

Final Exam

Thursday, December 15

1 hours, 30 minutes

Name: _____

Instructions

1. This is closed book, closed notes exam.
2. No calculators of any kind are allowed.
3. Show all the calculations.
4. If you need more space, use the back of the page.
5. Fully label all graphs.

Good Luck ☺

1. (15 points). Suppose that GDP in the U.S. is twice as large as that of China. Also suppose that the U.S. GDP is not growing at all, while Chinese GDP grows at 7% per year.
- a. How many years would it take China to catch up with the U.S. in terms of GDP? (**Instructions:** simplify your equations up to the point when you must use a calculator).

$$GDP_{US} (1+0)^t = GDP_{CHN} (1+0.07)^t$$

$$2GDP_{CHN} = GDP_{CHN} (1+0.07)^t$$

$$2 = (1+0.07)^t$$

$$\ln(2) = t \ln(1.07)$$

$$t = \frac{\ln(2)}{\ln(1.07)}$$

- b. Using the "rule of 70", give approximate answer to the previous question.

$$t \approx \frac{70}{7} = 10 \text{ years}$$

- c. Using the "rule of 70" again, how many years approximately would it take China to catch up with the U.S., if the U.S. GDP grows at 3.5% per year?

$$t \approx \frac{70}{(7-3.5)} = 20 \text{ years}$$

Now China is catching up with the U.S. at 3.5% rate.

2. (15 points). Consider the Classical model studied in class, and briefly described as follows. The consumer derives utility from consumption C and leisure l according to $U(C, l) = \alpha \ln C + (1 - \alpha) \ln l$. He is endowed with h hours which he can allocate between leisure and work L_S . The real wage is w . The consumer owns a firm and receives dividend income (profit) π . The firm produces output Y using technology $Y = AK^\theta L_D^{1-\theta}$, where A is productivity parameter (TFP), K is the capital owned by the firm, and L_D is labor employed by the firm. The government taxes labor income at the rate of t_w and dividend income at the rate of t_π .
- a. Suppose that labor income and dividends are taxed at the same rate $t = t_w = t_\pi = 20\%$. The capital share in the economy is 0.35, equilibrium output is 1000 and equilibrium employment is 40. Find the equilibrium private consumption C^* , government consumption G^* , dividend income π^* and unemployment rate in this economy.

$$C^* = (1 - t)Y^* = 0.8 \cdot 1000 = 800$$

$$G^* = tY^* = 0.2 \cdot 1000 = 200$$

$$\pi^* = \theta Y^* = 0.35 \cdot 1000 = 350$$

$$UR^* = 0, \text{ always in this model}$$

- b. Now suppose that the government raised the tax rate, and it becomes $t = t_w = t_\pi = 25\%$. Find the new equilibrium output, employment, private consumption, government consumption dividends and the unemployment rate. ($Y^*, L^*, C^*, G^*, \pi^*, UR^*$).

