

A Different Treatment of the Group-Size Paradox

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Abstract

The present paper analyzes situations in which groups compete for rents. A major result in the literature has been that there are both cases where larger groups have advantages and cases where they have disadvantages. The paper provides two intuitive criteria which for groups with homogenous valuations of the rent determine whether there are advantages or disadvantages for larger groups. For groups with heterogenous valuations the complementarity of group members' efforts is shown to play a role as a further factor.

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1 Introduction

In many economic situations like R&D races, military conflicts, lobbying, or sports, groups compete for economic rents. In most cases, these groups will differ in many ways, including size, heterogeneity of the members' valuations of the rent, or productivity. Naturally, all these factors should have an effect on the relative success of each group to acquire the rent. The influence of these factors on the relative success of each group however depends on the way in which groups compete. Focussing in military conflicts, Carl von Clausewitz held the position that “...in modern war one will search in vain for a battle in which the winning side triumphed over an army twice its size.” (Clausewitz, 1943). He acknowledged that there are other factors like technological superiority (temporarily) influencing the outcome of a conflict, but if all these variables were stripped away, numbers would determine victory. Further research has shown that the advantages from having a larger army have varied greatly over time (Hirshleifer, 1995). In lobbying contests, there may even be disadvantages from larger group sizes, as Olson (1965) argued. His arguments gave rise to a debate about the so-called group-size paradox, which Esteban and Ray (2001) define as: “larger groups may be less successful than smaller groups in furthering their interests” (p.663).

Informal observation shows that individuals pervasively organize in groups in conflicts. In hunter-gatherer societies, individuals were apparently better equipped to be successful in predator-prey conflicts (either as predator or as prey) as well as in conflicts with rivals as they organized in groups (Tainter (1990)).¹ Clausewitz's observation shows that similar forces exist in modern warfare. But also individuals engaging in lobbying and rent-seeking activities often organize in groups. Esteban and Ray (2001) develop one explanation for these facts: larger groups may profit from cost advantages. If the costs of effort are sufficiently convex, *ceteris paribus*, members of larger groups face sufficiently lower marginal costs that reverse the group-size paradox. This is a very important insight that helps to explain the prevalence of groups in conflicts. The starting point of our paper is to ask if there are additional properties of the conflict environment that add to the explanation of the relative

¹The same is true for animals who organize in flocks, swarms, packs, etc., see the discussion at the end of this paper.

advantage or disadvantage of larger compared to smaller groups by focussing on the properties of the “production” of group impact. We believe that those properties are an important factor for the explanation of the effect of group size in conflicts.

We show that two intuitive properties of the ways in which the groups aggregate their efforts are responsible for its occurrence. The first property is an inherent advantage that may be given by the contest structure to larger groups: If there are two groups with the same total amount of effort but different numbers of individuals, one cannot in general expect them to have the same lobbying impact. For example, there may be a difference in impact whether 10 000 people demonstrate for 10 hours or 100 000 people demonstrate for 1 hour. The different demonstrations may receive very different media attention which in turn may lead to very different impacts on policymaking. If the 10 000 people would have a higher impact, we would expect the group size paradox to appear more likely. This property of the contest structure will be called *group-size biasedness*.

The second important property is *returns to scale*. Suppose group members increase their efforts by some factor and their relative strength increases by less than this factor (decreasing returns to scale). This may cause the group-size paradox to occur even if group size biasedness does not hold. Larger groups tend to be disadvantaged by this because they are facing a larger problem of free riding. If returns to scale are decreasing, the marginal return of investing more effort is not large enough to make up for the better possibilities for free riding.

It turns out that if group members have homogenous valuations of winning the contest (which may differ *between* groups), these two properties completely determine whether the group-size paradox occurs or not.² Homogenous valuations are, however, rarely the case in reality. In general, we would expect to encounter groups where group members differ in many features: Not only the valuations of winning may be different, but also abilities, qualifications, or affections. Empirical research emphasizes that the level of heterogeneity in the group is an important mediator for the impact of group size (Hardin, 1982; Ostrom, 1997). Once we introduce heteroge-

²This claim may appear to be at odds with Esteban and Ray (2001) who focus on convexities in the cost-of effort functions. We will show that their model is isomorphic to a model with linear costs and nonlinear impact functions that is a special case of our model.

nous group members, other factors may start to play a role, such as complementarity between group-members' efforts: Groups where agents are heterogenous will often have the feature that group members have specialized according to their comparative advantage. Alchian and Demsetz (1972) see such non-additivity as constitutive for group or team production (pp. 777): *“Resource owners increase productivity through cooperative specialization. [...] With team production it is difficult, solely by observing total output, to either define or determine each individual’s contribution to this output of the cooperating inputs. The output is yielded by a team, by definition, and it is not a sum of separable outputs of each of its members.”* Despite the fact that there is a growing interest on the influence of heterogeneity within and between groups,³ with only a few exceptions the literature on group contests⁴ has focused attention on situations where the effort levels of group members are perfect substitutes, i.e. are aggregated by summation. In order to analyze the group-size paradox for heterogenous valuations, this paper will introduce different degrees of complementarity for the case of a CES-type impact function. If we hold the other properties – group-size biasedness and returns to scale – at a neutral level, the complementarity between the efforts of group members determines a minimum valuation a new group member must have in order for the group-size paradox not to occur.

