

Money and Barter under Private Information ^{*}

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Abstract

This paper examines the role of money when private information about the quality of the goods is present. In the private information environment, barter exchange for high-quality goods is rare since people have incentive to produce low-quality goods and attempt to cheat uninformed trading partners. This environment gives money a role in mitigating informational frictions. I consider two environments, one where traders can signal their quality of goods and one where they cannot, and two types of informational problems – adverse selection and moral hazard – in a search-theoretic framework. Both environments support the notion that money reduces the adverse selection problem and increases welfare. However, with moral hazard, money is less effective in overcoming informational frictions. Because low-quality goods producers can still consume as long as they hold money even when their products are recognized as low quality, agents have incentive to produce low-quality goods. I conduct several policy analyses, and find that the role of money is very sensitive to the inflation rate. While the Friedman rule is the optimal monetary policy in my environment, it cannot generate a first-best allocation unless traders are able to signal their quality of goods.

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1 Introduction

This paper investigates the role of money in an environment where producers have private information about the quality of goods that they can produce. Qualitative uncertainty can provide an incentive for some producers to produce low-quality outputs and attempt to cheat uninformed trading partners, and the trading partners have to decide whether or not to purchase a commodity of unknown quality. This environment gives money a function as a medium of exchange and a role in mitigating informational frictions associated with commodity transactions.

The above idea was found by Alchian (1977), who argues that overcoming the double-coincidence problem is at most a minor part of what money accomplishes, and private information is the principal friction underlying the institution of monetary exchange. Williamson and Wright (1994) use a search-theoretic model to formalize Alchian's idea and show that qualitative uncertainty can by itself make media of exchange necessary, even if there is no double-coincidence-of-wants problem.¹ Their paper has prompted other analyses of search-theoretic monetary economies with private information problems. For example, Kim (1996) studies the endogenous acquisition of information, and Trejos (1999) investigates how a lemon-problem affects the purchasing power of money. However, the above models put strong restrictions on individual money holdings and inventory and some even require indivisibility of commodities. These restrictions lose some insights of money and make the analyses of some policy issues difficult at best.

More recent studies by Berentsen and Rocheteau (2004) have relaxed the above limitations by using a large-number-of-household-members assumption proposed by Shi (1997) to make money and goods perfectly divisible. Here I use a different approach to solve the tractability problem with divisibility and consider different pricing mechanisms to reexamine monetary and barter exchange in the private information problem. My approach is based on the interaction between a decentralized market

¹Before Williamson and Wright (1994), models where money helps ameliorate lemons problems in the exchange of goods are constructed by Brunner and Meltzer (1971) and King and Plosser (1986). All these models are in reduced form and quite different from search-based framework. The advantage of search-based monetary model is straightforward: It provides strong micro foundations with explicit descriptions of specialization, the pattern of meetings, the information structure, and so on, which makes the role for money more explicitly than reduced form models.

and a centralized market which was first studied by Lagos and Wright (2005). Money and goods are perfectly divisible in my model.² There are no inventory restrictions in the model, where people can hold money or goods, or both, and as much as they want. The terms of trade are determined by bilateral bargaining and all agents are allowed to finance their consumption with money, real production, or both. I consider two different bargaining mechanisms, one where traders can signal their quality of goods and one where they cannot.

The information structure is similar to Williamson and Wright (1994). There is a positive probability that the trader cannot identify his trading partner's quality of goods. I consider two private information problems: the adverse selection problem and the moral hazard problem. In the adverse selection problem, each agent faces two types of productivity shocks and these shocks are privately observed. In this case, money is very useful to reduce the adverse selection problem and increase the ratio of high-quality goods consumption to total consumption. By increasing the real value of money, one can improve social welfare because money promotes the trading of high-quality goods and reduces the trading of lemons. While in the moral hazard problem, each person can choose to be a high-quality producer or a low-quality producer before matching, money is less effective in alleviating the information problem except in some limiting cases. If the probability being detected as a lemon producer is high, I show that the monetary equilibrium is dominated by the active non-monetary equilibrium. This result is due to the fact that money allows low-quality producers to consume even when they are recognized as lemon producers. Money provides incentive to cheat and increases the fraction of low-quality producers compared to the barter exchange.

I conduct some policy analyses in the private information environment. I find that the role of money is very sensitive to the inflation rate because the real value of money is negatively related to the inflation rate. Thus, fighting inflation can help to ameliorate the informational frictions and improve welfare unambiguously. Not surprisingly, the Friedman rule, which is optimal in other monetary search models, is the optimal monetary policy rule in this paper. However, whether the Friedman rule can generate a first-best allocation depends on the particular trading mechanism.

²Although both Shi's and Lagos and Wright's approach can generate very simple (degenerate) distribution of money holdings, the reasons for that are different. Comparison between two approaches and the merit for L-W framework can be found in Lagos and Wright (2005) for detail.

The first-best allocation is defined as the optimal high-quality goods production when there is no informational friction. In the bargaining game without signaling the quality of goods, the Friedman rule cannot restore the social optimum, while in the signaling mechanism it can.

The paper is organized as follows. In Sections 2 and 3 the economic environment and the information structure are presented. Section 4 considers a particular pricing mechanism and characterizes the equilibrium. Section 5 analyzes the adverse selection problem and the moral hazard problem respectively. In section 6, I consider another pricing mechanism to check the robustness of the results I found. Section 7 is conclusion. Most of the proofs can be found in Appendix.

2 Environment

The economy is similar to Williamson and Wright (1994) but with divisible goods and divisible money. Time is discrete and goes on forever. There is a continuum of infinitely-lived agents with unit mass. There are two types of nonstorable goods: high-quality goods and low-quality goods. An agent can derive positive utility from consuming all high-quality goods and can get zero utility from consuming the low-quality good. There are no costs for producing low-quality goods. However, producing q units of high-quality goods yields the instantaneous disutility q . Let $u(q)$ be the instantaneous utility of consuming q units of high-quality goods, and I assume that $u(q)$ is increasing and twice differentiable, and satisfies $u(0) = 0$, $u'(0) = \infty$, and $u''(q) < 0$. According to Lagos and Wright (2005), to guarantee our model has nontrivial results, I assume that u' is log-concave, i.e. $u''' \leq \frac{(u'')^2}{u'}$.³ Furthermore, there exists $q^* > 0$ such that $u'(q^*) = 1$. Hence q^* can be interpreted as the first-best production.

Besides consumption goods, there is another object called fiat money. Money is universally recognizable and cannot be produced or consumed by any private agents. Money is perfectly divisible and storable, and agents can carry any non-negative

³I maintain this assumption even though I only consider the take-it-or-leave-it bargaining mechanism in this paper, while in Lagos and Wright (2005), this assumption is not necessary if the terms of trade are determined by take-it-or-leave-it offer. This assumption is not so strong in the sense that many standard utility functions satisfy it, e.g. CRRA utility.

quantity of money. Let $F_t(m)$ denote the CDF of money holdings across agents, where $\int m_t dF_t(m_t) = M_t$ is the total amount of money, at date t . The money supply changes over time according to $M_{t+1} = (1 + \tau_t)M_t$. New money is injected in the form of lump-sum transfers or taxes.

Each period is divided into two sub-periods — day and night. Money transfers occur at the end of the second sub-period. Agents discount between periods at rate β , but not between day and night within a period. There are two types of markets in the economy: a decentralized market and a centralized market. The centralized market is a perfectly competitive market where all agents can consume and produce goods as in the standard macro model. In contrast to the centralized market, in the decentralized market it is assumed that market participants cannot consume their own output as in standard search theory.⁴

During the day, agents holding some amounts of money enter the decentralized market with bilateral random matching. Each agent can only produce a good desired by his trading partner which he cannot consume by himself during the match. But the quality of the good is private information. Thus there is no absence of double coincidence of wants problem here, but money could potentially be used in trade although it has no intrinsic value. More precisely, there are two types of quality of a special good, high-quality and low-quality. In the moral hazard problem, each agent can choose to be a high-quality producer or a low-quality producer before the match and his type is private information. Once he decides his type, he will not have an opportunity to produce other type of goods in the match. While in the adverse selection problem, an agent's type is exogenously given. When agents meet, they can assess other's quality with probability θ and they know whether other agents recognize their goods or not.⁵ The terms of trade in random matching are determined by bargaining which I will discuss later.

⁴In Lagos-Wright setup, they assume that there are two types of consumption goods: a general good and a set of special goods. The only thing that makes them different is that the agent cannot consume their own product of special goods. This feature is actually characterized by the market friction which I incorporate it into the decentralized market. Therefore I don't distinguish general goods and special goods in this paper.

⁵The purpose to make this assumption is to coincide with the bargaining process. If I allow the agent don't know their trading partner being informed or not, I need to consider other bargaining games.

During the night, all agents enter into the frictionless centralized market. Since there is no uncertainty concerned with quality of goods, all agents must produce high-quality goods and consume them or trade them for money. I normalize the price of goods in night market to 1 and let one dollar buy ϕ_t units of goods. All agents take the price of goods as given.

3 Random Matching and Uncertainty

As mentioned before, in bilateral random matching, each agent can assess his trading partner's quality with probability θ and cannot recognize the quality with probability $1 - \theta$.

Consider a random match consisting of two agents i and j . Assume P is the fraction of high-quality producers in the economy. When agents meet, after they assess their trading partner's quality, they have some (prior) beliefs about the type of their trading partner. Let $\varepsilon_i \in [0, 1]$ ($\varepsilon_j \in [0, 1]$) be the belief of agent $i(j)$ about his trading partner $j(i)$'s quality. So $\varepsilon_i = \alpha$ means that agent i believes that agent j is a high-quality producer with probability α . In some matches, agent i can recognize his partner to be a high-quality producer ($\varepsilon_i = 1$) or a low-quality producer ($\varepsilon_i = 0$). In other matches, agent i cannot distinguish the quality of goods produced by his partner j , by the law of large numbers, in this case, his belief is $\varepsilon_i = P$. In general, the prior beliefs during the match are $(\varepsilon_i, \varepsilon_j) \in E \equiv \{0, P, 1\}^2$. Depending on the match type, the pricing mechanism is characterized by complete information ($(\varepsilon_i, \varepsilon_j) \in \{0, 1\}^2$), one-sided incomplete information (only one of agent's belief is P , e.g. $(\varepsilon_i, \varepsilon_j) = (P, 1)$) or two-sided incomplete information ($(\varepsilon_i, \varepsilon_j) = (P, P)$).

When people are matched, the terms of trade must depend on what their beliefs are concerning their trading partner's quality. Let $f_H(\varepsilon)$ be the probability that conditional on agent j being a high-quality producer, agent i 's belief is ε . Similarly, $f_L(\varepsilon)$ is the probability that conditional on agent j being a low-quality producer, agent i 's belief is ε . Using the detection probability, define the conditional distribution of

beliefs:

$$f_H(\varepsilon) = \begin{cases} 0 & \text{if } \varepsilon = 0 \\ 1 - \theta & \text{if } \varepsilon = P \\ \theta & \text{if } \varepsilon = 1 \end{cases}, \quad f_L(\varepsilon) = \begin{cases} \theta & \text{if } \varepsilon = 0 \\ 1 - \theta & \text{if } \varepsilon = P \\ 0 & \text{if } \varepsilon = 1 \end{cases}. \quad (1)$$

The unconditional distribution of prior belief is defined by:

$$f(\varepsilon) = Pf_H(\varepsilon) + (1 - P)f_L(\varepsilon).$$

Therefore

$$f(0) = \theta(1 - P), \quad f(P) = 1 - \theta, \quad f(1) = \theta P. \quad (2)$$