$$Y^* = 1000, \text{ unchanged by } t$$

$$L^* = 40, \text{ unchanged by } t$$

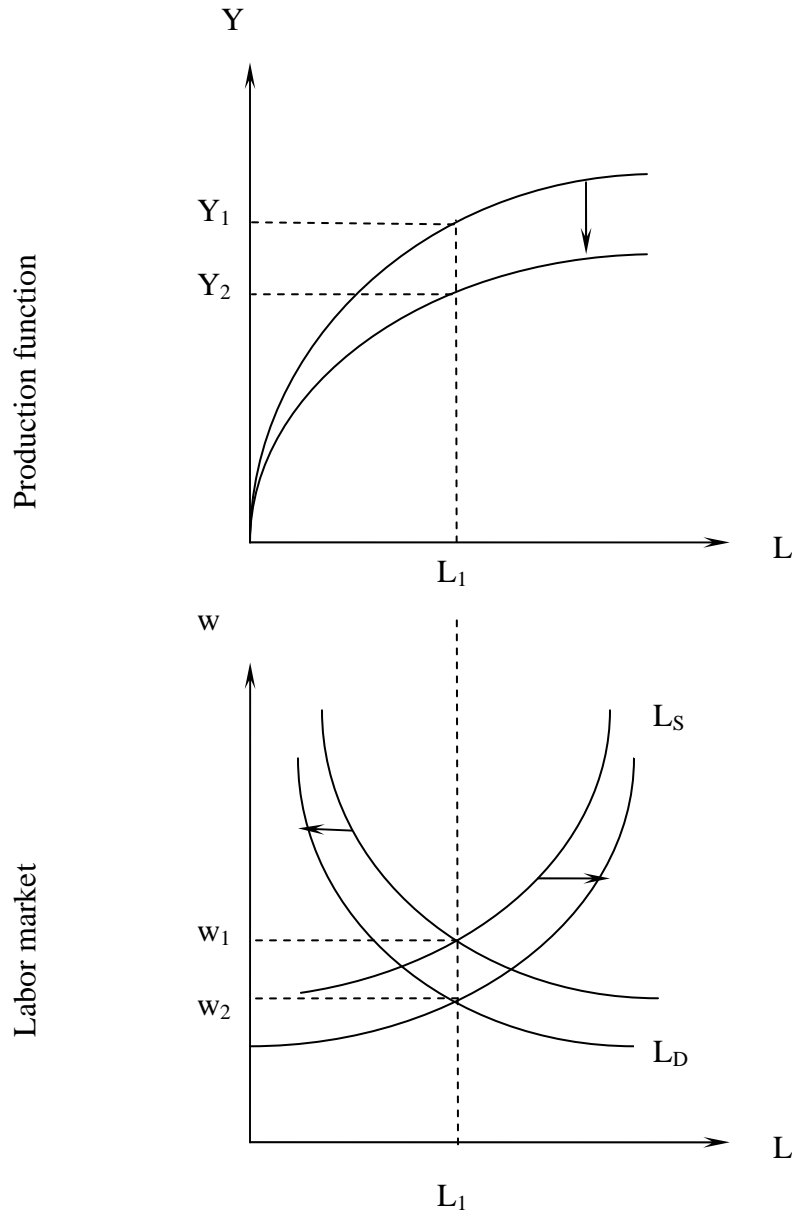
$$C^* = (1-t)Y^* = 0.75 \cdot 1000 = 750$$

$$G^* = tY^* = 0.25 \cdot 1000 = 250$$

$$\pi^* = 350, \text{ unchanged by } t$$

$$UR^* = 0, \text{ always in this model}$$

- c. Suppose that the economy of Japan is described by the classical model. Using fully labeled graphs of the production function and labor market, illustrate the effect of destruction of physical capital by a tsunami ($K \downarrow$) on equilibrium output (Y^*), equilibrium real wage (w^*) and equilibrium employment (L^*).



3. (20 points). Consider the two-period model of consumption and saving discussed in class. There are N identical consumers that live for two periods (1 and 2) and derive utility from consumption c_1 and c_2 in the two periods: $U(c_1, c_2)$.

Consumers receive income y_1 and y_2 in the two periods and pay a lump sum tax t_1 and t_2 to the government. The consumers decide how much to consume in each period and how much to save in the first period. We denote the saving in the first period by s . Consumers can borrow and lend at real interest rate r , which is assumed exogenously given. Thus the budget constraints in the two periods are

$$BC_1: c_1 + s = y_1 - t_1$$

$$BC_2: c_2 = y_2 - t_2 + (1+r)s$$

The government collects tax revenues $T_1 = N \cdot t_1$ and $T_2 = N \cdot t_2$, and spends G_1 and G_2 in the two periods. The government can borrow and lend at real interest rate r with the constraint that the present value of spending = present value of taxes

$$G_1 + \frac{G_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

- a. (5 points). Suppose that the real interest rate is $r = 7\%$ and the government increases taxes in the first period by 200. Find the necessary change in the second period's taxes that would keep the present value of taxes unchanged. Show your calculations.

$$T_1 + 200 + \frac{T_2 + \Delta T_2}{1+r} = T_1 + \frac{T_2}{1+r}$$

$$200 + \frac{\Delta T_2}{1+r} = 0$$

$$\Delta T_2 = -200(1+r) = -200 \cdot 1.07 = -214$$

- b. (5 points). Suppose that the consumer's utility is $U(c_1, c_2) = \alpha \ln(c_1) + (1 - \alpha) \ln(c_2)$. Write the consumer's demand for consumption in both periods and his supply of saving.