The group-size paradox has been extensively discussed in the theoretical and as well as the empirical literature. Agrawal and Goyal (2001) point to the inconclusive evidence: *“[...] scholars writing on the subject have remarked on the ambiguities in Olsons argument and suggested that the relationship between group size and collective action is not very straightforward. [...] research on irrigation groups in South India suggests that small size is not necessary to facilitate successful collective action.”* Most prominently, Esteban and Ray (2001) show in an important contribution that the group-size paradox does not always hold.⁵ However, up to now it is unclear whether the specific cause for the possible reversal of the group-size paradox on which Esteban and Ray (2001) focus, namely convexities in costs, is exhaustive.

³See Esteban and Ray (2008, 2010).

⁴The literature on contests between groups has recently been surveyed by Corchón, 2007, Section 4.2, Garfinkel & Skaperdas, 2007, Section 7, and Konrad, 2009, Chapters 5.5 and 7.

⁵See also Marwell and Oliver (1993); Pecorino and Temini (2008); Nitzan and Ueda (2009, 2010).

The current state of the literature is best summarized by Hwang (2009): “A variety of empirical or experimental studies have also examined the group size hypothesis [...] and many of them found that the size of a group is positively related to its level of collective action. [...] In sum, even though various empirical and experimental studies suggest that large groups may perform better, few theoretical works provide the logic and reasoning of how larger groups can overcome an aggravated free-rider problem.”⁶

The paper is organized as follows. We introduce the model in Section 2 and cover the case of homogenous group members in Section 3. In Section 4 we allow for heterogeneity of agents and use a CES type impact function to aggregate group members’ efforts. We characterize the simultaneous Nash equilibrium of the CES model in Subsection 4.1. In Subsection 4.2 we will show the effect of complementarity on the group-size paradox for heterogenous agents. There will be an extended discussion of findings from related strands of the literature in Section 5. Section 6 concludes.

2 The model

Assume that n groups compete for a given rent R . m_i is the number of individuals in group i and k is the index of a generic member of this group. The rent is a group-specific public good that has a value $v_i^k > 0$ to individual k of group i . p_i represents the probability of group $i = 1, \dots, n$ to win the contest. Individuals can influence the winning probability by contributing effort x_i^k . The members’ efforts of a group are then aggregated by a function $q_i(x_i^1, \dots, x_i^{m_i})$. We assume that $q_i(\cdot)$ is at least twice continuously differentiable and (weakly) monotonic in $x_i^k \forall i, k$. p_i is then a function of these aggregated efforts. Following the literature, we will call q_i *impact function* and p_i *contest-success function*. We focus on Tullock-form contest-success functions

⁶Hwang (2009) focusses on within-group incentive mechanisms to understand the effects of group size. This path is also taken by Nitzan and Ueda (2010) and Kolmar and Wagener (2010). The alternative approach used by Esteban and Ray (2001) is to stick to some voluntary-contributions mechanism within a group and to focus on the “technological” properties of the contest environment instead. Our paper follows this latter line of research.

where the winning probability of a group i is defined as:⁷

Assumption 1. $p_i(q_1, \dots, q_n) = \frac{q_i(x_i^1, \dots, x_i^{m_i})}{\sum_{j=1}^n q_j(x_j^1, \dots, x_j^{m_j})}$, $i = 1, \dots, n$.

Further, we impose the following assumptions on the individuals:

Assumption 2. Individuals are risk neutral, face linear costs, and maximize their net rent.

Assumptions 1 and 2 imply that we can write expected utility as:

$$\pi_i^k = \frac{q_i(x_i^1, \dots, x_i^{m_i})}{\sum_{j=1}^n q_j(x_j^1, \dots, x_j^{m_j})} v_i^k - x_i^k \quad (1)$$

We are looking for a Nash equilibrium of this game where individuals choose their effort x_i^k simultaneously to maximize their expected utility,

$$x_i^{k*} \in \arg \max_{x_i^k} \pi_i^k(x_i^k, x_{-x_i^k}^*) \quad \forall i, k. \quad (2)$$

where “*” refers to equilibrium values and $x_{-x_i^k}^*$ to the vector of efforts by all individuals except k in group i . In order to facilitate the analysis, we will focus on situations where a unique Nash equilibrium exists with respect to the total effort produced of each group. Formally,

Assumption 3. The Jacobian matrix of the first-order conditions of individuals 1 of each group is negative definite and the Jacobian matrix of all first-order conditions is negative semidefinite and symmetric among members of the same group.

This still allows for multiple equilibria within groups as they may arise when effort levels are for example perfect substitutes.

3 Homogenous valuations within groups

The group size paradox was first discussed by Olson (1965), who stated that *“the larger the group, the farther it will fall short of providing an optimal amount of*

⁷An axiomatic foundation for the Tullock function for group contests can be found in Münster (2009). An interpretation of the Tullock contest as a perfectly discriminatory noisy ranking contest can be found in Fu and Lu (2008).

a *collective good*” (p. 35). One particular interpretation of the statement has been given by Esteban and Ray (2001): In a contest environment in which different groups compete for a rent, larger groups should win with lower probability if the group size paradox was true. One could however also take a comparative-static perspective on the group size paradox, which seems to underscore its relevance even better:

Definition 1. (Group-size paradox) Suppose in a contest there are n groups i competing for a prize and have m_i individuals with equal valuations v_i . Then, the group-size paradox holds strictly (weakly) if and only if adding an individual with valuation v_i to group i will decrease (decrease or leaves constant) the probability of the group to win the prize.