4 Construction of Equilibrium

4.1 Value Functions

Let $V(m, \eta)$ be the value function of an agent entering the day market with money holding m and the aggregate state variable η . The state variable is specified by $\eta = (\phi, F, P)$, and the agent will take the law of motion of aggregate state as given. To simplify the notation, I suppress the state variable from now on. Before an agent meets his trading partner, he can choose to be a high-quality producer or a lemon producer. Let $V_H(m)$, $V_L(m)$ be the value function of being a high-quality and a low-quality agent in the decentralized market, respectively. Therefore, an agent's value function is:

$$V(m) = \max_{p \in [0,1]} pV_H(m) + (1 - p)V_L(m), \quad (3)$$

and in equilibrium, $p = P$. Note that if I don't endogenize the fraction of high-quality producers, the exogenous probability $p = P$ can be interpreted as a productivity shock, i.e., the agent has a probability P to produce high-quality goods and $1 - P$ to produce low-quality goods before he meets his partner, and this shock is again private information. Let $W(m)$ be the value function of an agent entering the night market. Consider two agents i, j who are matched, with agent i holding m units of money and agent j holding \tilde{m} units of money, and let “ \sim ” denote the variables for agent j . The prior belief before trading is given by $(\varepsilon, \tilde{\varepsilon})$. Therefore the terms of trade for the match is a triple $(q_{\varepsilon\tilde{\varepsilon}}(m, \tilde{m}), \tilde{q}_{\varepsilon\tilde{\varepsilon}}(m, \tilde{m}), d_{\varepsilon\tilde{\varepsilon}}(m, \tilde{m}))$ that specifies the

quantity consumed by agent i , $q_{\varepsilon\tilde{\varepsilon}}$, the quantity produced by him, $\tilde{q}_{\varepsilon\tilde{\varepsilon}}$ (that is the quantity consumed by agent j) and the money transfer $d_{\varepsilon\tilde{\varepsilon}}$ from i to j (Positive $d_{\varepsilon\tilde{\varepsilon}}$ means that the money transfers from i to j , while negative means that the money flows from j to i .) Bellman's equations for agent i at date t are:

$$V_{Ht}(m_t) = \sum_{(\varepsilon, \tilde{\varepsilon}) \in E} f_H(\tilde{\varepsilon})f(\varepsilon) \int [\varepsilon u(q_{\varepsilon\tilde{\varepsilon}}(m_t, \tilde{m}_t)) - \tilde{q}_{\varepsilon\tilde{\varepsilon}}(m_t, \tilde{m}_t) + W_t(m_t - d_{\varepsilon\tilde{\varepsilon}}(m_t, \tilde{m}_t))] dF_t(\tilde{m}_t), \quad (4)$$

$$V_{Lt}(m_t) = \sum_{(\varepsilon, \tilde{\varepsilon}) \in E} f_L(\tilde{\varepsilon})f(\varepsilon) \int [\varepsilon u(q_{\varepsilon\tilde{\varepsilon}}(m_t, \tilde{m}_t)) + W_t(m_t - d_{\varepsilon\tilde{\varepsilon}}(m_t, \tilde{m}_t))] dF_t(\tilde{m}_t). \quad (5)$$

The problem of an agent in the centralized market is:

$$W_t(m_t) = \max_{x, y, \Delta} u(x_t) - y_t + \beta V_t(\Delta_t + \tau_t M_t), \quad (6)$$

$$s.t. \quad x_t = \phi_t(m_t - \Delta_t) + y_t,$$

$$x_t \geq 0, \quad \Delta_t \geq 0, \quad (7)$$

where x_t is the consumption and y_t is the production in the centralized market, and $\Delta_t + \tau_t M_t = m_{t+1}$ is next period money holding, where Δ_t is money left over after trading.

Solving the optimization problem in the centralized market, I have:

$$W(m) = u(x^*) - x^* + \phi m + \max_{\Delta} \{-\phi\Delta + \beta V(\Delta + \tau M)\}. \quad (8)$$

Hence, $W(m)$ is linear in m and for all agents and all t ; $x_t = x^*$, where x^* satisfies $u'(x^*) = 1$. Moreover, Δ is independent of m and $W_m = \phi$.

4.2 Trading Mechanism

I now consider the terms of trade in the decentralized market, which are determined by bargaining games. There are three types of bargaining games: complete information, one-sided incomplete information and two-sided incomplete information. In all bargaining games with incomplete information, I consider the trading mechanism

that excludes the possibility that producers signal their goods's quality. I will come back and discuss the signaling issue in section 6.

For complete information, in an asymmetric meeting ($\varepsilon \neq \tilde{\varepsilon}$), assume that the low-quality producer makes a take-it-or-leave-it offer for simplicity. While in a symmetric meeting ($\varepsilon = \tilde{\varepsilon}$), each of the traders maximizes the product of expected surplus⁶. The threat point of an agent is given by his continuation value $W(m)$. For example, if agent i is recognized as a low-quality producer ($\tilde{\varepsilon} = 0$) and agent j is recognized as a high-quality producer ($\varepsilon = 1$), then agent i can make an offer. The bargaining solution $(q_{\varepsilon\tilde{\varepsilon}}(m, \tilde{m}), \tilde{q}_{\varepsilon\tilde{\varepsilon}}(m, \tilde{m}), d_{\varepsilon\tilde{\varepsilon}}(m, \tilde{m}))$ must satisfy:

$$\max_{q, \tilde{q}, d} \varepsilon u(q_{\varepsilon\tilde{\varepsilon}}) - \tilde{q}_{\varepsilon\tilde{\varepsilon}} + W(m - d_{\varepsilon\tilde{\varepsilon}}) - W(m),$$

$$s.t. \quad \tilde{\varepsilon} u(\tilde{q}_{\varepsilon\tilde{\varepsilon}}) - q_{\varepsilon\tilde{\varepsilon}} + W(\tilde{m} - d_{\varepsilon\tilde{\varepsilon}}) - W(\tilde{m}) \geq 0, \quad (9)$$

$$-\tilde{m} \leq d_{\varepsilon\tilde{\varepsilon}} \leq m, \quad (10)$$

where (9) is the market participation constraint. $\tilde{q}_{\varepsilon\tilde{\varepsilon}}$ must be zero because $\tilde{\varepsilon} = 0$. Since W is linear in m , I can rewrite the problem (and the following bargaining problem will use the same trick to simplify) into:

$$\max u(q_{\varepsilon\tilde{\varepsilon}}) - \phi d_{\varepsilon\tilde{\varepsilon}}, \quad (11)$$

$$s.t. \quad -q_{\varepsilon\tilde{\varepsilon}} + \phi d_{\varepsilon\tilde{\varepsilon}} \geq 0 \quad (12)$$

$$d_{\varepsilon\tilde{\varepsilon}} \leq m \quad (13)$$

The solution to (11) will depend on whether the constraint (13) binds or not. Observe that the participation constraint is always binding here. If (13) binds, then $d_{\varepsilon\tilde{\varepsilon}} = m$ and $q_{\varepsilon\tilde{\varepsilon}} = \phi m \equiv q_1(m)$, otherwise $d_{\varepsilon\tilde{\varepsilon}} = q^*/\phi \equiv m_1^*$ and $q_{\varepsilon\tilde{\varepsilon}} = q^*$. The following lemma states the result for this bargaining game:

Lemma 1 *Under complete information, the terms of trade are given by:*

$$(i) \text{ if } \varepsilon = 0, \quad \tilde{\varepsilon} = 0, \text{ then } q_{\varepsilon\tilde{\varepsilon}} = \tilde{q}_{\varepsilon\tilde{\varepsilon}} = 0 \text{ and } d_{\varepsilon\tilde{\varepsilon}} = 0$$

$$(ii) \text{ if } \varepsilon = 1, \quad \tilde{\varepsilon} = 1, \text{ then } q_{\varepsilon\tilde{\varepsilon}} = \tilde{q}_{\varepsilon\tilde{\varepsilon}} = q^* \text{ and } d_{\varepsilon\tilde{\varepsilon}} = 0$$

⁶This assumption implies that the set of outcomes are incentive-feasible and bilaterally efficient. If I maintain the make-it-or-take-it offer under symmetric case, it is not clear why one of the player can extract all the surplus when both players know each other be the same type.

(iii) if $\varepsilon = 1$, $\tilde{\varepsilon} = 0$, then $\tilde{q}_{\varepsilon\tilde{\varepsilon}} = 0$,

$$q_{\varepsilon\tilde{\varepsilon}} = \begin{cases} q_1(m) & \text{if } m < m_1^* \\ q^* & \text{if } m \geq m_1^* \end{cases} \quad \text{and} \quad d_{\varepsilon\tilde{\varepsilon}} = \begin{cases} m & \text{if } m < m_1^* \\ m^* & \text{if } m \geq m_1^* \end{cases}$$

(iv) if $\varepsilon = 0$, $\tilde{\varepsilon} = 1$, then $q_{\varepsilon\tilde{\varepsilon}} = 0$,

$$\tilde{q}_{\varepsilon\tilde{\varepsilon}} = \begin{cases} q_1(\tilde{m}) & \text{if } \tilde{m} < m^* \\ q^* & \text{if } \tilde{m} \geq m^* \end{cases} \quad \text{and} \quad d_{\varepsilon\tilde{\varepsilon}} = \begin{cases} -\tilde{m} & \text{if } \tilde{m} < m_1^* \\ -m_1^* & \text{if } \tilde{m} \geq m_1^* \end{cases}$$

where $q_1(m) = \phi m$ and $m_1^* = \frac{q^*}{\phi}$.

According to Lemma 1 (iii) and (iv), buyers can finance their consumption with money even though they can be recognized as lemon producers in the monetary economy. In contrast, the agents who are recognized as low-quality producers cannot consume in a barter economy because no one will trade with them. This feature gives money a role in providing consumption insurance for low-quality producers. Berentsen and Rocheteau (2004) call this the *insurance effect* of money.

For the one-sided incomplete information case, I assume that the uninformed agent who cannot recognize the quality of the good produced by his trading partner makes a take-it-or-leave-it offer. By making such an assumption, I prevent any signaling issue from money holdings because the quality of goods produced by the uninformed agent is common-knowledge, the offer made by him does not convey additional information.

Assume agent j is the uninformed player, when he is assessed as a low-quality producer, i.e. the beliefs $(\varepsilon, \tilde{\varepsilon}) = (0, P)$, then agent i will not accept agent j 's product, $q_{\varepsilon\tilde{\varepsilon}} = 0$. Also agent j maximizes his expected payoff subject to the market participation constraint of agent i . There are two participation constraints: (a) $-\tilde{q}_{\varepsilon\tilde{\varepsilon}} - \phi d_{\varepsilon\tilde{\varepsilon}} \geq 0$ if i is high type, and (b) $-\phi d_{\varepsilon\tilde{\varepsilon}} \geq 0$ if i is low type. Obviously if the former constraint is satisfied then the latter will be automatically satisfied. Hence agent j makes the offer that must be acceptable by a high-quality trading partner. Given this, the bargaining outcome satisfies:

$$\max_{q, \tilde{q}, d} Pu(\tilde{q}_{\varepsilon\tilde{\varepsilon}}) + \phi d_{\varepsilon\tilde{\varepsilon}}, \quad (14)$$

$$\begin{aligned} s.t. \quad & -\tilde{q}_{\varepsilon\tilde{\varepsilon}} - \phi d_{\varepsilon\tilde{\varepsilon}} \geq 0, \\ & -\tilde{m} \leq d_{\varepsilon\tilde{\varepsilon}}. \end{aligned}$$

The solution for this problem depends on whether the constraint $d_{\varepsilon\tilde{\varepsilon}} \geq -\tilde{m}$ binds or not. Note that the side condition $-\tilde{q}_{\varepsilon\tilde{\varepsilon}} - \phi d_{\varepsilon\tilde{\varepsilon}} \geq 0$ is always binding here. Hence if the cash-in-advance constraint of agent j binds, then $d_{\varepsilon\tilde{\varepsilon}} = -\tilde{m}$ and $\tilde{q}_{\varepsilon\tilde{\varepsilon}} = \phi\tilde{m}$. If the money holding does not bind, then the first order condition implies that $\tilde{q}_{\varepsilon\tilde{\varepsilon}} = q_P^*$, where q_P^* satisfies $Pu'(q_P^*) = 1$, and $d_{\varepsilon\tilde{\varepsilon}} = -m_2^* = -q_P^*/\phi$.

When agent j is recognized as a high-quality producer, so the beliefs $(\varepsilon, \tilde{\varepsilon}) = (1, P)$, then the bargaining problem becomes:

$$\max_{q, \tilde{q}, d} Pu(\tilde{q}_{\varepsilon\tilde{\varepsilon}}) - q_{\varepsilon\tilde{\varepsilon}} + \phi d_{\varepsilon\tilde{\varepsilon}}, \quad (15)$$

$$\begin{aligned} s.t. \quad u(q_{\varepsilon\tilde{\varepsilon}}) - \tilde{q}_{\varepsilon\tilde{\varepsilon}} - \phi d_{\varepsilon\tilde{\varepsilon}} &\geq 0, \\ d_{\varepsilon\tilde{\varepsilon}} &\leq m. \end{aligned}$$

Again, the participation constraint always binds which means the expected payoff of agent i if he is high type is 0. If money holding binds, then $d_{\varepsilon\tilde{\varepsilon}} = m$, and the quantities $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}})$ must be a solution of following Euler equation and the participation constraint:

$$Pu'(\tilde{q}_{\varepsilon\tilde{\varepsilon}})u'(q_{\varepsilon\tilde{\varepsilon}}) = 1 \quad (16)$$

$$u(q_{\varepsilon\tilde{\varepsilon}}) - \tilde{q}_{\varepsilon\tilde{\varepsilon}} - \phi m = 0 \quad (17)$$

If money holding does not bind, then $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}) = (q^*, q_P^*)$ and $d_{\varepsilon\tilde{\varepsilon}} = m_3^* = [u(q^*) - q_P^*]/\phi$.

