

Final Exam

Tuesday, May 19

2 hours, 30 minutes

Name: _____

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (30 points). Consider the Classical model studied in class, and briefly described as follows. The consumer derives utility from consumption C and leisure l according to $U(C,l) = \alpha \ln C + (1 - \alpha) \ln l$. He is endowed with h hours which he can allocate between leisure and work L_S . The real wage is w . The consumer owns a firm and receives dividend income (profit) π . The firm produces output Y using technology $Y = AK^\theta L_D^{1-\theta}$, where A is productivity parameter (TFP), K is the capital owned by the firm, and L_D is labor employed by the firm. The government taxes labor income at the rate of t_w and dividend income at the rate of t_π .

- a. (5 points). Write the consumer's problem.

$$\begin{aligned} & \text{Consumer's problem} \\ & \max_{C,l} \alpha \ln C + (1 - \alpha) \ln l \\ & \text{s.t.} \\ & C = w(h-l)(1-t_w) + \pi(1-t_\pi) \end{aligned}$$

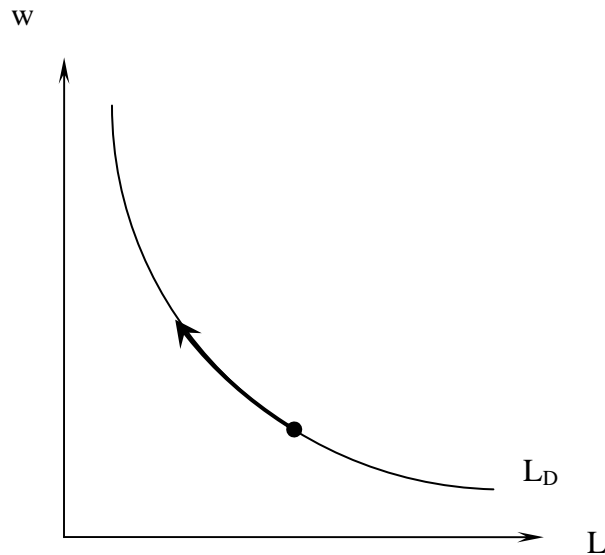
- b. (5 points). Write the firm's problem.

$$\max_{L_D} \pi = AK^\theta L_D^{1-\theta} - wL_D$$

- c. (5 points). Derive the firm's demand for labor.

$$\begin{aligned} \frac{\partial \pi}{\partial L_D} &= (1-\theta)AK^\theta L_D^{-\theta} - w = 0 \\ L_D &= \left(\frac{(1-\theta)AK^\theta}{w} \right)^{1/\theta} \end{aligned}$$

- d. (5 points). Illustrate graphically the impact on the labor demand curve of an increase in real wage, and provide economic intuition for it.



With higher wage, the firm wants to employ less labor. Remember that competitive firm employs labor up to the point where the marginal product of labor is equal to the wage. Since the marginal product is decreasing, higher wage can be paid only when fewer workers are employed.

- e. (5 points). In the classical model, unemployment rate is (circle the correct answer):
- i. always zero
 - ii. always positive
 - iii. can be positive
 - iv. none of the above
- f. (5 points). **"In the classical model the government cannot affect output and employment with fiscal policies"**. This statement is (circle the correct answer):
- i. always true
 - ii. true if labor and non-labor income are taxed at different rates
 - iii. true if labor and non-labor income are taxed at the same rate
 - iv. never true

2. (15 points). Consider the two-period model of consumption and saving discussed in the class. There are N identical consumers that live for two periods (1 and 2) and derive utility from consumption c_1 and c_2 in the two periods: $U(c_1, c_2)$. Consumers receive income y_1 and y_2 in the two periods and pay a lump sum tax t_1 and t_2 to the government. The consumers decide how much to consume in each period and how much to save in the first period. We denote the saving in the first period by s . Consumers can borrow and lend at real interest rate r , which is assumed exogenously given. Thus the budget constraints in the two periods are

$$BC_1: c_1 + s = y_1 - t_1$$

$$BC_2: c_2 = y_2 - t_2 + (1+r)s$$

The government collects tax revenues $T_1 = N \cdot t_1$ and $T_2 = N \cdot t_2$, and spends G_1 and G_2 in the two periods. The government can borrow and lend at real interest rate r with the constraint that the present value of spending = present value of taxes:

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

- a. (5 points). Suppose that the real interest rate is $r = 3\%$ and the government gives a tax cut of 1000 in the first period. Find the necessary change in the second period's taxes that would keep the present value of taxes unchanged. Show your calculations.

$$T_1 - 1000 + \frac{T_2 + \Delta_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

$$-1000 + \frac{\Delta_2}{1+r} = 0$$

$$\Delta_2 = 1000(1 + 0.03) = 1030$$

- b. (5 points). Assuming that consumption in both periods is a normal good, an increase in current income y_1 will increase the current consumption by the same amount (i.e. $\Delta c_1 = \Delta y_1$). True/false, circle the correct answer and provide a short proof.