Next we formulate two intuitive criteria that will turn out to be able to explain the occurrence of the group-size paradox if individuals of a group have identical valuations of the rent.

Definition 2. (Group-size bias) A class of impact functions $q_m(x)$ with m being the number of elements in the effort vector $x = (x^1, \dots, x^m)$ is said to be *group-size unbiased* if for all numbers $\xi > 1$ such that $\xi \cdot m$ is a natural number, it holds that $q_m(x) = q_{m \cdot \xi}(\frac{1}{\xi} \cdot x)$. The impact function is said to be *positively group-size biased* or *negatively group-size biased* if $q_m(x) < q_{m \cdot \xi}(\frac{1}{\xi} \cdot x)$ or $q_m(x) > q_{m \cdot \xi}(\frac{1}{\xi} \cdot x)$.

The reason why this property is interesting is that it reflects whether a redistribution of the same total amount of effort to more (heterogenous) group members will lead to an increase, a decrease, or no effect on the impact of the group. For example, the simple sum of efforts of all group members, $\sum_{k=0}^m x^k$, is group-size unbiased: If all group members exert the same effort x^1 , then $\sum_{k=0}^m x^k = m \cdot x^1$. Multiplying m by ξ and dividing x^1 by ξ will then naturally lead to the same result.

Another property of an impact function is whether it has increasing or decreasing returns to scale:⁸

⁸The following example shows that both concepts are in fact independent from each other. Assume that the impact functions have the generalized CES-form $q_{m_i}(x) = m_i^\delta (\sum (x_i^k)^\gamma / m_i)^\beta / \gamma$. In this case, $q_{\xi m_i}(x/\xi) = \xi^{\frac{\beta(1-\gamma)}{\gamma} - \delta} q_{m_i}(x)$ and $q_{m_i}(\lambda x) = \lambda^\beta q_{m_i}(x)$, which shows that returns to scale and group-size bias can be independently chosen.

Definition 3. (Returns to scale) A class of impact functions $q_m(x)$ is said to have *constant returns to scale* if $\forall m : q_m(\xi x) = \xi \cdot q_m(x)$, *decreasing returns to scale* if $\forall m : q_m(\xi x) < \xi \cdot q_m(x)$, *increasing returns to scale* if $\forall m : q_m(\xi \cdot x) > \xi \cdot q_m(x)$.

To show the relationship between our approach and the one employed by Esteban and Ray (2001)⁹, note that their model would translate into

$$\pi_i^k = v_i^k \frac{\sum_l y_i^l}{\sum_j \sum_l y_j^l} - c(y_i^k),$$

with $c(\dots)$ being an increasing, strictly convex function. The central result is that if the elasticity of the marginal rate of substitution between effort y_i^k and $v_i^k \frac{\sum_l y_i^l}{\sum_j \sum_l y_j^l}$ is sufficiently high, the winning probability will strictly increase with group size. Note that we can write $y_i^k = c^{-1}(x_i^k)$ where c^{-1} is the inverse of c . Since by their assumptions the cost function is a bijection, the maximization problem

$$\pi_i^k = v_i^k \frac{\sum_l c^{-1}(x_i^l)}{\sum_j \sum_l c^{-1}(x_j^l)} - x_i^k$$

yields the same solutions as the original problem. We can now focus again on the case with $c(y) = y^\alpha$, from which we obtain $q(x_i) = \sum_l (x_i^l)^{1/\alpha}$ as the input to the Tullock impact function.

In the special case that the costs of effort function is equal to $(x_i^l)^\alpha$, the group-size paradox does (not) occur for $\alpha < (>)2$. Since $q_{m_i}(x_i) = \sum_l (x_i^l)^{1/\alpha}$ yields an isomorphic optimization problem, we can look for group-size bias and returns to scale of $q_{m_i}(x_i)$. It is easy to check that positive group-size bias as well as decreasing returns to scale exist if and only if $\alpha > 1$. In addition, a decrease in the returns to scale (as measured by an increase in α) reduces any positive group-size bias: changing the convexity of costs is equivalent to a simultaneous change in returns to scale and groups size bias in the isomorphic problem. The result by Esteban and Ray (2001) imply that if $\alpha > 2$ the group-size-bias/returns to scale combinations work in favor of large groups, whereas the opposite is true for $\alpha < 2$. Given that this result has been derived in a situation where group-size bias and returns to scale cannot be disentangled, the question arises as to whether further insights can be gained if both effects are treated separately.

The following propositions hold (all proofs can be found in the appendix).

⁹Their model allows for rival as well as nonrival elements of the rent.

Proposition 1. *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions are group-size unbiased. Then for constant or decreasing returns to scale the group-size paradox holds weakly if all groups' members have equal valuations.*

Proposition 2. *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions have constant returns to scale. Then for negative (positive) group-size biased impact functions the group-size paradox holds (does not hold) weakly if all groups' members have equal valuations.*

These results provide a very intuitive explanation when the group-size paradox arises if group members have the same valuation of the rent. Returns to scale and group-size biasedness are indeed the main driving forces behind the different results on the group-size paradox by Olson (1965) and Esteban and Ray (2001). If one is willing to accept the assumption that the impact functions are homogenous, the above results can be generalized in the following way.