Notice in both of meetings, when the true type of informed player (agent i in here) is a high-quality producer, his expected payoff is 0, but when he is a low-quality producer, his expected payoff is $\phi\tilde{m} \geq 0$ in the $(0, P)$ match and $u(q_{\varepsilon\tilde{\varepsilon}}) - \phi m \geq 0$ in the $(1, P)$ match. This reflects the fact that under this pricing mechanism, due to private information, the lemon producer can always mimic the high-quality producer at no cost and extract a rent equal to the production cost of the high-quality producer. This result is worthwhile to notice because it gives the incentive to produce lemons and in turn it affects the social welfare. Also it gives a role for money to mitigate the adverse selection problem which is the major theme to be analyzed in this paper.

By symmetry, I can get a similar result when agent i is an uninformed player. Therefore I summarize the above results and put them into Lemma 2.

Lemma 2 *In one-sided incomplete information match,*

(i) *if $\varepsilon = P$, $\tilde{\varepsilon} = 0$, then $\tilde{q}_{\varepsilon\tilde{\varepsilon}} = 0$,*

$$q_{\varepsilon\tilde{\varepsilon}} = \begin{cases} q_2(m) & \text{if } m < m_2^* \\ q_P^* & \text{if } m \geq m_2^* \end{cases} \quad \text{and} \quad d_{\varepsilon\tilde{\varepsilon}} = \begin{cases} m & \text{if } m < m_2^* \\ m_2^* & \text{if } m \geq m_2^* \end{cases}$$

where $q_2(m) = \phi m$ and $m_2^* = q_P^*/\phi$;

(ii) *if $\varepsilon = P$, $\tilde{\varepsilon} = 1$, then*

$$(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}) = \begin{cases} (q_s(\tilde{m}), q_b(\tilde{m})) & \text{if } \tilde{m} < m_3^* \\ (q_P^*, q^*) & \text{if } \tilde{m} \geq m_3^* \end{cases} \quad \text{and} \quad d_{\varepsilon\tilde{\varepsilon}} = \begin{cases} -\tilde{m} & \text{if } \tilde{m} < m_3^* \\ -m_3^* & \text{if } \tilde{m} \geq m_3^* \end{cases}$$

where $(q_s(\tilde{m}), q_b(\tilde{m}))^7$ solves

$$\begin{aligned} Pu'(q_s)u'(q_b) &= 1 \\ u(q_b) - q_s - \phi\tilde{m} &= 0 \end{aligned}$$

and $m_3^* = [u(q^*) - q_P^*]/\phi$;

(iii) *if $\varepsilon = 0$, $\tilde{\varepsilon} = P$, then $q_{\varepsilon\tilde{\varepsilon}} = 0$,*

$$\tilde{q}_{\varepsilon\tilde{\varepsilon}} = \begin{cases} q_2(\tilde{m}) & \text{if } \tilde{m} < m_2^* \\ q_P^* & \text{if } \tilde{m} \geq m_2^* \end{cases} \quad \text{and} \quad d_{\varepsilon\tilde{\varepsilon}} = \begin{cases} -\tilde{m} & \text{if } \tilde{m} < m_2^* \\ -m_2^* & \text{if } \tilde{m} \geq m_2^* \end{cases};$$

(iv) *if $\varepsilon = 1$, $\tilde{\varepsilon} = P$, then*

$$(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}) = \begin{cases} (q_b(m), q_s(m)) & \text{if } m < m_3^* \\ (q_P^*, q^*) & \text{if } m \geq m_3^* \end{cases} \quad \text{and} \quad d_{\varepsilon\tilde{\varepsilon}} = \begin{cases} m & \text{if } m < m_3^* \\ m_3^* & \text{if } m \geq m_3^* \end{cases};$$

For two-sided incomplete information, neither of the two agents in the match can recognize their trading partner's quality. If I assume that each of them can make a take-it-or-leave-it offer with equal probability, then this game has a signaling structure because the sender of information who makes the offer can privately observe his type and the receiver of information decides to accept or reject the offer. The

⁷Here I use subscript s to denote the "seller" who accepts money to exchange his outputs, and use subscript b to denote the "buyer" who use money to purchase his trading partner's goods.

signaling game will be discussed later. Therefore, instead of studying the take-it-or-leave-it offer, I look at the Nash bargaining game which is without signaling structure. Consider the following symmetric Nash bargaining problem:

$$\begin{aligned} \max_{q, \tilde{q}, d} & [Pu(q_{\varepsilon\tilde{\varepsilon}}) - \tilde{q}_{\varepsilon\tilde{\varepsilon}} - \phi d_{\varepsilon\tilde{\varepsilon}}][Pu(\tilde{q}_{\varepsilon\tilde{\varepsilon}}) - q_{\varepsilon\tilde{\varepsilon}} + \phi d_{\varepsilon\tilde{\varepsilon}}] \\ \text{s.t.} & \quad -\tilde{m} \leq d_{\varepsilon\tilde{\varepsilon}} \leq m \end{aligned}$$

It is straightforward to show that the solution is: $q_{\varepsilon\tilde{\varepsilon}} = \tilde{q}_{\varepsilon\tilde{\varepsilon}} = q_P^*$ and $d_{\varepsilon\tilde{\varepsilon}} = 0$.

4.3 Degenerate Monetary Distribution

From Lemmas 1 and 2, the money transfer $d_{\varepsilon\tilde{\varepsilon}}$ will depend on the money holding m if and only if $\varepsilon > \tilde{\varepsilon}$. If $\varepsilon < \tilde{\varepsilon}$, then the money transfer will only depend on \tilde{m} . Also in a symmetric meeting, $\varepsilon = \tilde{\varepsilon}$, no money transfer at hand. Given this fact, and by (1), (2), (4), and (5), I can insert the bargaining outcomes into the value function of (3) and rewrite it as:

$$\begin{aligned} V(m) &= \max_{p \in [0,1]} (1-p)f(1)f_L(0)[u(q_1(m)) - \phi d_{1,0}(m)] \\ &\quad + (1-p)f(P)f_L(0)[Pu(q_2(m)) - \phi d_{P,0}(m)] \\ &\quad + (1-p)f(1)f_L(P)[u(q_b(m)) - \phi d_{1,P}(m)] \\ &\quad + V_0 + W(m), \end{aligned} \tag{18}$$

where

$$\begin{aligned} V_0 &= p \int f(P)f_H(1)[Pu(q_s(\tilde{m})) - q_b(\tilde{m}) - \phi d_{P,1}(\tilde{m})]dF(\tilde{m}) \\ &\quad + pf(1)f_H(1)[u(q^*) - q^*] + pf(P)f_H(P)[Pu(q_P^*) - q_P^*] \\ &\quad + (1-p)f(0)f_L(P) \int -\phi d_{0,P}(\tilde{m})dF(\tilde{m}) + (1-p)f(P)f_L(P)Pu(q_P^*). \end{aligned}$$

is independent of m and Δ . Thus, the value function depends critically on the functional forms of q_1 , q_2 and q_b . q_1 and q_2 are linear in m by Lemmas 1 and 2, but q_b is more complicated.

First consider the first order conditions of bargaining outcomes in Lemma 2:

$$Pu'(q_s(m))u'(q_b(m)) = 1, \tag{19}$$

$$u(q_b(m)) - \phi m = q_s(m). \tag{20}$$

By implicit function theorem, for all $m < m_3^*$, I can get:

$$q'_b(m) = \frac{\phi u''(q_s) u'(q_b)}{u''(q_s) [u'(q_b)]^2 + u'(q_s) u''(q_b)} > 0, \quad (21)$$

$$q'_s(m) = \frac{-\phi u'(q_s) u''(q_b)}{u''(q_s) [u'(q_b)]^2 + u'(q_s) u''(q_b)} < 0. \quad (22)$$

This result says that if money holding is constrained, the high-quality producer (the uninformed seller) is willing to produce more if the buyer has more money available. On the other hand, increasing the amount of money holdings by the buyer will decrease the quantity of uncertain goods that he can use to finance his purchase. Therefore it illustrates the *recognizability effect* of money: In the monetary economy, a high-quality producer is more willing to trade his product if payment is in money instead of goods of uncertain quality, and that will crowd out the payments with real production.

Next notice that the value function will depend on the constraints on money transfers $d_{\varepsilon\bar{\varepsilon}}$ bind or not. Hence the three critical values of money holdings m_1^* , m_2^* and m_3^* will affect the results. Notice that m_1^* is always greater than m_2^* , but it is ambiguous when comparing m_3^* with m_1^* or m_2^* .⁸ However, if I assume that $u(q^*) > 2q^*$, then I will have $m_3^* > m_1^* > m_2^*$.⁹ In Figure 1 I show the relations between the bargaining solutions and money holdings.

For $m \geq m_3^*$, the first three terms in (18) are constant. Let V_m denote the derivative of V in respect to m , then $V_m = \phi > 0$. When $m_1^* \leq m < m_3^*$, the first two terms in (18) are constant but the third one is not,

$$V_m = (1-p)f(1)f_L(P)[u'(q_b(m))q'_b(m) - \phi] + \phi, \quad (23)$$

by(20)

$$\underline{=} (1-p)f(1)f_L(P)q'_s(m) + \phi.$$

By (22), $\tilde{q}'_3(m) > -\phi$, implies $V_m > 0$. And since $\tilde{q}'_3(m)$ is negative, for all $m_1^* \leq m < m_3^*$, $V_m < \phi$. So V has a kink at m_3^* . When $m_2^* \leq m < m_1^*$, the second term in (18)

⁸Because $1 = u'(q^*) = Pu'(q_P^*)$, and u is concave, $q^* > q_P^*$, then it implies that $m_1^* = q^*/\phi > q_P^*/\phi = m_2^*$

⁹That is in CRRA utility, the coefficient of relative risk aversion is between 0.5 to 1.

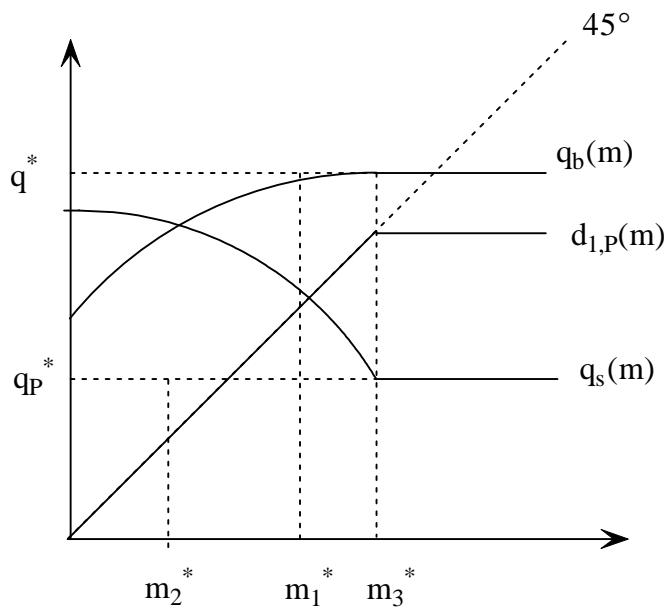
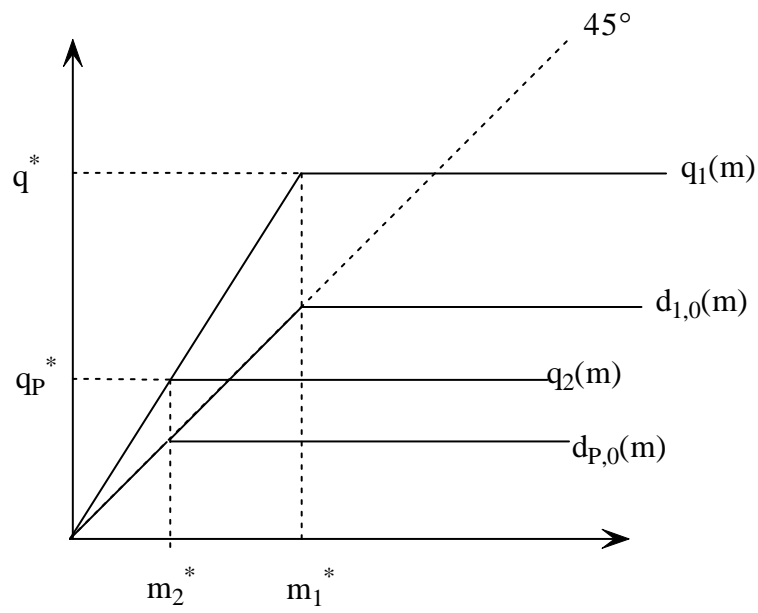


Figure 1: Bargaining Solutions

remains constant

$$\begin{aligned}
V_m &= (1-p)f(1)f_L(0)[u'(q_1(m))\phi - \phi] \\
&\quad + (1-p)f(1)f_L(P)q'_s(m) + \phi, \\
&> 0.
\end{aligned} \tag{24}$$

As $m \rightarrow m_1^*$ from below, the first term on the RHS of (24) goes to 0, the limit of $V_m \rightarrow (1-p)f(1)f_L(P)q'_s(m) + \phi$, which is exactly equal to the limit as $m \rightarrow m_1^*$ from above. Thus V is smooth at m_1^* . When $m < m_2^*$, I have

$$\begin{aligned}
V_m &= (1-p)f(1)f_L(0)[u'(q_1(m))\phi - \phi] \\
&\quad + (1-p)f(P)f_L(0)[Pu'(q_2(m))\phi - \phi] \\
&\quad + (1-p)f(1)f_L(P)q'_s(m) + \phi, \\
&> 0.
\end{aligned} \tag{25}$$

Use the same argument, I can show that V is smooth at m_2^* .