Writing the consumer's problem with the lifetime budget constraint helps for this section.

$$\begin{cases} \max_{c_1, c_2} \alpha \ln(c_1) + (1 - \alpha) \ln(c_2) \\ s.t. \\ c_1 + \frac{c_2}{1+r} = y_1 - t_1 + \frac{y_2 - t_2}{1+r} \end{cases}$$

Now we can see that since the preferences are of the Cobb-Douglas form, the consumer will spend a fixed fraction of his lifetime income on c_1 and c_2 :

$$c_1^* = \alpha \left(y_1 - t_1 + \frac{y_2 - t_2}{1+r} \right)$$

$$c_2^* = (1 - \alpha) \left(y_1 - t_1 + \frac{y_2 - t_2}{1+r} \right) (1+r)$$

The saving from BC_1 :

$$s^* = y_1 - t_1 - \underbrace{\alpha \left(y_1 - t_1 + \frac{y_2 - t_2}{1+r} \right)}_{c_1^*} = (1 - \alpha)(y_1 - t_1) - \alpha \left(\frac{y_2 - t_2}{1+r} \right)$$

- c. (5 points). Suppose that income in the first period increases by \$1000. Find the resulting change in consumption of both periods, and the change in saving, if the real interest rate is $r = 7\%$ and $\alpha = 0.6$.

$$\Delta c_1^* = \alpha \Delta y_1 = 0.6 \cdot 1000 = 600$$

$$\Delta c_2^* = (1 - \alpha)(1 + r)\Delta y_1 = 0.4 \cdot 1000 \cdot 1.07 = 428$$

$$\Delta s^* = (1 - \alpha)\Delta y_1 = 0.4 \cdot 1000 = 400$$

- d. (5 points). Is the result in the last section consistent with consumption smoothing? Explain briefly.

Yes, $\Delta c_1^* < \Delta y_1$ ($600 < 1000$).

4. (15 points). Consider the quantity theory of money equation: $MV = PY$.
- a. (5 points). Write this equation in approximate growth rates.

$$\hat{M} + \hat{V} = \hat{P} + \hat{Y}$$

- b. (5 points). Suppose that velocity is constant, the growth rate of real GDP is 8% and the central bank wants to achieve inflation of 0%. What is the required growth of the money supply?

$$\begin{aligned} \hat{M} + \hat{V} &= \hat{P} + \hat{Y} \\ ? \quad 0\% \quad 0\% \quad 8\% \\ \Rightarrow \hat{M} &= 8\% \end{aligned}$$

- c. (5 points). Now suppose that velocity growth is decreasing in the growth rate of money: $\hat{V} = -0.5\hat{M}$, other things being the same as in the previous section. How would your answer the part b change?

$$\begin{aligned} \hat{M} + \hat{V} &= \hat{P} + \hat{Y} \\ ? \quad 0\% \quad 0\% \quad 8\% \\ \hat{M} - 0.5\hat{M} &= \hat{Y} \\ 0.5\hat{M} &= \hat{Y} \\ 8\% \\ \Rightarrow \hat{M} &= 16\% \end{aligned}$$

5. (20 points). Suppose that the public wants to hold currency/deposit ratio of $cd = 0.2$, and the required reserve/deposit ratio is $rd = 0.4$. The initial consolidated balance sheet of commercial banks is:

Assets	Capital + Liabilities
$R = 80$	$D = 200$
$B_G = 15$	
$L = 105$	
200	200

- a. (5 points). Find the monetary base, the money supply and the money multiplier in this economy.