Proposition 3. *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions are homogenous, negatively group-size biased or group-size unbiased, and have constant or decreasing returns to scale. Then, the group-size paradox holds weakly if all groups' members have equal valuations.*

The following corollary of the above propositions establishes an interesting special case.

Corollary 1. *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions have constant returns to scale and are group-size unbiased. Then, the group-size paradox holds weakly but not strictly if all groups' members have equal valuations.*

Corollary 1 establishes a link to the case of additively linear impact functions, which is a special case of an group-size unbiased impact function with constant returns to scale and that have been standard in the literature so far (see, for example, Baik (2008) and Konrad, 2009, Chapters 5.5 and 7). In this case, equilibrium group impact and therefore winning probability is independent of group size as the maximum valuation remains unchanged. Corollary 1 shows that this finding carries over to a larger class of impact functions.

The next result covers the case of positive group-size bias combined with decreasing returns to scale. Propositions 1 and 2 which cover the boundary cases as well as Esteban and Ray (2001) suggest that we cannot expect clear-cut results for this area which in fact turns out to be true. Nevertheless, we get a nice monotonicity property for the case of homogenous impact functions and equilibria where members of the same group behave identically. Note that Definition 2 can be used to define a measure of group-size bias as $b_i(\xi) = q_{m_i\xi}(x_i/\xi)/q_{m_i}(x_i)$ in this case.

Definition 4. (Increase in group-size bias) Take to classes of impact functions, $\{q_{m_i}(\vec{x}_i), q_{m_i\xi}(\vec{x}_i/\xi)\}$, and $\{\hat{q}_{m_i}(\vec{x}_i), \hat{q}_{m_i\xi}(\vec{x}_i/\xi)\}$ with associated group-size bias measures $b_i(\xi), \hat{b}_i(\xi)$. The first class has more (equal, less) group-size bias than the second if $b_i(\xi) > (=, <) \hat{b}_i(\xi)$.

Definition 5. (Increase in returns to scale) Let $q_m(x, r)$ be a class of homogenous decreasing returns to scale impact functions and $r \in R \subset \mathbf{R}^+$ the degree of homogeneity of $q_m(\cdot)$. An impact function $q_m(x, r')$ has more (equal, less) decreasing returns to scale than an impact function $q_m(x, r)$ if $r' < (=, >) r$.

The following Proposition holds.

Proposition 4. *Suppose a contest fulfills Assumptions 1, 2, and 3, and the impact functions are homogenous, positively group-size biased and have decreasing returns to scale. Suppose we have two impact functions $q_{m_i}, q_{m_i\xi}$ for $m_i, m_i \cdot \xi$ group members with returns to scale $r_i < 1$ and group-size bias $b_i(\xi) > 1$. a) If the group-size paradox holds for $q_{m_i}, q_{m_i\xi}$, it must also hold for any two impact functions $\hat{q}_{m_i}, \hat{q}_{m_i\xi}$, for which the returns to scale are $\hat{r}_i \leq r_i$ and the group-size bias is $\hat{b}_i(\xi) \leq b_i(\xi)$. b) If the group-size paradox does not hold for $q_{m_i}, q_{m_i\xi}$, it does not hold for any two impact functions $\hat{q}_{m_i}, \hat{q}_{m_i\xi}$, for which the returns to scale are $1 > \hat{r}_i \geq r_i$ and the group-size bias is $\hat{b}_i(\xi) \geq b_i(\xi)$.*

4 Heterogenous valuations within groups

We now turn to the analysis of heterogenous groups and how the complementarity of the impact functions affect the group-size paradox. Notice first that Definition 1 cannot be used in a framework where group members have different valuations of

the rent. For groups that consist of members with different valuations, it is not at all clear what valuation a new member should have. In this case, it is more interesting to see what the minimum valuation of a new group member has to be in order to increase the winning probability of the group. Second, we may want to remove the effects of changing returns to scale and group-size biasedness and introduce a parameter (γ) to account for different degrees of complementarity of the group members' efforts. These properties are fulfilled by the following CES-type impact function:

Assumption 4. $q_i(x_i^1, \dots, x_i^{m_i}) = m_i \left(\sum_{l=1}^{m_i} \frac{1}{m_i} (x_i^l)^\gamma \right)^{1/\gamma}$, $\gamma \in (-\infty, 1]$, $i = 1, \dots, n$.

It is easy to check that this function has constant returns to scale and is group-size unbiased. Note that we obtain a closed-form solution only if $\gamma \neq 0$. The Cobb-Douglas case $\gamma \rightarrow 0$ will be covered by a limit result. It follows from Assumptions 1, 2, and 4 that the individual expected utility functions are as follows:

$$\pi_i^k(x_1^1, \dots, x_n^{m_n}) := \pi_i^k(x_i^k, x_{/x_i^k}) = v_i^k \frac{m_i (\sum_l \frac{1}{m_i} (x_i^l)^\gamma)^{1/\gamma}}{\sum_j m_j (\frac{1}{m_j} \sum_l (x_j^l)^\gamma)^{1/\gamma}} - x_i^k, \quad (3)$$

where $x_{/x_i^k}$ refers to the vector $x_1^1, \dots, x_n^{m_n}$ without x_i^k . In order to have a lean notation, let $y_i^k = (x_i^k)^\gamma$ and $Y_i = (\sum_l y_i^l)$. Further, $Q = \sum_j Q_j = \sum_j Y_j^{1/\gamma} = Y_i^{1/\gamma} + \sum_{j \neq i} Y_j^{1/\gamma} = Q_i + Q_{/i}$ in the following. While deriving the equilibrium strategies, we will omit the parameters of these functions for better readability (e.g y_i^k instead of $y_i^k(\gamma, x_i^k)$).