Last but not least I need to address the concavity of V so that the unique solution exists. Lemma 3 shows that V is indeed concave under the assumption that u' is log-concave.

Lemma 3 *For all $m < m_3^*$, if $u''' \leq \frac{(u'')^2}{u'}$, then q_s is strictly concave, which implies $V_{mm} < 0$.*

Figure 2 illustrates the shape of V .

Given the value function V and W , I can solve the problem of deciding how much money to take out of the centralized market at date t , $\max_{m_{t+1}} \{-\phi m_{t+1} + \beta V(m_{t+1}, \eta_{t+1})\}$. It turns out that the necessary condition for the solution exists is $\phi_t \geq \beta \phi_{t+1}$ (see LW lemma 3). It is clear that the optimal money holding m_{t+1} is less than $m_{1,t+1}^*$ and by concavity of V , this money holding is unique.¹⁰ I can conclude that the distribution F is degenerate and each agent will choose the same $m_{t+1} = \Delta_t + \tau_t M_t < m_{1,t+1}^*$ for all t .

¹⁰The optimal money holding cannot lie in $[m_{1,t+1}^*, m_{3,t+1}^*]$ because by (23) $V_m(m_{t+1}, \eta_{t+1}) < \phi_t$ for all $m \in [m_{1,t+1}^*, m_{3,t+1}^*]$.

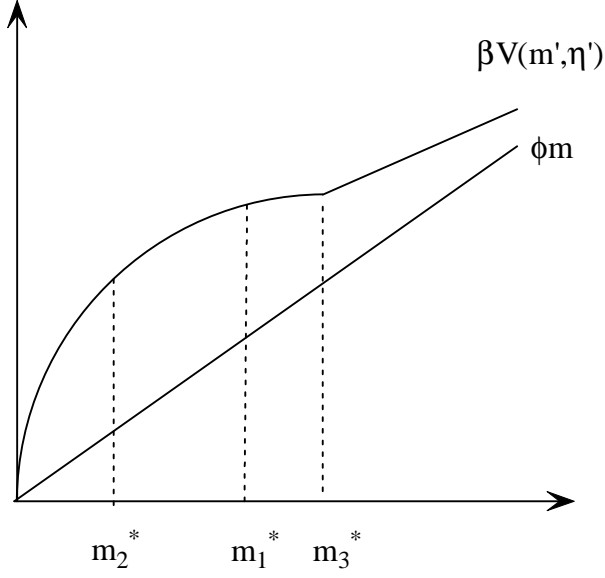


Figure 2: Value Function

4.4 Symmetric Steady State Equilibrium

In the previous subsection, I have shown that the distribution of money holdings is degenerate and the monetary equilibrium, if it exists, is unique.¹¹ But I didn't pin down the equilibrium yet, so the main task in this subsection is to take a close look at this monetary equilibrium.

First notice that if $m < m_2^*$, then $q_2 < q_P^*$ which implies $Pu'(q_2) > Pu'(q_P^*) = 1$. Therefore I can combine (24) and (25) and rewrite them into one equation:

$$\begin{aligned}
 V_m &= (1-p)f(1)f_L(0)\phi[u'(q_1(m)) - 1] \\
 &\quad + (1-p)f(P)f_L(0)\phi \max\{Pu'(q_2(m)) - 1, 0\} \\
 &\quad + (1-p)f(1)f_L(P)q'_s(m) + \phi.
 \end{aligned}$$

The bargaining solutions $q_2 = \phi m = q_1$, (19) and (20) can be used to compute values of q_2 , q_s and q_b in terms of q_1 . Define $e(q_1) \equiv q'_s(q_1) = q'_s(m)/\phi$, and insert all the

¹¹The uniqueness in here means given θ and P , the money holdings and hence the terms of trade are unique. However, if P is endogenously determined, then different P may correspond to the same θ . Multiple monetary equilibria can exist.

bargaining outcomes into the above equation, I can get:

$$\begin{aligned}
V_m &= (1-p)f(1)f_L(0)\phi[u'(q_1(m)) - 1] \\
&\quad + (1-p)f(P)f_L(0)\phi \max\{Pu'(q_1(m)) - 1, 0\} \\
&\quad + (1-p)f(1)f_L(P)e(q_1(m))\phi + \phi.
\end{aligned} \tag{26}$$

In any monetary equilibrium ϕ must strictly greater than zero. At date t , The first order condition for m_{t+1} is:

$$\begin{aligned}
\phi_t &= \beta V_m(m_{t+1}, \eta_{t+1}), \\
&= \beta(1-p)f(1)f_L(0)\phi_{t+1}[u'(q_{1,t+1}) - 1] \\
&\quad + \beta(1-p)f(P)f_L(0)\phi_{t+1} \max\{Pu'(q_{1,t+1}) - 1, 0\} \\
&\quad + \beta(1-p)f(1)f_L(P)e(q_{1,t+1})\phi_{t+1} + \beta\phi_{t+1}.
\end{aligned} \tag{27}$$

Confine the attention to symmetric steady state equilibrium where all the real variables remain unchanged and the money growth rate is constant over time. The symmetric equilibrium implies $\phi_{t+1} = \phi_t/(1+\tau)$, where the inflation rate is the same as the money growth rate. Because in equilibrium ϕ_{t+1} is not less than $\beta\phi_t$, it implies $\beta \leq 1 + \tau$. The monetary policy is restricted: one cannot contract the money supply faster than the Friedman Rule, otherwise the monetary equilibrium will break down.

In symmetric steady state equilibrium, by setting $p = P$, I can rearrange the terms and reduce (27) to:

$$\begin{aligned}
f(P) + f(1) + \frac{1 + \tau - \beta}{\beta(1-P)f_L(0)} &= f(P) \max\{Pu'(q_1), 1\} \\
&\quad + f(1)u'(q_1) + \frac{f(1)f_L(P)}{f_L(0)}e(q_1).
\end{aligned} \tag{28}$$

Definition 1 *A symmetric monetary steady state equilibrium is a price $\phi > 0$, and the terms of trade $\{(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}})\}_{(\varepsilon, \tilde{\varepsilon}) \in E}$ satisfies Lemma 1, Lemma 2 and (28).*

5 Money and Welfare

5.1 Inflation, Adverse Selection and Welfare

In the previous section I have defined a unique monetary equilibrium, which allows me to investigate how money affects social welfare and the role of money in providing incentives for agents to produce high-quality goods.¹² To compare my results with other similar studies, I first look at the case where the fraction of high-quality producers P is exogenous. The private information problem is a typical adverse selection problem in this case. I use the definition of quality mix to measure the severity of the adverse selection problem. The quality mix is the ratio of high-quality goods being traded to total consumption goods and denoted as:

$$\begin{aligned}\omega &= \frac{P \sum_{(\varepsilon, \tilde{\varepsilon}) \in E} \tilde{q}f(\varepsilon)f_H(\tilde{\varepsilon})}{\sum_{(\varepsilon, \tilde{\varepsilon}) \in E} \tilde{q}f(\varepsilon)f(\tilde{\varepsilon})}, \\ &= \frac{P \sum_{(\varepsilon, \tilde{\varepsilon}) \in E} \tilde{q}f(\varepsilon)f_H(\tilde{\varepsilon})}{P \sum_{(\varepsilon, \tilde{\varepsilon}) \in E} \tilde{q}f(\varepsilon)f_H(\tilde{\varepsilon}) + (1 - P) \sum_{(\varepsilon, \tilde{\varepsilon}) \in E} \tilde{q}f(\varepsilon)f_L(\tilde{\varepsilon})}.\end{aligned}\quad (29)$$

An increase in ω means an alleviation of adverse selection problem.

Insert the value of distributions of prior beliefs into (28) and get:

$$1 - \theta + \theta P + \frac{1 + \tau - \beta}{\beta(1 - P)\theta} = (1 - \theta) \max\{Pu'(q_1), 1\} + \theta Pu'(q_1) + P(1 - \theta)e(q_1). \quad (30)$$

As it can be shown in the appendix that $\partial q_1 / \partial \tau < 0$ and $\partial q_1 / \partial \theta > 0$, the real value of money holdings increases when the inflation rate decreases or the level of information θ increases. An increase in the real value of money will have positive effect on increasing the quality mix ω , because of the recognizability effect and insurance effect, (i.e., an increase in real money balance will increase the purchase of high-quality goods). This fact implies that reducing the inflation rate will alleviate the adverse selection problem.

Increase in the level of information has two effects on the quality mix. First, changes in θ will change the distribution of the prior beliefs which directly modifies the lifetime utility. More information generates more matches where trader's true types

¹²The welfare I measure here is referred to average life time utility (expected surplus), or utilitarian. Because I only consider the steady state equilibrium, the expected surplus V is not changing over time.

can be detected. It reduces the production of lemons and increases the production of high-quality goods. Therefore, more information can reduce the adverse selection problem. Second, the change of θ alters the real value of money holdings. Since here the real value of money will become larger when information is more abundant, the adverse selection problem is less severe. Thus, increase in the level of information will increase the quality mix through the recognizability effect and insurance effect.

In summary, money and information are two effective ways to eliminate the adverse selection problem either by decreasing the inflation rate τ for given information level θ , or by increasing the information level θ for given inflation rate τ .

Because the monetary growth rate cannot be less than the Friedman rule level, I may guess that the Friedman rule is the optimal monetary policy in here. The following proposition confirms this conjecture.

Proposition 1 *Given the detection probability θ , reducing the inflation rate can improve social welfare, and the social welfare is maximized at the Friedman rule.*

It is worthwhile to notice that even though the Friedman rule is optimal policy in here, it can only generate the second best allocation which means $q_1 < q^*$. It achieves the efficient outcome q^* if and only if $\theta = 1$. In Lagos and Wright (2005) and other literatures, the inefficiency is generated by a *hold-up* problem — buyers cannot obtain the full return that an additional unit of money provides to the match. Thus to achieve efficiency, it is required that the buyers have all the bargaining power. However, in our paper even though the buyers make a take-it-or-leave-it offer, the inefficiency still exists. My intuition to explain this is that the inefficiency comes from the asymmetric information problem and corresponds to pooling trading mechanism. Think of an uninformed buyer who carries one unit of money by making an investment with cost ϕ . When he meets a lemon producer, since the pooling mechanism is incentive-compatible that lemon producers can extract a rent equal to the production cost of high-quality producers, his trading partner will steal part of the surplus. It turns out that the agent cannot reap all of the returns to his investment, which will reduce the incentive to hold money and lower the marginal value of money. Therefore, the Friedman rule cannot achieve first-best allocation.