$$CU = 0.2 \cdot 200 = 40$$

$$MB = CU + R = 40 + 80 = 120$$

$$M = CU + D = 40 + 200 = 240 \text{ (or } M = mm \cdot MB)$$

$$mm = \frac{cd + 1}{cd + rd} = \frac{0.2 + 1}{0.2 + 0.4} = 2$$

- b. (10 points). Suppose that the central bank reduces the required reserve/deposit ratio to 20%. Find the new monetary base, money multiplier, the money supply and present the new balance sheet of the commercial banks.

$$MB = 120$$

$$mm = \frac{cd + 1}{cd + rd} = \frac{0.2 + 1}{0.2 + 0.2} = 3$$

$$M = mm \cdot MB = 3 \cdot 120 = 360$$

$$D = \left(\frac{1}{rd + cd} \right) \cdot MB = \left(\frac{1}{0.2 + 0.2} \right) \cdot 120 = 300$$

$$R = \left(\frac{rd}{rd + cd} \right) \cdot MB = \left(\frac{0.2}{0.2 + 0.2} \right) \cdot 120 = 60$$

$$CU = \left(\frac{cd}{rd + cd} \right) \cdot MB = \left(\frac{0.2}{0.2 + 0.2} \right) \cdot 120 = 60$$

Assets	Capital + Liabilities
$R = 60$	$D = 300$
$B_G = 15$	
$L = 225$	
300	300

- c. (5 points). Suppose the FYM bank (which stands for "Forget Your Money") has the following balance sheet:

Assets	Capital + Liabilities
Toxic Assets = 40	Capital = 20
Good assets = 80	Liabilities = 100
120	120

Circle the correct answer.

- i. This bank is **balance sheet insolvent**.
- ii. This bank could become **balance sheet insolvent** if toxic assets will turn out to be worth more than 20.
- iii. In order to prevent **balance sheet insolvency** for this bank, the government can buy all the toxic assets for a price lower than 20.
- iv. This bank could become **balance sheet insolvent** if toxic assets will turn out to be worth less than 20.
- v. None of the above.

6. (15 points). Let P and P^* be the price indexes in the domestic economy and foreign economy respectively. Suppose that the price index is a weighted average of traded goods (indexed by T) and non-traded goods (indexed by N):

$$P = \alpha P^T + (1 - \alpha) P^N \quad 0 \leq \alpha \leq 1$$

$$P^* = \beta P^{*T} + (1 - \beta) P^{*N} \quad 0 \leq \beta \leq 1$$

- a. (10 points). Assuming that: (1) the weights on traded and non-traded goods in the price index are fixed for both countries, (2) the ratio of prices of non-traded to traded goods is fixed in both countries, and (3) the PPP holds for traded goods only, prove that the relationship between the growth of the exchange rate (\hat{e}), the domestic inflation (π) and foreign inflation (π^*) is: $\hat{e} = \pi^* - \pi$.

$$e^r = e \frac{P}{P^*} = e \left[\frac{\alpha P^T + (1 - \alpha) P^N}{\beta P^{*T} + (1 - \beta) P^{*N}} \right] = e \frac{P^T}{P^{*T}} \left[\frac{\alpha + (1 - \alpha) P^N / P^T}{\beta + (1 - \beta) P^{*N} / P^{*T}} \right]$$

The term $e \frac{P^T}{P^{*T}} = 1$ because of assumption (3), i.e. the PPP holds for traded goods. The

term in the brackets is constant because of assumptions (1) and (2). Thus, the real exchange rate must be constant.

$$e \frac{P}{P^*} = \text{const}$$

$$\hat{e} + \pi - \pi^* = 0$$

$$\hat{e} = \pi^* - \pi$$

- b. (5 points). For several years, China pegged its currency to the U.S. dollar, mainly in order to achieve low inflation. Using the model described in this question, demonstrate how China can achieve low inflation by pegging its currency to the U.S. dollar.

Fixing the exchange rate means that $\hat{e} = 0$ and we have

$$\hat{e} = \pi^* - \pi = 0$$

Thus, the domestic inflation becomes the same as the foreign inflation to which the currency is pegged.