4.1 Nash equilibrium

We will now determine the Nash equilibrium of the given model. Hillman and Riley (1987) and Stein (2002) have shown that groups/individuals may prefer to stay inactive if the size of all groups is equal to 1. Baik (2008) has shown that only group members with maximum valuation participate in a contest. Hence, it is possible that some individuals and/or groups will stay inactive in our setup. We therefore start with an analysis of active individuals and groups.

Definition 6. An individual k of group i is said to participate if $x_i^k > 0$. A group i is said to *participate* if there exists some k such that $x_i^k > 0$. A group is said to *fully participate* if $\forall k : x_i^k > 0$.

Lemma 1. *In a Nash equilibrium of a contest fulfilling Assumptions 1, 2, and 4 if a group participates, it fully participates.*

Lemma 1 implies that in order to determine whether an individual participates, it is sufficient to determine whether its group participates and vice versa. Let $V_i(\gamma) \equiv \left(1/m_i \sum_l v_l^{\frac{\gamma}{1-\gamma}}\right)^{\frac{1-\gamma}{\gamma}}$. Without loss of generality, suppose the groups are ordered such that $V_i(\gamma) \geq V_{i+1}(\gamma)$ for a given γ . $Q_i^*(\gamma)$ and $Q^*(\gamma)$ shall denote Q_i and Q in equilibrium. The following Lemma determines the groups that participate in equilibrium.

Lemma 2. *a) There exist best-response strategies of the members of a group, if and only if the following group best-response function is fulfilled:*

$$\hat{Q}_i(\gamma, Q_{/i}) = \max\left(0, \sqrt{Q_{/i}V_i(\gamma)} - Q_{/i}\right). \quad (4)$$

b) Groups $1 \dots n^(\gamma)$ participate, where $n^*(\gamma) \equiv \arg \max_i i$ such that $V_i(\gamma) > Q^*(\gamma)$.
c) If the Nash equilibrium is unique, $Q_i^*(\gamma)$ and $Q^*(\gamma)$ are continuous functions for $\gamma \neq 0$.*

Lemma 2.c is useful for the comparative-static analysis. Given that the number and identity of active groups depends on γ , it is *a priori* not clear that aggregate effort and indirect utilities are continuous in γ . The Lemma reveals that continuity is in fact guaranteed except at $\gamma = 0$. The economic intuition is as follows: Assume that $\hat{\gamma}$ is a point where a formerly active group becomes inactive or a formerly inactive group becomes active. The aggregate group effort of the active group is continuously reduced to zero as γ approaches $\hat{\gamma}$, and the formerly inactive group continuously increases its effort from 0 as γ increases from $\hat{\gamma}$. Hence, there is a “smooth” fade out or fade in of groups at those points.

The following proposition characterizes the unique Nash equilibrium of the game. For readability, the strategies x_i^k are defined as functions of $Q^*(\gamma)$ and $V_i(\gamma)$.

Proposition 5. *The unique Nash equilibrium of the game characterized by Assumptions 1, 2, and 4 is given by strategies $x_i^{k*}(\gamma)$ that fulfill*

$$x_i^{k*}(\gamma) = \begin{cases} Q^*(\gamma) \left(1 - \frac{Q^*(\gamma)}{V_i(\gamma)}\right) \frac{1}{m_i} \frac{(v_i^k)^{\frac{1}{1-\gamma}}}{V_i(\gamma)^{\frac{1}{1-\gamma}}}, & V_i(\gamma) > Q^*(\gamma) \\ 0, & V_i(\gamma) \leq Q^*(\gamma) \end{cases}, \quad (5)$$

where $Q^*(\gamma) = \frac{n^*(\gamma)-1}{\sum_{i=1}^{n^*(\gamma)} V_i(\gamma)^{-1}}$ and $n^*(\gamma)$ is defined in Lemma 2.a and groups are ordered such that $V_i(\gamma) \geq V_{i+1}(\gamma)$.

Proof. To obtain $Q^*(\gamma)$ we sum (4) over all $i \leq n^*(\gamma)$:

$$Q^*(\gamma) = \frac{n^*(\gamma) - 1}{\sum_{i=1}^{n^*(\gamma)} V_i(\gamma)^{-1}}. \quad (6)$$

With an explicit solution for $Q^*(\gamma)$, we can now determine individual expenditures $x_i^{k^*}(\gamma)$ by solving equation (4) using (6). The participation condition of a group is given by Lemma 2, while Lemma 1 ensures that there does not exist an incentive for any group member to deviate to $x_i^k = 0$. It was further shown that the first-order conditions return local maxima. Since the system of equations given by the first-order conditions of the participating groups has a unique solution this is indeed the unique Nash equilibrium. \square

It is of course interesting to see whether different degrees of complementarity have an effect on the equilibrium if all individuals have the same valuations, i.e. $v_i^k = v_i \forall k \forall i$. The following corollary of Proposition 5 can then be established.