To illustrate the relationship among money, information and welfare, I also provide a numerical example. I calibrate the model using CRRA utility function $u(q) =$

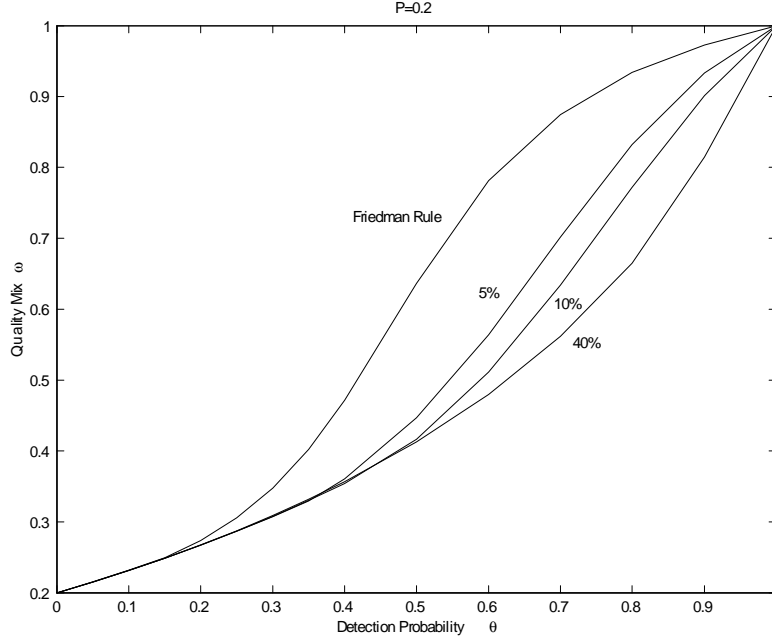


Figure 3: Inflation, Information and Adverse Selection

$\sigma^{-1}q^\sigma$ with the rate of relative risk aversion $\sigma = 0.5$. For other parameters, I choose the discount factor β equal to the inverse of real interest rate, and set current real interest rate to be 0.8%. I set the probability being a high-quality producer equal to 0.2. Figure 3 displays the relation between the quality mix and information level under different inflation rates. I can see that as information becomes more abundant, the fraction of high-quality output becomes larger, and lowering the inflation rate improves the quality mix as what I expected from previous discussion.

Figures 4 and 5 plot the welfare as a function of θ for various rates of inflation under different fraction of high-quality producers. First observation is that for any value of θ , the Friedman rule maximizes the welfare as predicted in Proposition 1. Second when the fraction of high-quality producers is low, in monetary economy, the welfare is strictly increasing in the level of information θ . But when the economy consists of a large amount of high-quality producers and the rate of inflation is relative high, the welfare may be higher when information is scarce. In barter economy, the welfare is decreasing in θ . An increase in information will cause welfare decreasing can be interpreted as following. When the information problem is severe, most of

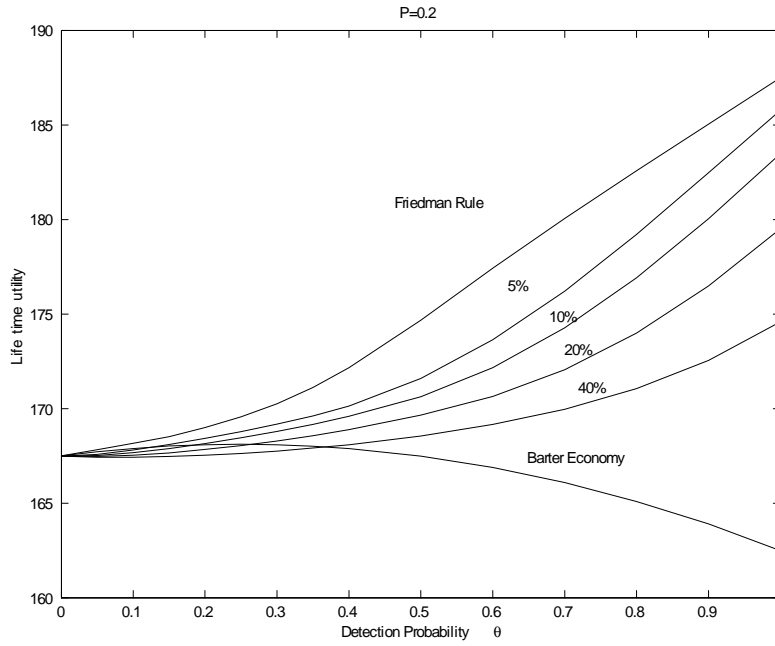


Figure 4: Inflation, Information and Welfare ($P = 0.2$)

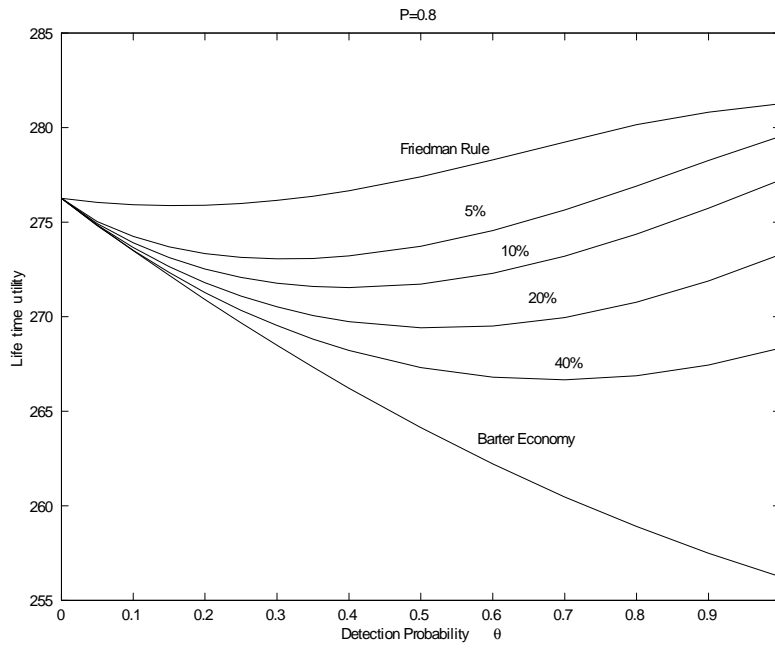


Figure 5: Inflation, Information and Welfare ($P = 0.8$)

meetings are dominated by two-sided incomplete information matching. In this case agents trade frequently even though the trading quantities are small. Increase in the information level will reduce the frequency of trades although it will increase the amount of high-quality goods exchanging at hand. The tradeoff may result in decreasing the welfare. When P is high, this tradeoff becomes more significant. Because the trading quantities are larger than that P is small, the agents are prefer to trade more frequently.

5.2 Money, Barter and Moral Hazard

Now I consider the case when the fraction of high-quality producers is endogenously determined so that I can investigate how money affects an agent's incentive to produce high-quality goods. Recall that when an agent chooses the probability p to produce high-quality goods, he will take other agents' choice of P as given. In symmetric equilibrium, all agents will choose the same probability to be a high-quality producer.

An agent's decision problem is given by (3), which implies that the agent chooses to produce high-quality goods or lemon products depends on the value of being a high-quality producer or low-quality producer. Let $D(P) = V_H - V_L$, the difference between the expected gains from high-quality producer and low-quality producer. The optimal decision of P must satisfy:

$$\begin{aligned} P &= 1 && \text{if } D(1) \geq 0, \\ P &= 0 && \text{if } D(0) \leq 0, \\ P &\in [0, 1] && \text{if } D(P) = 0. \end{aligned} \tag{31}$$

As in Williamson and Wright (1994), I refer to the active equilibrium as an equilibrium with $P > 0$, and it can be either a barter equilibrium or a monetary equilibrium. Notice that the non-monetary equilibrium $\phi = 0$ always exists in our model economy. To characterize the non-monetary equilibrium and monetary equilibrium, I provide following proposition:

Proposition 2 *If the fraction of high-quality producers is endogenously determined, then in symmetric steady state equilibrium, I have:*

1. *A non-active non-monetary equilibrium always exists for any $\theta \in [0, 1]$, specifically, when $\theta = 0$, the unique equilibrium is non-active.*

2. For any $\theta > 0$, there exists $\theta_B \in (0, 1]$, such that for any $\theta \geq \theta_B$, non-monetary equilibrium with $P = 1$ exists, and for any $\theta < \theta_B$, non-monetary equilibrium with $P < 1$ exists.
3. Given any inflation rate τ , there exists $\theta_\tau \in (0, 1]$, such that the active monetary equilibrium exists iff $\theta \leq \theta_\tau$, moreover $\theta_\tau \geq \theta_B$.

Proposition 2 is an extension of proposition 1 in Berentsen and Rocheteau (2004), because they only consider the monetary equilibrium under the Friedman rule. The first statement in proposition 2 is very common when informational frictions are present. It says that if agents cannot obtain any information about their trading partner's quality before trading, no one will have incentive to produce high-quality goods, thus market collapses. Money cannot be valued if everyone believes that other agents are lemon producers, in this case the best response for an agent is to be a lemon producer too. Therefore, these beliefs can cause a market collapse.

In the barter economy as well as in the monetary economy, if the probability of being recognized is high, then it is hard for agents to cheat. Each agent has no incentive to produce a lemon. The only active equilibrium exists in this case is a non-monetary equilibrium where all agents produce high-quality goods. The critical value of the detection probability θ which prevents agents from cheating is larger in the monetary economy than in the barter economy, which implies that if the information level is relatively high ($\theta \in [\theta_B, \theta_\tau]$), the fraction of high-quality producers is larger in the barter economy ($P = 1$) than in the monetary economy ($P < 1$). One possible explanation for this is: money allows low-quality producers to consume even when they are recognized as lemon producers; this *insurance effect* provides incentive for agents to choose a riskier behavior by increasing the fraction of low-quality producers compared to their choices in the non-monetary equilibrium. When the informational friction is not severe, the insurance effect dominates, thus, there are less high-quality producers in the monetary equilibrium than in the barter equilibrium.

The uniqueness of monetary equilibrium in this model is not guaranteed when the fraction of high-quality producers is endogenously determined. For given θ , there may be different values of P corresponding to it, hence, multiple equilibria exist in the monetary economy. Two issues about multiple equilibria are worth noting. First, because $P(\theta)$ is not single valued, whether an increase in the information level will

enlarge or diminish the fraction of high-quality producers is not easy to see. Thus, gaining more information may not necessarily alleviate the moral hazard problem in the monetary economy. Second, the multiple equilibria limits us to compare monetary equilibria and non-monetary equilibria with respect to the fraction of high-quality producers in the market. Proposition 2 only tells us that this fraction is higher in the non-monetary equilibrium than in the monetary equilibrium when information level is high. Not much can be said for other θ .

To further investigate the effects of money on the incentive to produce high-quality goods, I simulate the model using CRRA utility and the same parameters in previous subsection. I find that for small value of θ the monetary equilibrium is unique, which allows me to compare the effectiveness of money to reduce moral hazard problem under different environment.¹³

In Figure 6, I plot the fraction of high-quality producers when $\theta \ll \theta_B$ under barter economy and monetary economy with different inflation rates. First, observe that all three curves are strictly increasing in the level of information θ , this means an increase in information can provide more incentive for agents to produce high-quality goods. The reason is straightforward: more information allows cheaters to be caught more easily, thus, it brings more risk to lemon producers and reduce agent's incentive to cheat. Second, when the inflation rate is close to the Friedman rule, if the problem of information is severe, the fraction of high-quality producers is larger in monetary equilibrium than in non-monetary equilibrium. Therefore, valued fiat money can help to disciplines producers and reduce the moral hazard problem. However, if information becomes more abundant, the fraction of high-quality producers will be smaller in monetary equilibrium. Third, given the same information level, an increase in inflation rate will reduce the incentive to be a high-quality producer substantially.

As I mentioned before, since money is universally recognizable, the *recognizability effect* of money can raise an agent's incentive to be a high-quality producer. In the monetary economy, when a high-quality producer cannot verify his trading partner's type, he can accept to trade his product with money instead of goods of uncertain

¹³Actually except under the Friedman rule, for any $\theta < \theta_B$, the monetary equilibrium is unique. For $\theta \in [\theta_B, \theta_\tau]$, there always exist two monetary equilibria. Under the Friedman rule, the monetary equilibrium is unique for all θ . Also the active non-monetary equilibrium is unique when I choose CRRA utility in our model. All these results are something that cannot be shown analytically.