Suppose income in period 1 goes up by Δy_1 . Since income in both periods is a normal good, the individual wants to increase both c_1 and c_2 . The only way to increase c_2 is by saving some of the increase in first period's income. Therefore we must have $\Delta c_1 < \Delta y_1$.

Same can be written in concise mathematical language:

$$y_1 \uparrow \Rightarrow c_1 \uparrow, c_2 \uparrow \Rightarrow s \uparrow \Rightarrow \Delta c_1 < \Delta y_1$$

- c. (5 points). State the **Ricardian Equivalence Theorem**.

Theorem (Ricardian equivalence):

If the present value of government spending remains unchanged, then changes in the taxes do not affect the households' optimal consumption choice (c_1^*, c_2^*) .

3. (15 points). Consider the model of optimal investment, briefly described as follows. A firm can produce output in two periods according to

$$Y_1 = A_1 K_1^\theta L_1^{1-\theta}$$

$$Y_2 = A_2 K_2^\theta L_2^{1-\theta}$$

where A_1, A_2 are productivity parameters, K_1, K_2 are physical capital, and L_1, L_2 are labor in the two periods. The firm owns the capital stock and the consumers own the firm. The capital stock evolves according to

$$K_2 = (1 - \delta)K_1 + I$$

where δ is depreciation and I is investment. The capital stock is exogenously given, and the firm can choose L_1, L_2, K_2, I . The dividends in each period are

$$\pi_1 = Y_1 - wL_1 - I$$

$$\pi_2 = Y_2 + (1 - \delta)K_2 - w_2L_2$$

- a. (10 pt). Derive the optimal investment condition and provide economic interpretation of it.

Firm's problem

$$\left\{ \begin{array}{l} \max_{L_1, L_2, I, K_2} V = A_1 K_1^\theta L_1^{1-\theta} - w_1 L_1 - I + \frac{A_2 K_2^\theta L_2^{1-\theta} + (1 - \delta)K_2 - w_2 L_2}{1 + r} \\ s.t. \\ K_2 = (1 - \delta)K_1 + I \end{array} \right.$$

Substituting the constraint into the objective gives

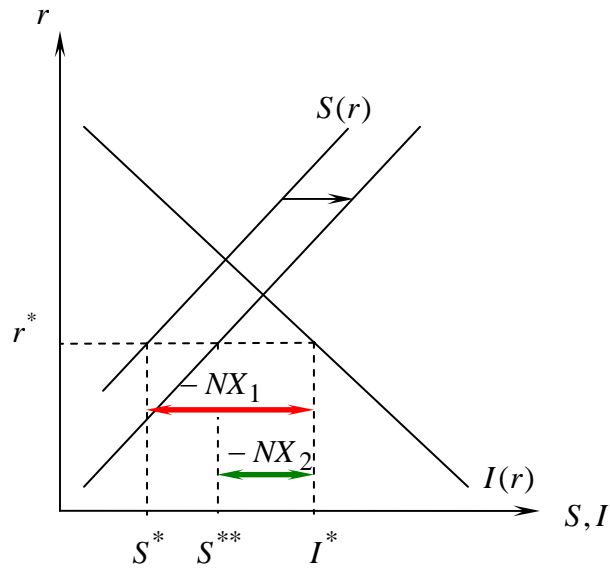
$$\max_{L_1, L_2, K_2} V = A_1 K_1^\theta L_1^{1-\theta} - w_1 L_1 - K_2 + (1 - \delta)K_1 + \frac{A_2 K_2^\theta L_2^{1-\theta} + (1 - \delta)K_2 - w_2 L_2}{1 + r}$$

F.O.C. for K_2 :

$$\frac{\partial V}{\partial K_2} = -1 + \frac{\theta A_2 K_2^{\theta-1} L_2^{1-\theta} + 1 - \delta}{1 + r} = 0$$

The cost of increasing future capital by 1 is a decline in current dividends by 1 unit (the first term in the derivative). The benefit in the next period consists of the marginal product of capital and the non-depreciated value of the extra unit of capital. Dividing the next period's benefit by $1 + r$ gives the present value of the benefit.