Corollary 2. *If $v_i^k = v_i \forall k \forall i$ the equilibrium efforts of all groups are independent of γ .*

Proof. The corollary directly follows from inserting $v_i^k = v_i$ into the above definitions, since then $V_i = v_i$. \square

This finding implies that an increase in complementarity between group members' effort *per se* has no effect on the within-group free-rider problem, as could have been conjectured from Hirshleifer (1983). A further implication of the result is that the results on group contests that have been derived in the literature for the case of perfect substitutes or perfect complements carry over to arbitrary elasticities of substitution if groups differ only in their valuations of the rent. In particular, different elasticities of substitution will not affect the occurrence of the group size paradox for homogenous groups.

Most interestingly, individual valuations in relation to the valuations of the other group members define the individuals' share of the amount of effort spent by the

group, x_i^{k*}/X_i^* . The valuation of other groups have no effect on these shares. As was to be expected, a larger elasticity of substitution γ increases *ceteris paribus* the dispersion of these shares, since the exponent discriminates more strongly between differences in valuations, which is expressed by the following corollary.

Corollary 3. *The share of an individual of its group's effort, $\frac{x_i^k}{X_i}$, increases (decreases) strictly in the elasticity of substitution among efforts if the valuation of the individual is strictly larger (smaller) than the weighted geometric mean $\prod_l v_i^l \left(\frac{v_i^l \frac{1}{1-\gamma}}{\sum_s v_i^s \frac{1}{1-\gamma}} \right)$ of the group members' efforts.*

Proof. It is straightforward to derive the following equation from (5):

$$\frac{x_i^{k*}(\gamma)}{X_i^*(\gamma)} = \frac{(v_i^k)^{\frac{1}{1-\gamma}}}{\sum_l (v_i^l)^{\frac{1}{1-\gamma}}}. \quad (7)$$

Taking the derivative of (7) with respect to γ yields

$$\frac{\partial \frac{x_i^k}{X_i}}{\partial \gamma} = \frac{v_i^k \frac{1}{1-\gamma}}{\sum_l v_i^l \frac{1}{1-\gamma}} \frac{1}{(1-\gamma)^2} \left(\ln v_i^k - \frac{\sum_l v_i^l \frac{1}{1-\gamma} \ln v_i^l}{\sum_l v_i^l \frac{1}{1-\gamma}} \right). \quad (8)$$

The RHS of the above equation is positive whenever the term in brackets is positive. Setting $\ln v_i^k \geq \frac{\sum_l v_i^l \frac{1}{1-\gamma} \ln v_i^l}{\sum_l v_i^l \frac{1}{1-\gamma}}$ and rearranging yields:

$$\frac{\partial \frac{x_i^k}{X_i}}{\partial \gamma} \geq 0 \Leftrightarrow v_i^k \geq \prod_l v_i^l \left(\frac{v_i^l \frac{1}{1-\gamma}}{\sum_s v_i^s \frac{1}{1-\gamma}} \right). \quad (9)$$

□

4.2 Group-size paradox for heterogenous valuations within groups

It is already clear that the group-size paradox will hold weakly but not strictly in this model if agents within groups have identical valuations. For heterogenous agents we can however no longer rely on Definition 1, since agents with different valuations can be added to the group. It therefore makes more sense to look at how high the valuation of a new member of the group needs to be to increase the winning probability of the group.

Proposition 6. *For groups with heterogenous valuations, there exists a minimum valuation a new group member must have in order to raise the winning probability of its group. This minimum valuation is increasing in the elasticity of substitution among the efforts of the group members.*

This result shows that for heterogenous valuations, a third property of a contest plays an important role with respect to the effect of the group size on the winning probabilities: The more complementary the efforts of the group members are, the lower the necessary valuation of a new member has to be in order to raise the winning probability. Since heterogeneity in valuations is a rather common feature of interest groups, the returns to scale, the group size biasedness and the effort complementarity of the impact function should play a role when analyzing the relative strength of interest groups.

5 Relationship of our results with other fields of the literature

The studies mentioned in the introduction – like Olson’s original argument and contrary to the specific interpretation given to it by Esteban and Ray (2001) – do not specifically focus attention to situations of conflict between groups, and there are only few strands of the literature that deal with contest-like situations.

The first one studies the effect of group size for non-human species¹⁰, especially bird groups and predation. The empirical evidence points in the direction that – for several reasons and up to a certain limit – larger bird groups are more successful than smaller ones, for example because larger groups may detect predators sooner than solitary individuals. Bertram (1980) found that the percentage of time a single ostrich has its head up is decreasing in group size, but the percentage of time one

¹⁰Which of course implies that one has to be sufficiently cautious with respect to the implications for models based on the utility maximization paradigm. The seminal contribution to game theoretic application to animal behavior is Smith and Price (1972). For a critical comparison of models of utility maximization and Nash equilibrium as solution concept with models of fitness maximization and evolutionary stable strategies as solution concept in contests see Leininger (2003).