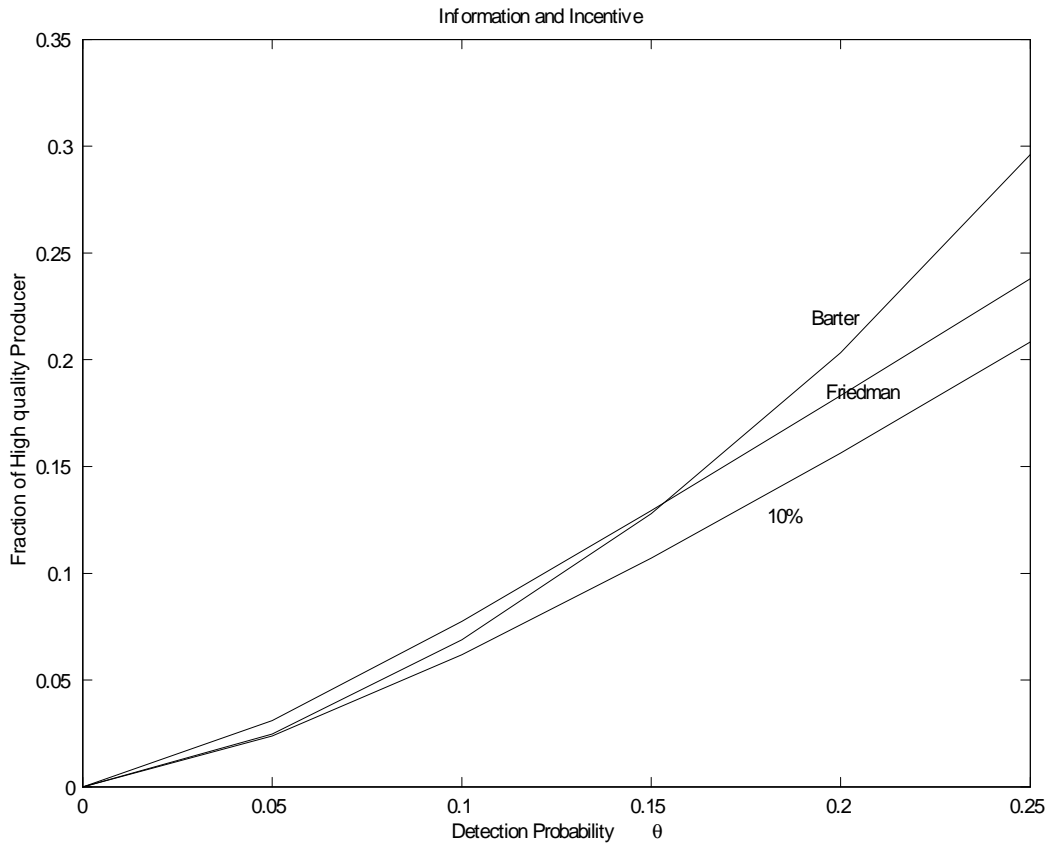


Figure 6: Inflation, Information and Moral Hazard

quality. While in the barter economy, high-quality producers are always paid with commodities of uncertain quality. Therefore, the recognizability effect of money improves the gains of high-quality producers through trading, which induces agents to become high-quality producers.

When the information level is low and the inflation rate is low enough, the recognizability effect of money dominates the insurance effect, which cause the fraction of high-quality producers is larger in the monetary equilibrium than in the non-monetary equilibrium. But the welfare gain in the Friedman rule is quite small from Figure (6). When the information level is high, the insurance effect dominates the recognizability effect, a reverse result presents. However, the effects of money are very sensitive to the inflation rate. As the level of inflation increases, the real value of money decreases, which lowers the recognizability effect and in turn results in a smaller fraction of high-quality producers. Figure 6 shows that if inflation rate is higher than the Friedman rule, the role of money is limited such that money cannot provide enough incentive to produce high-quality goods.

6 Signaling Mechanism

In this section, I modify the trading mechanism in the incomplete information match so that each agent is able to signal his product's quality to his trading partner.

6.1 Bargaining Games

I consider the bargaining games with a simple signaling structure¹⁴. Instead of the uninformed agent making a take-it-or-leave-it offer, let the informed agent who can recognize the quality of the good produced by his trading partner make an offer, and the uninformed agent who cannot distinguish his trading partner's type decide whether to accept or reject the offer.

Consider a match where $(\varepsilon, \tilde{\varepsilon}) = (1, P)$. That is agent j is the uninformed player who is assessed as high-quality producer and agent i is the informed player. Agent i makes a take-it-to-leave-it offer specifying $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}})$, and agent j can accept or reject the offer. By making an offer agent i sends a signal (or message) to agent j .

¹⁴These games were first proposed by Berentsen and Rocheteau (2003).

Therefore, agent i is the *sender* and agent j is the *receiver*. Let T be the true type of agent i . If he is a high-quality producer then $T = H$, and if he is a low-quality producer then $T = L$. The set of actions that the receiver can take is $a \in \{1, 0\}$. $a = 1$ means agent j accepts the offer whereas $a = 0$ means he rejects the offer. Let I be the indicator function, the payoff for the sender and receiver are $U^i(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}, T, a)$ and $U^j(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}, T, a)$, where

$$\begin{aligned} U^i(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}, T, a) &= a[u(q_{\varepsilon\tilde{\varepsilon}}) - I_{\{T=H\}}\tilde{q}_{\varepsilon\tilde{\varepsilon}} - \phi d_{\varepsilon\tilde{\varepsilon}}] \\ U^j(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}, T, a) &= a[I_{\{T=H\}}u(\tilde{q}_{\varepsilon\tilde{\varepsilon}}) - q_{\varepsilon\tilde{\varepsilon}} + \phi d_{\varepsilon\tilde{\varepsilon}}] \end{aligned}$$

Sequential rationality requires that the action agent j taken after receiving an offer must be optimal given agent j 's beliefs. That is,

$$a^* \in \arg \max_{a \in \{0,1\}} \sum_{T \in \{H,L\}} \mu(T|q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}) U^j(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}, T, a)$$

where $\mu(T|q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}})$ is the belief of the receiver that the sender is a T -type producer if sender makes the offer $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}})$.

Suppose that in a particular equilibrium both types send the message $(q_{\varepsilon\tilde{\varepsilon}}^e, \tilde{q}_{\varepsilon\tilde{\varepsilon}}^e, d_{\varepsilon\tilde{\varepsilon}}^e)$ with probability one. Then the message $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}) \neq (q_{\varepsilon\tilde{\varepsilon}}^e, \tilde{q}_{\varepsilon\tilde{\varepsilon}}^e, d_{\varepsilon\tilde{\varepsilon}}^e)$ is off equilibrium path, so agent j 's beliefs after observing $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}})$ cannot be derived from Bayes' rule. Use the fact that in a signaling game, *any* belief off the equilibrium path satisfies Kreps and Wilson (1982) definition of consistency, I assume that after receiving the off equilibrium message, agent j believes that i is a lemon producer with certainty. That is, $\mu(L|q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}) = 1$ for any $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}) \neq (q_{\varepsilon\tilde{\varepsilon}}^e, \tilde{q}_{\varepsilon\tilde{\varepsilon}}^e, d_{\varepsilon\tilde{\varepsilon}}^e)$. Consequently, any strategy profile $\{(q_{\varepsilon\tilde{\varepsilon}}^e, \tilde{q}_{\varepsilon\tilde{\varepsilon}}^e, d_{\varepsilon\tilde{\varepsilon}}^e), a = 1\}$ with $(q_{\varepsilon\tilde{\varepsilon}}^e, \tilde{q}_{\varepsilon\tilde{\varepsilon}}^e, d_{\varepsilon\tilde{\varepsilon}}^e) \in \{u(q_{\varepsilon\tilde{\varepsilon}}^e) - \tilde{q}_{\varepsilon\tilde{\varepsilon}}^e - \phi d_{\varepsilon\tilde{\varepsilon}}^e \geq 0, Pu(\tilde{q}_{\varepsilon\tilde{\varepsilon}}^e) - q_{\varepsilon\tilde{\varepsilon}}^e + \phi d_{\varepsilon\tilde{\varepsilon}}^e \geq 0, \text{ and } d_{\varepsilon\tilde{\varepsilon}}^e \leq m\}$ is a sequential equilibrium.

I, then, use the concept of "intuitive criterion" proposed by Cho and Kreps (1987) to refine the set of sequential equilibria. Any equilibrium that does not satisfy the criterion will be rejected. The idea of intuitive criterion is based on the following two supposition: Suppose that

1. No matter what belief the receiver holds, the resulting action a^* makes type L (or H) worse off than L (or H) in the equilibrium.
2. If the receiver infers from $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}}) \neq (q_{\varepsilon\tilde{\varepsilon}}^e, \tilde{q}_{\varepsilon\tilde{\varepsilon}}^e, d_{\varepsilon\tilde{\varepsilon}}^e)$ that sender is type H (or L), then receiver's optimal action will make H (or L) better off than L (or H)

is in the equilibrium. Then if the sender is type H (or L), the following speech given by the sender should be believed by the receiver:

I am a high (or low) quality producer. To prove this, I am sending $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}})$ instead of the equilibrium $(q_{\varepsilon\varepsilon}^e, \tilde{q}_{\varepsilon\varepsilon}^e, d_{\varepsilon\varepsilon}^e)$. Note that if I were a low (or high) quality producer. I would not want to do this, no matter what you might infer from $(q_{\varepsilon\tilde{\varepsilon}}, \tilde{q}_{\varepsilon\tilde{\varepsilon}}, d_{\varepsilon\tilde{\varepsilon}})$, since it will hurt me undoubtedly. And, as a high (or low) quality producer, I have an incentive to do this provided it convinces you that I am not low (high) type.

Thus, given the two suppositions above, H (or L) should deviate from the sequential equilibrium in which $(q_{\varepsilon\varepsilon}^e, \tilde{q}_{\varepsilon\varepsilon}^e, d_{\varepsilon\varepsilon}^e)$ is sent with probability one.

When using intuitive criterion to check the sequential equilibria, clearly, the lemon producers have no incentive to break the pooling outcome since they can always gain by imitating. However, the high-quality producer would like to signal his type by reducing the quantity he consumes and the quantity he produces. By doing so, he can make him better off and will make a low type worse off. For example, instead of proposing $(q_{\varepsilon\varepsilon}^e, \tilde{q}_{\varepsilon\varepsilon}^e, d_{\varepsilon\varepsilon}^e)$, he can propose:

$$\begin{aligned} q_{\varepsilon\tilde{\varepsilon}} &= q_{\varepsilon\varepsilon}^e - \delta \\ u(\tilde{q}_{\varepsilon\tilde{\varepsilon}}) &= Pu(\tilde{q}_{\varepsilon\varepsilon}^e) \end{aligned}$$

for δ is arbitrarily small so that

$$\text{For low-quality producers: } u(q_{\varepsilon\tilde{\varepsilon}}) - \phi d_{\varepsilon\tilde{\varepsilon}}^e < u(q_{\varepsilon\varepsilon}^e) - \phi d_{\varepsilon\varepsilon}^e$$

$$\text{For high-quality producers: } u(q_{\varepsilon\tilde{\varepsilon}}) - \tilde{q}_{\varepsilon\tilde{\varepsilon}} - \phi d_{\varepsilon\tilde{\varepsilon}}^e > u(q_{\varepsilon\varepsilon}^e) - \tilde{q}_{\varepsilon\varepsilon}^e - \phi d_{\varepsilon\varepsilon}^e$$

and the participation constraint of agent j must be satisfied:

$$u(\tilde{q}_{\varepsilon\tilde{\varepsilon}}) - q_{\varepsilon\tilde{\varepsilon}} + \phi d_{\varepsilon\tilde{\varepsilon}}^e > Pu(\tilde{q}_{\varepsilon\varepsilon}^e) - q_{\varepsilon\varepsilon}^e + \phi d_{\varepsilon\varepsilon}^e \geq 0$$

Hence, the unique sequential equilibrium¹⁵ that survives the intuitive criterion is

¹⁵Here is actually a pooling equilibrium, but as shown in Berentsen and Rocheteau (2003), the separating equilibria are equivalent to the unique pooling equilibrium that satisfies the intuitive criterion in terms of the payoffs.

$(q_{\varepsilon\tilde{\varepsilon}}^e, \tilde{q}_{\varepsilon\tilde{\varepsilon}}^e, d_{\varepsilon\tilde{\varepsilon}}^e)$ such that $\tilde{q}_{\varepsilon\tilde{\varepsilon}}^e = 0$ and

$$\begin{aligned} (q_{\varepsilon\tilde{\varepsilon}}^e, \tilde{q}_{\varepsilon\tilde{\varepsilon}}^e, d_{\varepsilon\tilde{\varepsilon}}^e) &= \arg \max_{q_{\varepsilon\tilde{\varepsilon}}^e, d_{\varepsilon\tilde{\varepsilon}}^e} [u(q_{\varepsilon\tilde{\varepsilon}}^e) - \phi d_{\varepsilon\tilde{\varepsilon}}^e] \\ \text{s.t.} \quad & -q_{\varepsilon\tilde{\varepsilon}}^e + \phi d_{\varepsilon\tilde{\varepsilon}}^e = 0 \\ & d_{\varepsilon\tilde{\varepsilon}}^e \leq m \end{aligned} \tag{32}$$

The bargaining result tells us a producer of unknown quality cannot use his production to finance his consumption. It implies that no trade can take place in the barter economy, and the trade can take place in the monetary economy if and only if the uninformed producer is a high-quality producer, and his production is only exchanged for money.

