parameters, K_1, K_2 are physical capital, and L_1, L_2 are labor in the two periods. The firm owns the capital stock and the consumers own the firm. The capital stock evolves according to

$$K_2 = (1-\delta)K_1 + I$$

where δ is depreciation and *I* is investment. The capital stock is exogenously given, and the firm can choose L_1, L_2, K_2, I . The dividends in each period are

$$\pi_1 = Y_1 - wL_1 - I$$

$$\pi_2 = Y_2 + (1-\delta)K_2 - w_2 L_2$$

   a. (10 pt). Derive the optimal investment condition and provide economic interpretation of it.

<u>Firm's problem</u>

$$\begin{cases} \max\limits_{L_1,L_2,I,K_2} V = A_1 K_1^{\theta} L_1^{1-\theta} - w_1 L_1 - I + \dfrac{A_2 K_2^{\theta} L_2^{1-\theta} + (1-\delta)K_2 - w_2 L_2}{1+r} \\ s.t. \\ K_2 = (1-\delta)K_1 + I \end{cases}$$

Substituting the constraint into the objective gives

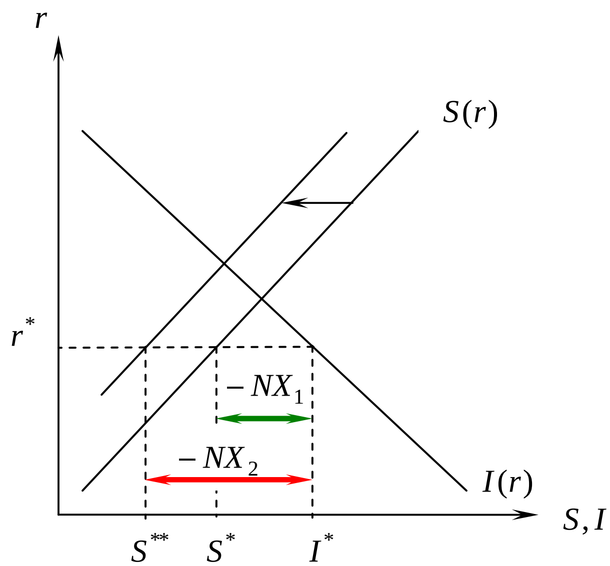
$$\max_{L_1,L_2,I,K_2} V = A_1 K_1^{\theta} L_1^{1-\theta} - w_1 L_1 - K_2 + (1-\delta)K_1 + \frac{A_2 K_2^{\theta} L_2^{1-\theta} + (1-\delta)K_2 - w_2 L_2}{1+r}$$

F.O.C. for K_2:

$$\frac{\partial V}{\partial K_2} = -1 + \frac{\theta A_2 K_2^{\theta-1} L_2^{1-\theta} + 1 - \delta}{1+r} = 0$$

The cost of increasing future capital by 1 is a decline in current dividends by 1 unit (the first term in the derivative). The benefit in the next period consists of the marginal product of capital and the non-depreciated value of the extra unit of capital. Dividing the next period's benefit by 1 + *r* gives the present value of the benefit.

b. (5 pt). Suppose the government increases its deficit. Illustrate graphically the impact of this event on the capital market in an economy with trade deficit. State what happens to the equilibrium saving, investment, and trade deficit.



Equilibrium saving declines from S^* to S^{**}, equilibrium investment stays the same, and trade deficit increases from $-NX_1$ to $-NX_2$ (or from $(I^* - S^*)$ to $(I^* - S^{**})$).

4. (15 points). Suppose that the public wants to hold currency/deposit ratio of $cd = 0.2$, and the required reserve/deposit ratio is $rd = 0.1$. The initial consolidated balance sheet of commercial banks is:

Assets	Liabilities
$R = 5$	$D = 50$
$B_G = 15$	
$L = 30$	
50	50

a. (5 pt). Find the monetary base, the money supply and the money multiplier in this economy.

$$CU = 0.2 \cdot 50 = 10$$

$$MB = CU + R = 10 + 5 = 15$$

$$M = CU + D = 10 + 50 = 60 \ \ (or\ M = mm \cdot MB)$$

$$mm = \frac{cd + 1}{cd + rd} = \frac{0.2 + 1}{0.2 + 0.1} = 4$$

b. (10 pt). Now suppose that the central bank increases the required reserve/deposit ratio to 0.4. Find the new monetary base, the new money multiplier, the new money supply, the new currency held by the public, and show the new balance sheet of the commercial banks.

Assets	Liabilities
$R = 10$	$D = 25$
$B_G = 15$	
$L = 0$	
25	25

$MB = 15$ (unchanged)

$$\text{mm} = \frac{cd + 1}{cd + rd} = \frac{0.2 + 1}{0.2 + 0.4} = 2$$

$$M = mm \cdot MB = 2 \cdot 15 = 30$$

$$CU = \frac{cd}{cd + rd} \cdot MB = \frac{0.2}{0.2 + 0.4} \cdot 15 = 5$$

$$D = M - CU = 30 - 5 = 25$$

$$R = rd \cdot D = 0.4 \cdot 25 = 10$$

5. (15 points). Let P and P^* be the price indexes in the domestic economy and foreign economy respectively. Suppose that the price index is a weighted average of traded goods (indexed by T) and non-traded goods (indexed by N):

$$P = \alpha P^T + (1-\alpha)P^N \quad 0 \leq \alpha \leq 1$$

$$P^* = \beta P^{*T} + (1-\beta)P^{*N} \quad 0 \leq \beta \leq 1$$

   a. (5 points). Assuming that: (1) the weights on traded and non-traded goods in the price index are fixed for both countries, (2) the ratio of prices of non-traded to traded goods is fixed in both countries, and (3) the PPP holds for traded goods, write the relationship between the growth of the exchange rate ($\hat{e}$), the domestic inflation (π) and foreign inflation (π^*).