or more heads are up is increasing. For the case of wood-pigeons, Kenward (1978) found that the percentage of successful hawk attacks is decreasing in group size. An explanation for this positive effect of a larger group-size is the so-called ‘many eyes hypothesis:’ All members of a group are alerted if at least one member detects a potential predator¹¹ or the fact that it is harder for the predator to focus on a specific prey if the group is large. Cresswell and Quinn (2004) and Kenward (1978) analyzed Sparrowhawk attack success when hunting Redshanks and found that the probability of capture of Sparrowhawks increased when the group size of their prey decreased. This finding suggests that there are positive effects of group size for the prey in predator-prey contests. Translated into the formal language of our model and with the necessary prudence, the group advantage can be interpreted as a specific property of the contest that cannot be reduced to some form of strictly convex costs-of effort function but can better be explained by impact functions leading to some form of group-size bias and/or economies of scale. The main driving force for these positive effects, however, does not necessarily stem from the contest situation; the decrease in successful attacks may be a useful byproduct of some other positive effects. In order to test this, Beauchamp (2004) examined flock sizes of species living on islands where predation risk is either absent or negligible with flock sizes of the same species on the mainland (with higher predation risk). Controlled for other potential explanatory factors like population density, habitat type, food type, etc., mean and maximum flock size were smaller on islands than on the mainland. The results suggest that predation is a significant factor in the evolution of flocking in birds. The ‘many eyes hypothesis’ and the increasing difficulty of the predator to focus on prey in larger groups closely resemble a type of non-additive impact function with positive group-size bias.

There is also a lively discussion about the prevalence of a group-size paradox in contest environments in the sociological as well as the strategic-management literature with (as can be expected from Esteban and Ray (2001) as well as our paper) mixed empirical evidence. Siegel (2009) argues that in large groups, the large number of ties between group members can hamper collective action. Larger groups require

¹¹Pulliam, Pyke, and Caraco (1982) found that individuals in groups use a ‘conditional vigilance strategy’ where individuals are cooperatively vigilant as long as all other group members remain so too (tit-for-tat); this implies that individuals monitor the behavior of others.

more specialization to effectively manage the increasing complexity McCarthy and Zald (1977) and to allow for effective decision-making procedures (Benbasat and Lim (1993)). The in general smaller diversity of members of smaller groups makes it easier to coordinate on shared goals and collective action (e.g. Gamson (1995), Klandermans and deWeerd (n.d.), Monge et al. (1998)). It is also easier to speak with one voice if the group is smaller, avoiding inconsistent messages which are counterproductive for the success in any lobbying process (Dominelli (1996)), which implies that there must be a certain degree of complementarity between the group members' efforts. Using data from Swedish firms, Wincent, rtqvist, Eriksson, and Autio (2010) found evidence for the predicted adverse effect of group size on the amount of fundraising. Economies of scale and group-size bias therefore tend to favor smaller groups in these situations. However, there are also opposite findings. For example, the mere number of group members may give the group more media coverage and/or political power (McAdam (1882)), pointing to a positive group-size bias. Larger groups may also be able to provide more funds (Oliver (1993), Zald and Ash (1966)). A study by McCarthy and Wolfson (1996) showed that in fact the size of task committees had a positive effect on the amount of funding obtained in campaigns for local government. Finally, Dejean, Penard, and Suire (2009) find a positive relation between the size of a community and the amount of collective good provided (with decreasing individual propensity to cooperate) in an empirical analysis of P2P file-sharing communities, a result that is strikingly similar to the findings about individual and group vigilance of birds mentioned above.

6 Concluding remarks

According to our model one can expect that four crucial “technological” factors determine the role of group size on the outcome of a group contest, group-size biasedness, returns to scale, and complementarity between group members' efforts as well as the composition of their valuations in case of heterogenous valuations within groups. These findings complement and extend the results by Esteban and Ray (2008, 2010) who have shown that the convexity of individual cost functions may explain the reversal of the group-size paradox, and it turns out that convexity in costs is a special case of positive group-size bias.

Empirical findings support the existence of a group-size paradox, but as noted by Marwell and Oliver (1993), it also stands in contrast to a significant body of empirical findings pointing to a positive relationship between group size and group performance in conflicts. Our analysis reveals that this diverse empirical pattern may not be reduced to only one explanatory variable, namely the degree of convexity in costs. To continue the example from the introduction, the success of political demonstrations may depend on media coverage which in turn may depend on the number of demonstrators. This is an example of group-size biasedness that cannot be reduced to convexities in costs.

Our analysis of within-group heterogeneity, a constellation that should empirically rather be the rule rather than the exception, shows first that the composition of individual valuations is in fact important for contest success. Second, the degree of complementarity in reaching impact becomes important. The higher the degree of complementarity, the lower the threshold an individual valuation has to reach in order to have a positive impact on group success. This finding sheds light on the empirical findings stressing that heterogeneity is likely to have adverse effects on the contribution to the group-specific public good (Bandiera, Barankay, & Rasul, 2005).