- b. (5 pt). Suppose the government decreases its deficit. Illustrate graphically the impact of this event on the capital market in an economy with trade deficit and **clearly state in words what happens to the equilibrium saving, investment, and trade deficit.**



Equilibrium saving **increases** from S^* to S^{**} , equilibrium investment stays the same, and trade deficit **decreases** from $-NX_1$ to $-NX_2$ (or from $(I^* - S^*)$ to $(I^* - S^{**})$).

Remark: it is possible that trade deficit will disappear or even become a trade surplus, after the change.

4. (15 points). Suppose that the public wants to hold currency/deposit ratio of $cd = 0.2$, and the required reserve/deposit ratio is $rd = 0.1$. The initial consolidated balance sheet of commercial banks is:

Assets	Liabilities
$R = 5$	$D = 50$
$B_G = 15$	
$L = 30$	
50	50

- a. (5 points). Find the monetary base, the money supply and the money multiplier in this economy.

$$CU = 0.2 \cdot 50 = 10$$

$$MB = CU + R = 10 + 5 = 15$$

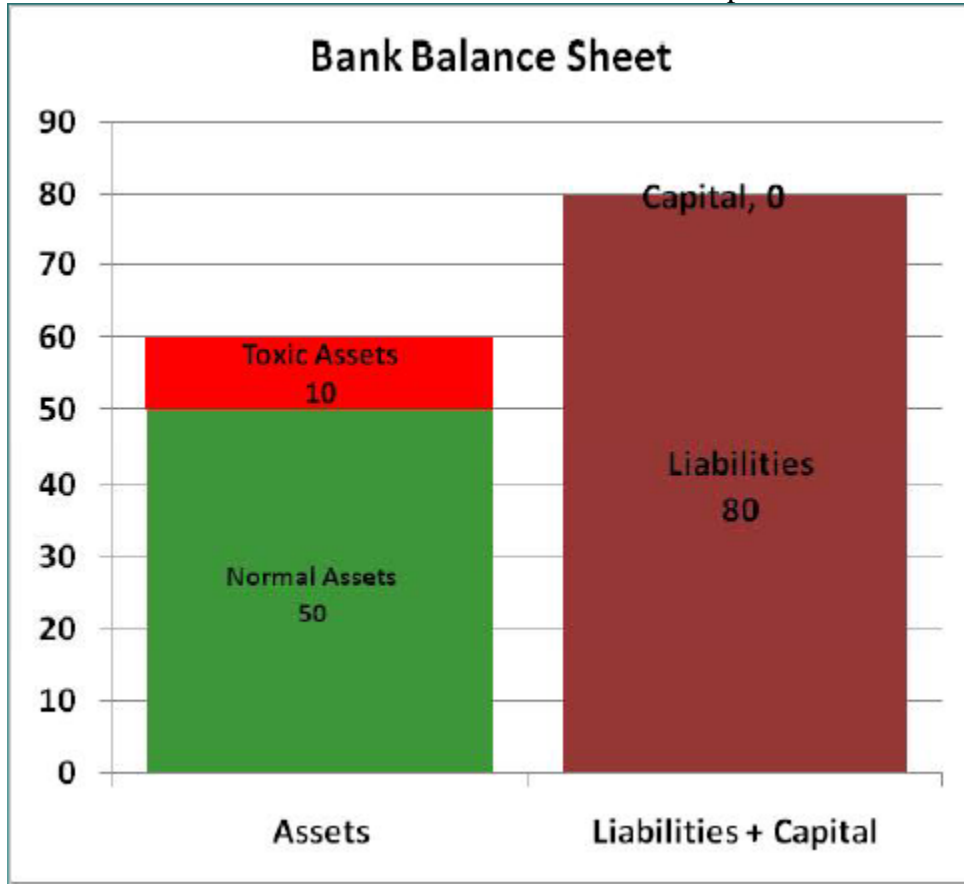
$$M = CU + D = 10 + 50 = 60 \text{ (or } M = mm \cdot MB)$$

$$mm = \frac{cd + 1}{cd + rd} = \frac{0.2 + 1}{0.2 + 0.1} = 4$$

- b. (5 points). If the central bank increases the monetary base by \$100, the money supply will increase by \$ 400 (write your answer in the blank space).

c. (5 points). Present a balance sheet of a bank experiences **Balance Sheet Insolvency**.