$$\hat{e} = \pi^* - \pi$$

   b. (5 points). Using the quantity theory of money and assuming that the money velocity is fixed, write the relationship between the growth of the exchange rate, the growth rates of the domestic money ($\hat{M}$) and foreign money ($\hat{M}^*$) and the growth rates of domestic real GDP ($\hat{Y}$) and foreign real GDP ($\hat{Y}^*$).

$$\hat{e} = (\hat{M}^* - \hat{M}) + (\hat{Y} - \hat{Y}^*)$$

c. (5 points). Some countries that experience high inflation, peg their currency to another "stable" currency. Using the model described in this question, explain how pegging the domestic currency helps reducing the domestic inflation.

Fixing the exchange rate means that $\hat{e} = 0$ and we have

$$\hat{e} = \pi^* - \pi = 0$$

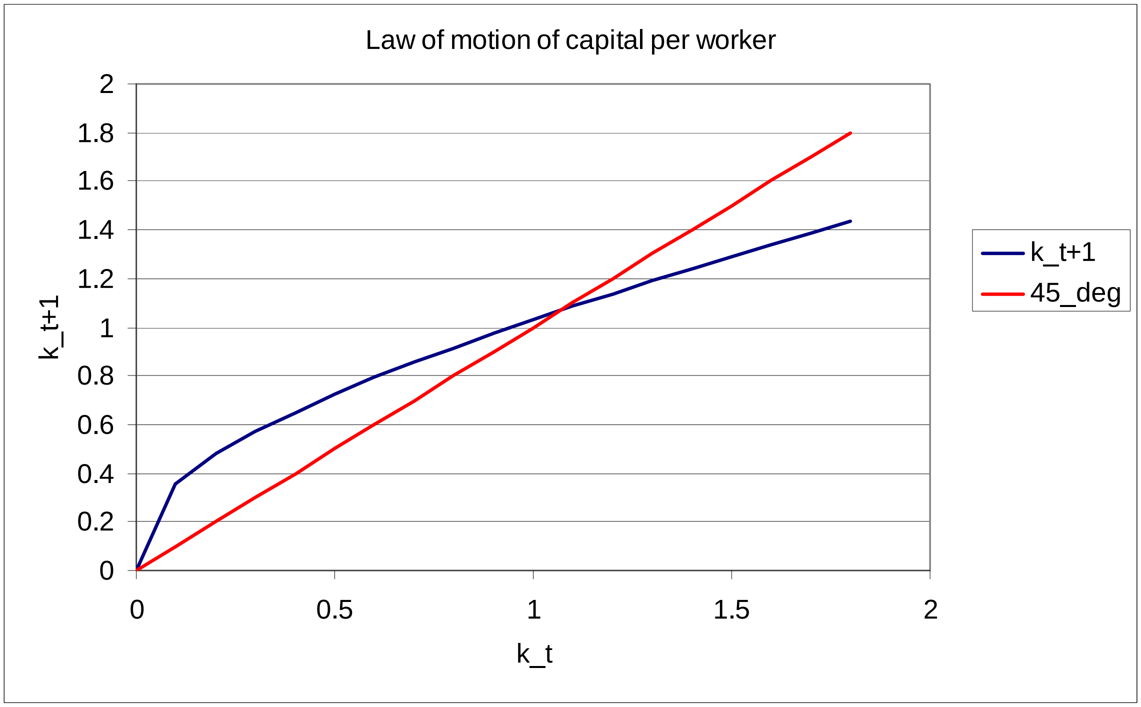
Thus, the domestic inflation becomes the same as the foreign inflation to which the currency is pegged.

6. (10 points). Consider the Solow model discussed in class. Output is produced according to $Y_t = A_t K_t^{\theta} L_t^{1-\theta}$, $0 < \theta < 1$. Capital evolves according to $K_{t+1} = K_t(1-\delta) + I_t$, where δ is depreciation rate and I_t is investment. People save a fraction *s* of their income, and the total saving and total investment in this (closed) economy is $S_t = I_t = sY_t$. The population of workers (and the total population) grows at rate *n*, i.e. $L_{t+1} = (1+n)L_t$.
   a. (5 points). Derive the law of motion of capital per worker and plot its graph for a fixed level of TFP.

$$K_{t+1} = (1-\delta)K_t + I_t$$

$$\frac{K_{t+1}}{L_{t+1}} = \frac{(1-\delta)K_t}{(1+n)L_t} + \frac{sAK_t^{\theta} L_t^{\theta}}{(1+n)L_t}$$

$$k_{t+1} = \frac{(1-\delta)k_t}{1+n} + \frac{sAk^{\theta}}{1+n}$$



b. (5 points). According to the Solow model, will the growth in standard of living continue forever? Explain with the help of a graph.

It depends. In the Solow model unless there is a growth in TFP (A) we showed that all the variables (capital per worker and therefore output per worker and consumption per worker) converge to a steady state level. So the key to sustained growth according to the Solow model is growth in productivity. The next figure shows the law of motion of capital as a result of an increase in productivity (from A_1 to A_2).