This pricing mechanism also illustrates the recognizability effect and insurance effect of money. A high-quality producer only accept to trade his production for money even if the good of unknown quality is very likely to be a high-quality good. The only chance to trade for the low-quality producer is by holding money.

Note also that the solution for (32) depends on the money holding binds or not like previous section. I can summarize the bargaining outcomes and put them into following lemma:

Lemma 4 *In a one-sided information bargaining game with take-it-or-leave-it offer by the informed party, the sequential equilibrium that satisfies intuitive criterion is unique and its outcome is given by:*

1. for any match with $(\varepsilon, \tilde{\varepsilon}) = (0, P)$ or $(P, 0)$, no trade happens;
2. if $(\varepsilon, \tilde{\varepsilon}) = (1, P)$ then $\tilde{q}_{\varepsilon\tilde{\varepsilon}}^e = 0$ and

$$q_{\varepsilon\tilde{\varepsilon}}^e = \begin{cases} q_1(m) & \text{if } m < m_1^* \\ q^* & \text{if } m \geq m_1^* \end{cases}, \quad \text{and} \quad d_{\varepsilon\tilde{\varepsilon}}^e = \begin{cases} m & \text{if } m < m_1^* \\ m^* & \text{if } m \geq m_1^* \end{cases};$$

3. if $(\varepsilon, \tilde{\varepsilon}) = (P, 1)$ then $q_{\varepsilon\tilde{\varepsilon}}^e = 0$, and

$$\tilde{q}_{\varepsilon\tilde{\varepsilon}}^e = \begin{cases} q_1(\tilde{m}) & \text{if } \tilde{m} < m^* \\ q^* & \text{if } \tilde{m} \geq m^* \end{cases} \quad \text{and} \quad d_{\varepsilon\tilde{\varepsilon}}^e = \begin{cases} -\tilde{m} & \text{if } \tilde{m} < m_1^* \\ -m_1^* & \text{if } \tilde{m} \geq m_1^* \end{cases},$$

where $q_1(m) = \phi m$ and $m_1^* = \frac{q^*}{\phi}$.

In the two-sided incomplete information match, as I mentioned in section 4, if each of traders to make a take-it-or-leave-it offer with equal probability, then the bargaining game has a simple signaling structure. Now I maintain this assumption, and the intuitive criterion can again be used to rule out any unreasonable sequential equilibrium. It turns out that any sequential equilibrium with positive production cannot survive the intuitive criterion, hence, in equilibrium there is no trade happened¹⁶.

6.2 Monetary Equilibrium

Similar to section 4, I can insert the bargaining outcomes into the value function (3) and get:

$$\begin{aligned}
V(m) &= \max_{p \in [0,1]} pf(1)f_H(P)[u(q_{1,P}(m)) - \phi d_{1,P}(m)] \\
&\quad + (1-p)f(1)f_L(P)[u(q_{1,P}(m)) - \phi d_{1,P}(m)] \\
&\quad + (1-p)f(1)f_L(0)[u(q_{1,0}(m)) - \phi d_{1,0}(m)] \\
&\quad + pf(1)f_H(1)[u(q^*) - q^*] + W(m).
\end{aligned}$$

Notice that terms of trade determined by Lemmas 1 and 4 require $d_{1,P} = d_{1,0} = d$ and $q_{1,P} = q_{1,0} = q_1$, rewrite above formula as:

$$\begin{aligned}
V(m) &= \max_{p \in [0,1]} [pf(1)f_H(P) + (1-p)f(1)f_L(P) + (1-p)f(1)f_L(0)][u(q_1(m)) - \phi d(m)] \\
&\quad + pf(1)f_H(1)[u(q^*) - q^*] + W(m). \tag{33}
\end{aligned}$$

Again, apply the same argument in section 4, I can show that the distribution of money holdings is still degenerate and the optimal money holding is unique and less than m_1^* . See figures 7 and 8 for detail.

The optimal money holdings are determined by the first order condition (27):

$$\begin{aligned}
\phi_t &= \beta V_m(m_{t+1}, \eta_{t+1}), \\
&= \beta [pf(1)f_H(P) + (1-p)f(1)f_L(P) + (1-p)f(1)f_L(0)][u'(q_1) - 1]\phi_{t+1} \\
&\quad + \beta \phi_{t+1}.
\end{aligned}$$

¹⁶For two-sided incomplete information match, see Berentsen and Rocheteau (2003) for detail discussion.

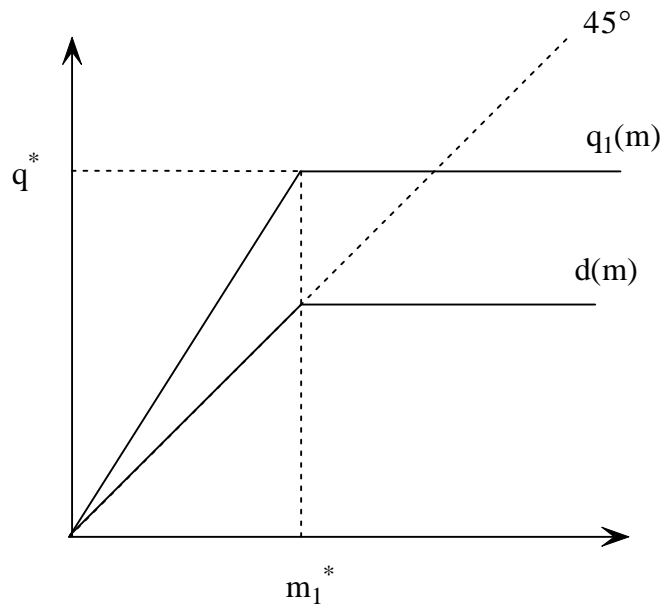


Figure 7: Bargaining Outcomes in the Signaling Mechanism

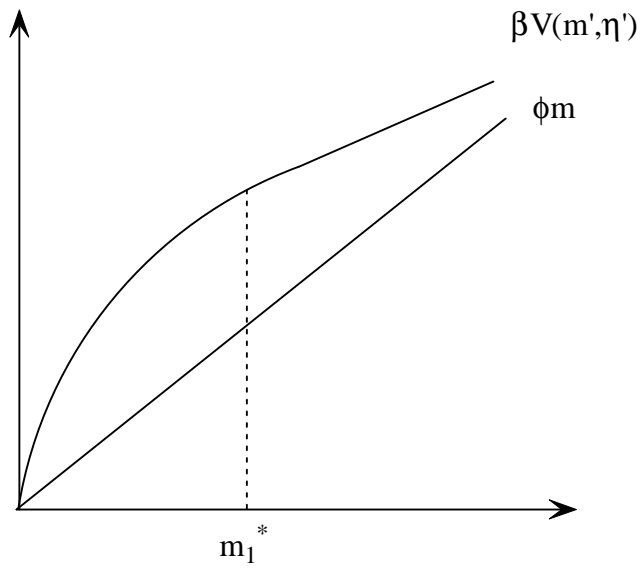


Figure 8: Value Function in the Signaling Mechanism

Consider the steady state symmetric equilibrium and insert the probabilities of prior beliefs, I can rewrite above equation into following Euler equation:

$$u'(q_1) = 1 + \frac{1 + \tau - \beta}{\beta[P\theta(1 - \theta) + P(1 - P)\theta^2]}. \quad (34)$$

From (34), it is easy to see that the real value of money increases as the inflation rate decreases. By (33) the Friedman rule is optimal policy and proposition 1 in section 5 still holds. However, unlike the previous result, the Friedman rule can generate the first best result in this case. Because the high-quality producers can signal their ability by impairing the low-quality producers, lemon producers cannot gain from cheating, thus, the purchasing power of money can be fully restored in equilibrium compared to previous result. Hence, money is useful in ameliorating the adverse selection problem and improving social welfare unambiguously.

If the fraction of high-quality producers is endogenously determined, then in the barter economy, the equilibrium can only be non-active or active without any low-quality producers. There is no non-monetary equilibrium with $P < 1$. Furthermore, if there is no information available before trading ($\theta = 0$), no high-quality goods can be produced and traded, even though a fraction of high-quality producers may exist because agents are indifferent between being high-quality and low-quality producers. In monetary economy, the difference between the expected gains from high-quality producer and low-quality producer is given by:

$$D(P) = \theta[(u(q^*) - q^*) - (u(q_1) - q_1)]$$

The monetary equilibrium exists if and only if the inflation rate is under the Friedman rule, and the equilibria are continuous. Under other inflation level, no monetary equilibrium can exist. Therefore, money is less effective to alleviate the moral hazard problem because the signaling mechanism already disciplines producers and prevent cheating from producing lemon, but the insurance effect of money can provide incentives for agent to become a low-quality producer.

7 Conclusion

This paper has analyzed the role of money under the private information environment concerning the quality of goods. Compared to other similar studies, I apply the Lagos

and Wright (2005) approach to model an economy with perfectly divisible goods and divisible money.

I consider two different trading mechanisms in this paper, and find that because money is a universally recognizable medium of exchange, there always exists two effects of money: the recognizability effect and the insurance effect. These two effects combined together will give us different results under different information problems. Money is very useful in ameliorating the adverse selection problem. When the fraction of high-quality producers is exogenously given, money has positive effects on both high-quality and low-quality producers, which can improve welfare unambiguously. However, money is not very effective in reducing the moral hazard problem except in some limiting cases. In the signaling mechanism, this ineffectiveness is more significant. The origin of ineffectiveness comes from the insurance effect which is beneficial for the lemon producers.

In this paper I also examine how inflation changes the equilibrium. Although money is neutral in our model, it is not super-neutral in general. Inflation can decrease the real value of money so that trading with money will be less desirable than goods. In the extreme case where producers can signal their goods' quality, money cannot be valued if the money growth rate is higher than the optimal level. Thus the effects of money are diminished by inflation.

This paper is a first attempt to apply L-W framework in the private information environment. Some possible future studies are worth considering. In this paper, credit has been ruled out by assumption. It might be reasonable to consider credit arrangements in private information environments by introducing financial intermediaries. One can study the implications of central bank intervention in the centralized market by modifying L-W model appropriately. Possible modifications may involve changing the timing and letting the search market and centralized market run simultaneously.

8 Appendix

- *Proof of Lemma 1.*

Proof. In the main text I have already shown that (iii) and (iv) holds. The remaining is to check (i) and (ii). For (i), it is straightforward to see that if

both of them are recognized as low-quality producer, no one will buy his trading partner's good, so no money transfer at hand. For (ii), when both of agents are recognized as high-quality producer, they try to maximize the product of each expected surplus. The bargaining solution is given by:

$$\begin{aligned} \max_{q, \tilde{q}, d} & [u(q_{\varepsilon\tilde{\varepsilon}}) - \tilde{q}_{\varepsilon\tilde{\varepsilon}} - \phi d_{\varepsilon\tilde{\varepsilon}}][u(\tilde{q}_{\varepsilon\tilde{\varepsilon}}) - q_{\varepsilon\tilde{\varepsilon}} + \phi d_{\varepsilon\tilde{\varepsilon}}] \\ \text{s.t.} & \quad -\tilde{m} \leq d_{\varepsilon\tilde{\varepsilon}} \leq m \end{aligned}$$

the solution is: $q_{\varepsilon\tilde{\varepsilon}} = \tilde{q}_{\varepsilon\tilde{\varepsilon}} = q^*$ and $d_{\varepsilon\tilde{\varepsilon}} = 0$. Hence the expected payoff of each agents is $[u(q^*) - q^*]$. ■

- *Proof of Lemma 3*

Proof. Take the derivative on both sides of (22) with respect to m , I can get:

$$\begin{aligned} q_s''(m) &= \{u''(q_s)[u'(q_b)]^2 + u'(q_s)u''(q_b)\}^{-2} \times \\ &\quad \{\phi[u'(q_b)]^2 u''(q_b)q_s'(u'(q_s)u'''(q_s) - [u''(q_s)]^2) \\ &\quad + \phi u'(q_s)u'(q_b)u''(q_s)q_b'(2[u''(q_b)]^2 - u'''(q_b)u'(q_b))\} \end{aligned} \quad (35)$$

Because $u''' \leq \frac{(u'')^2}{u'}$, $u''(q_b)q_s' > 0$ and $u''(q_s)q_b' < 0$, the second term and the third term in RHS of (35) are less than zero. Hence $q_s''(m) < 0$.