Any balance sheet with assets smaller than liabilities. For example:



5. (15 points). Let P and P^* be the price indexes in the domestic economy and foreign economy respectively. Suppose that the price index is a weighted average of traded goods (indexed by T) and non-traded goods (indexed by N):

$$P = \alpha P^T + (1 - \alpha) P^N \quad 0 \leq \alpha \leq 1$$

$$P^* = \beta P^{*T} + (1 - \beta) P^{*N} \quad 0 \leq \beta \leq 1$$

- a. (5 points). Define the concept of **Purchasing Power Parity**.

PPP is a way to compare the purchasing power of unit of currency (usually \$1) in different countries. PPP holds for particular bundle of goods if the domestic price of the bundle is equal to the foreign price, when compared in the same currency. More concise, PPP holds if

$$e \cdot P = P^*$$

where P and P^* are domestic and foreign prices respectively.

- b. (5 points). Assuming that: (1) the weights on traded and non-traded goods in the price index are fixed for both countries, (2) the ratio of prices of non-traded to traded goods is fixed in both countries, and (3) the PPP holds for traded goods, show that the relationship between the growth of the exchange rate (\hat{e}), the domestic inflation (π) and foreign inflation (π^*) is: $\hat{e} = \pi^* - \pi$.

$$e^r = e \frac{P}{P^*} = e \left[\frac{\alpha P^T + (1 - \alpha) P^N}{\beta P^{*T} + (1 - \beta) P^{*N}} \right] = e \frac{P^T}{P^{*T}} \left[\frac{\alpha + (1 - \alpha) P^N / P^T}{\beta + (1 - \beta) P^{*N} / P^{*T}} \right]$$

The term $e \frac{P^T}{P^{*T}} = 1$ because of assumption (3), i.e. the PPP holds for traded goods. The

term in the brackets is constant because of assumptions (1) and (2). Thus, the real exchange rate must be constant.

$$e \frac{P}{P^*} = \text{const}$$

$$\hat{e} + \pi - \pi^* = 0$$

$$\hat{e} = \pi^* - \pi$$

- c. (5 points). Some countries that experience high inflation, peg their currency to another "stable" currency. Using the model described in this question, explain how pegging the domestic currency helps reducing the domestic inflation.

Fixing the exchange rate means that $\hat{e} = 0$ and we have

$$\hat{e} = \pi^* - \pi = 0$$

Thus, the domestic inflation becomes the same as the foreign inflation to which the currency is pegged.

6. (10 points). Consider the Solow model discussed in class. Output is produced according to $Y_t = A_t K_t^\theta L_t^{1-\theta}$, $0 < \theta < 1$. Capital evolves according to $K_{t+1} = K_t(1 - \delta) + I_t$, where δ is depreciation rate and I_t is investment. People save a fraction s of their income, and the total saving and total investment in this (closed) economy is $S_t = I_t = sY_t$. The population of workers (and the total population) grows at rate n , i.e. $L_{t+1} = (1 + n)L_t$.
- a. (5 points). Derive the law of motion of capital per worker and solve for the steady state capital per worker (k_{ss}), output per worker (y_{ss}) and consumption per worker (c_{ss}), assuming constant TFP.

Law of motion of capital per worker:

$$K_{t+1} = (1 - \delta)K_t + I_t$$

$$\frac{K_{t+1}}{L_{t+1}} = \frac{(1 - \delta)K_t}{(1 + n)L_t} + \frac{sAK_t^\theta L_t^\theta}{(1 + n)L_t}$$

$$k_{t+1} = \frac{(1 - \delta)k_t}{1 + n} + \frac{sAk_t^\theta}{1 + n}$$

Steady state:

$$k_{ss} = \frac{(1 - \delta)k_{ss}}{1 + n} + \frac{sAk_{ss}^\theta}{1 + n}$$

$$k_{ss}(1 + n) = (1 - \delta)k_{ss} + sAk_{ss}^\theta$$

$$k_{ss}(n + \delta) = sAk_{ss}^\theta$$

$k_{ss} = \left(\frac{sA}{n + \delta} \right)^{\frac{1}{1-\theta}}$ $y_{ss} = Ak_{ss}^\theta$ $c_{ss} = (1 - s)y_{ss} = (1 - s)Ak_{ss}^\theta$

- b. (5 points). According to the Solow model, countries that have higher saving rate will necessarily enjoy higher steady state consumption per worker. True false, circle the correct answer and provide a short proof. (either mathematical **or** graphical).

The steady state consumption is

$$c_{ss} = \underbrace{(1-s)}_{\downarrow} \overbrace{Ak_{ss}^{\theta}}^{\uparrow}$$

Higher saving rate indeed increases the steady state capital per worker, and therefore the output per worker. At the same time, the **consumption rate**, $(1-s)$, decreases. Thus, it is not obvious which effect is stronger.