Take the derivative on both sides of (25), I will have:

$$\begin{aligned} V_{mm} &= (1-p)f(1)f_L(0)u''(q_1)\phi^2 \\ &\quad + (1-p)f(P)f_L(0)Pu''(q_2)\phi^2 \\ &\quad + (1-p)f(1)f_L(P)q_s'' \\ &< 0 \end{aligned}$$

So V is concave. ■

- *Proof of $\partial q_1/\partial \tau < 0$ and $\partial q_1/\partial \theta > 0$*

Proof. If $m < m_2^*$, i.e. $q_1 < q_P^*$, from (30)

$$\begin{aligned} \frac{\partial q_1}{\partial \tau} &= Pu''(q_1) + P(1-\theta)e'(q_1) \\ \frac{\partial q_1}{\partial \theta} &= -\frac{-Pe(q_1) + 1 - P + (1 + \tau - \beta)/[\beta(1 - P)\theta^2]}{Pu''(q_1) + P(1 - \theta)e'(q_1)} \end{aligned}$$

From the proof of lemma 3, I know that $q_s''(m) < 0$, hence $e'(q_1) = q_s''(m)/\phi < 0$. Note that $u''(q_1)$ and $e(q_1)$ are negative, I can infer that $\partial q_1/\partial\tau < 0$ and $\partial q_1/\partial\theta > 0$.

If $m \geq m_2^*$, i.e. $q_P^* \leq q_1 < q^*$, then

$$\frac{\partial q_1}{\partial\tau} = \theta Pu''(q_1) + P(1-\theta)e'(q_1)$$

$$\frac{\partial q_1}{\partial\theta} = -\frac{Pu'(q_1) - Pe(q_1) + 1 - P + (1 + \tau - \beta)/[\beta(1 - P)\theta^2]}{\theta Pu''(q_1) + P(1 - \theta)e'(q_1)}$$

I still have $\partial q_1/\partial\tau < 0$ and $\partial q_1/\partial\theta > 0$. ■

• *Proof of Proposition 1*

Proof. Since the money holdings are degenerate and each agent will hold the same amount of money in equilibrium, I can insert the bargaining outcome into V_0 and eliminate the integral signs. Therefore the value function of (18) becomes:

$$\begin{aligned} V = & v(m) + Pf(P)f_H(1)[Pu(q_s) - q_b - q_1] + (1 - P)f(0)f_L(P)q_2 \\ & + Pf(1)f_H(1)[u(q^*) - q^*] + Pf(P)f_H(P)[Pu(q_P^*) - q_P^*] \\ & + (1 - P)f(P)f_L(P)Pu(q_P^*) \end{aligned}$$

where

$$\begin{aligned} v(m) = & (1 - P)f(1)f_L(0)[u(q_1) - q_1] \\ & + (1 - P)f(P)f_L(0)[Pu(q_2) - q_2] \\ & + (1 - P)f(1)f_L(P)[u(q_b) - q_1] + W(m) \end{aligned}$$

The derivative of V with respect to q_1 is:

$$\frac{\partial V}{\partial q_1} = \frac{v'(m)}{\phi} + Pf(P)f_H(1)[Pu'(q_s)q_s' - q_b' + 1] + (1 - P)f(0)f_L(P)q_2'$$

by (19) , (21) and (22)

$$\begin{aligned} Pu'(q_s)q_s' - q_b' + 1 & = \frac{q_s'}{u'(q_b)} - q_b' + 1 \\ & = 1 - \frac{1}{u'(q_b)} \end{aligned}$$

Since $q_b < q^*$, $1 - \frac{1}{u'(q_b)} > 0$, I have $Pu'(q_s)q'_s - q'_b + 1 > 0$. Because $v'(m) > 0$ and $q'_2(q_1) > 0$, it implies that $\partial V/\partial q_1 > 0$. Also because $\partial q_1/\partial \tau < 0$, decrease in inflation rate can increase the welfare level.

The money growth rate cannot less than the Friedman rule in monetary equilibrium, so q_1 is maximized under the Friedman rule. Therefore for any information level, the Friedman rule is optimal. ■

• *Proof of proposition 2*

Proof. First insert the bargaining outcomes into the value functions of V_H and V_L , simplify (4) and (5), I will get:

$$\begin{aligned} V_H = & f(P)f_H(1)(Pu(q_s) - q_b + q_1) + f(1)f_H(1)(u(q^*) - q^*) \\ & + f(P)f_H(P)(Pu(q_P^*) - q_P^*) + W \end{aligned} \quad (36)$$

$$\begin{aligned} V_L = & f(1)f_L(0)(u(q_1) - q_1) + f(P)f_L(0)(Pu(q_2) - q_2) \\ & + f(1)f_L(P)(u(q_b) - q_1) + f(0)f_L(P)q_2 \\ & + f(P)f_L(P)Pu(q_P^*) + W \end{aligned} \quad (37)$$

Step 1. *For any $\theta \geq 0$, a non-active non-monetary equilibrium exists.* If $P = 0$, an agent who is not recognized is perceived as a low-quality producer. Accordingly, beliefs are either 0 or 1. In the barter economy, a trade takes place if and only if both agents believe each trading partner being a high-quality producer, i.e. $\varepsilon = \tilde{\varepsilon} = 1$. If $P = 0$, the whole economy consists only lemon producers, hence $f(1) = 0$. Therefore, I have $D(0) = 0$. Each agent should choose $p \in [0, 1]$, by symmetry, $p = P = 0$.

Step 2. *When $\theta = 0$, the unique equilibrium is non-active.* According to (1) and (2), if $\theta = 0$, then $f(P) = f_H(P) = f_L(P) = 1$ and $f(1) = f(0) = f_H(1) = f_L(0) = 0$. Hence

$$D(P) = \begin{cases} -q_P^* < 0 & \text{if } P > 0 \\ 0 & \text{if } P = 0 \end{cases}$$

Consequently, the unique solution is $P = 0$.

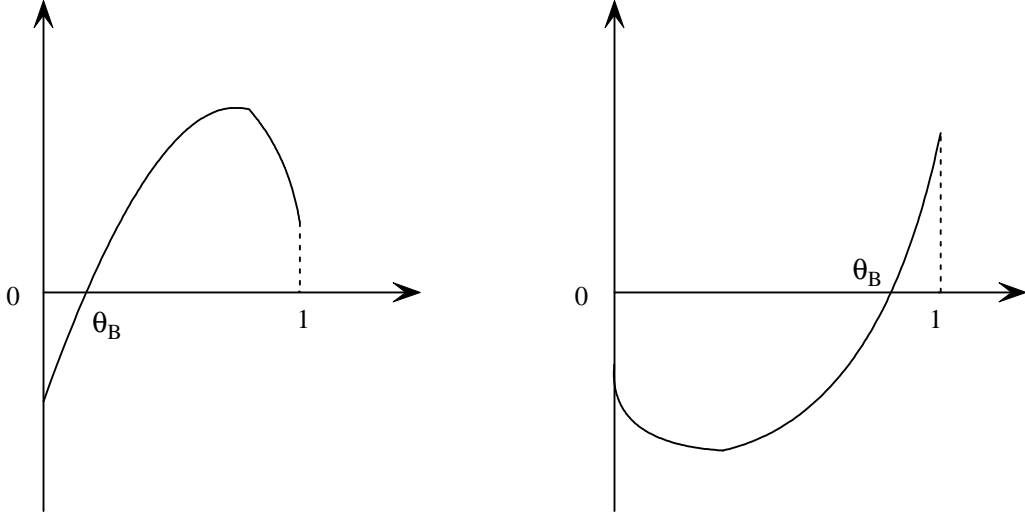


Figure 9: Function $G(\theta)$

Step 3. For any $\theta > 0$, there exists $\theta_B \in (0, 1]$, such that for any $\theta \geq \theta_B$, non-monetary equilibrium with $P = 1$ exists. In barter economy, money is not valued, thus $q_1 = q_2 = 0$. Insert (1), (2) into (36) and (37), I have:

$$D(P) = P[\theta(1 - \theta)(u(q_s) - q_s) + \theta^2(u(q^*) - q^*) - (1 - \theta)\frac{q_b}{P} - (1 - \theta)^2\frac{q_P^*}{P}] \quad (38)$$

An equilibrium with $P = 1$ exists if and only if $D(1) \geq 0$. Using the fact that $q_P^* \rightarrow q^*$, as $P \rightarrow 1$, the condition implies that

$$G(\theta) \equiv \theta(1 - \theta)(u(\hat{q}_s) - \hat{q}_s) + \theta^2(u(q^*) - q^*) - \theta(1 - \theta)\hat{q}_b - (1 - \theta)^2q^* = D(1) \geq 0 \quad (39)$$

where

$$\begin{aligned} u'(\hat{q}_s)u'(\hat{q}_b) &= 1 \\ u(\hat{q}_b) &= \hat{q}_s \end{aligned}$$

as $\theta \rightarrow 0$, $G(\theta) = -q^* < 0$, and as $\theta \rightarrow 1$, $G(\theta) = u(q^*) - q^* > 0$. Because $G(\theta)$ is a quadratic function of θ , there must exist $\theta_B \in (0, 1)$, such that for any $\theta \geq \theta_B$, $G(\theta) \geq 0$ and for any $\theta < \theta_B$, $G(\theta) < 0$. See Figure 9 for illustration.

Step 4. For all $\theta \in (0, \theta_B)$, an active non-monetary equilibrium with $P < 1$ exist. First I use the fact that as $P \rightarrow 0$, $q_P^*/P \rightarrow 0$. I omit the proof in here

since it has been shown in Berentsen and Rocheteau (2004)¹⁷. The bargaining outcome must satisfy the following first order conditions:

$$\begin{aligned} Pu'(\widehat{q}_s)u'(\widehat{q}_b) &= 1 \\ u(\widehat{q}_b) &= \widehat{q}_s \end{aligned}$$

as $P \rightarrow 0$, $\widehat{q}_b \rightarrow 0$ and $\widehat{q}_s \rightarrow 0$. When P is sufficiently small, $Pu'(\widehat{q}_b) = 1/u'(\widehat{q}_s) > 1 = Pu'(q_P^*)$. This implies $\widehat{q}_b < q_P^*$, therefore, $\widehat{q}_b/P < q_P^*/P$. Since $\lim_{P \rightarrow 0} \frac{q_P^*}{P} = 0$, I must have

$$\lim_{P \rightarrow 0} \frac{\widehat{q}_b}{P} = 0$$

Hence when P is arbitrarily small, the first term and the last two terms within the brackets of RHS of (38) go to zero, while the second term is strictly positive and independent of P . Consequently, in the neighborhood of $P = 0$, $D(P) > 0$ for any $\theta \in [0, 1]$.

Because $D(1) < 0$ for any $\theta < \theta_B$, by the continuity of D in P , there exists a $P \in (0, 1)$ such that $D(P) = 0$.

Step 5. For any inflation rate τ , the active monetary equilibrium exists iff $\theta < \theta_\tau$. In monetary equilibrium, by (36) and (37), I have

$$\begin{aligned} D(P) &= P[\theta(1 - \theta)(u(q_s) - q_s) + \theta^2(u(q^*) - q^*) + \theta(1 - \theta)\frac{q_b - q_1}{P} \\ &\quad - (1 - \theta)^2\frac{q_P^*}{P} - \theta(1 - \theta)(u(q_2) - q_2) - \theta^2(u(q_1) - q_1)] \end{aligned} \quad (40)$$

Note first that if $P = 1$, no monetary equilibrium exists, hence, by (31), the active monetary equilibrium exists if and only if $D(P) = 0$ for any $P \in (0, 1)$.

Given the inflation rate τ , for any $P > 0$, as $\theta \rightarrow 0$, $D(P)|_{\theta=0} = -q_P^* < 0$, and as $\theta \rightarrow 1$, $D(P)|_{\theta=1} = u(q^*) - q^* - (u(q_1) - q_1) \geq 0$. Since $D(P)$ is a continuous function of θ ¹⁸, there is a $\theta_\tau^P \in (0, 1]$, such that whenever $\theta = \theta_\tau^P$, $D(P) = 0$.

Choose $\theta_\tau = \max_{P \in (0, 1]} \theta_\tau^P$, then for any $\theta \leq \theta_\tau$, I can find a $P \in (0, 1)$ which satisfies $D(P) = 0$. Therefore the active monetary equilibrium exists iff $\theta \leq \theta_\tau$.

¹⁷The proof is based on the strict concavity and monotone increase of the utility function.

¹⁸Although $D(P)$ is not quadratic in θ because the change of θ will also change the value of money q_1 and other real variables, we can still confirm that $D(P)$ is continuous in θ .

To show that $\theta_\tau \geq \theta_B$, note that in step 3, at $\theta = \theta_B$, I have $D(1) = 0$. When $P = 1$, q_1 and q_2 are equal to zero, (40) will reduce to (38), hence $\theta_\tau^1 = \theta_B$. By definition of θ_τ , $\theta_\tau \geq \theta_\tau^1 = \theta_B$. ■

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