Appendix A: Proof of Proposition 1

Proof. Suppose we have for all groups a group-size unbiased impact function q . Denote by Q^* and $Q_{/i}^*$ the total impact of all groups and of all groups except group i in equilibrium. By Assumption 3 there exists a Nash equilibrium that is unique up to redistributions among the group members and that is characterized by the solution to the first-order conditions (FOCs):

$$\frac{\partial q(x_i^*)}{\partial x_i^k} \frac{Q_{/i}^*}{Q^{*2}} v_i - 1 = 0 \quad \forall i, k \quad (\text{A.1})$$

It is evident from the FOC that the equilibrium is symmetric among members of a group and we thus only need to focus on fulfilling the FOC of the first member of each group. By assumption, the impact function is group-size unbiased. For equal inputs we can then write $q(x_i) = g(x_i^1 \cdot m_i)$, since otherwise the functional equation induced by group-size unbiasedness cannot be fulfilled Aczél and Dhombres (1989).

Note that $\frac{\partial q(x_i)}{\partial x_i^k} = g'(x_i^1 \cdot m_i) \cdot m_i$ is not the correct partial derivative. However, we can employ the total differential:

$$\Delta q(x_i) = \sum_k \Delta x_i^k \frac{\partial q(x_i)}{\partial x_i^k} \quad (\text{A.2})$$

which becomes for symmetric agents:

$$\Delta g(x_i^1 m_i) = m_i \Delta x_i^1 \frac{\partial q(x_i)}{\partial x_i^k}, \quad (\text{A.3})$$

and thus

$$\frac{\partial q(x_i)}{\partial x_i^k} = \frac{\Delta g(x_i^1 \cdot m_i)}{m_i \cdot \Delta x_i^1} = g'(x_i^1 \cdot m_i) \quad (\text{A.4})$$

for $\Delta \rightarrow 0$. We can reinsert this expression into the FOC and replace Q_i^*/Q^* by $(1 - p_i^*)$ as this is the probability with which the other groups win,

$$g'(x_i^{1*} \cdot m_i) = \frac{Q^*}{v_i(1 - \frac{Q_i^*}{Q^*})} = \frac{Q^*}{v_i(1 - p_i^*)} \quad \forall k. \quad (\text{A.5})$$

We can now prove by contradiction that m_i and p_i^* cannot rise at the same time. Suppose this would be the case. For the behavior of Q^* , we can now distinguish three cases, Q^* increases, remains constant, or decreases.

Suppose Q^* increases. This implies by the definition of $p_i^* = Q_i^*/Q^*$ that Q_i^* increases, as otherwise p_i^* would not increase. Since Q_i^* increases if and only if $x_i^{k*} \cdot m_i$ increases, the left-hand side (LHS) of (A.5) must decrease, as $g'(\dots)$ is a decreasing function. But the fact that Q^* and p_i^* increase, implies that the right-hand side (RHS) increases, which means the FOCs cannot be fulfilled.

Second, suppose Q^* remains unchanged. This implies that the RHS of (A.5) increases, which in turn implies that $x_i^{k*} \cdot m_i$ falls. But if Q_i^* decreases, Q^* must decrease as well, since p_i^* increases. Q^* can thus not remain constant.

Third, consider Q^* decreasing. Since p_i^* is increasing, there must exist a group j where p_j^* is decreasing. Take the FOC of this group j : Since Q^* is decreasing and p_j^* is decreasing, the RHS is decreasing. This implies that $x_j^{k*} \cdot m_j$ is increasing which means Q_j^* is increasing and thus p_j^* as well. This contradicts the assumption that there exists a group where p_j^* is decreasing for decreasing Q^* . If there is no group with decreasing probability, p_i^* cannot be increasing.

Since for all possible cases of behavior for Q^* , there arises a contradiction from the assumption that p_i^* increases when m_i increases, it is established that p_i^* weakly decreases in m_i under the given assumptions. \square

Appendix B: Proof of Proposition 2

Proof. It is convenient to summarize the following properties in a Lemma.

Lemma B.1. *If $q_{m_i}(\cdot), q_{m_i+1}(\cdot)$ have constant returns to scale and negative (positive) group-size bias for all m_i , it follows for a symmetric equilibrium $x_i^k = x_i^l \forall k, l \forall i$ that*

$$\frac{\partial q_{m_i}(\hat{x}_i)}{\partial x_i^k} > (<) \frac{\partial q_{m_i+1}(\hat{x}_i)}{\partial x_i^k}.$$

and that $\frac{\partial q_{m_i}(\hat{x}_i)}{\partial x_i^k}$ is invariant in the level of effort of a group.

Proof of Lemma. Due to constant returns to scale, we have $q(a \cdot x_i) = a \cdot q(x_i)$, and it follows that $\frac{\partial q(x_i)}{\partial x_i^k}$ is homogenous of degree zero in x_i . Using the symmetry of an equilibrium, $q(\cdot)$ can be expressed by some function $h(\cdot)$ such that $q_{m_i}(x_i) = h(x_i^1, m_i)$, and $\frac{\partial q(x_i)}{\partial x_i^k}$ can only be a function of m_i . By Euler's homogenous function theorem,

$$\sum_k \frac{\partial q(x_i)}{\partial x_i^k} x_i^k = q(x_i),$$

we have in a symmetric equilibrium

$$\frac{\partial q(x_i)}{\partial x_i^k} = \frac{q(x_i)}{x_i^1 \cdot m_i} \equiv \frac{h(x_i^1, m_i)}{x_i^1 \cdot m_i}.$$

By homogeneity of degree one of h in its first argument, we know that $\frac{\partial q_{m_i}(\hat{x}_i)}{\partial x_i^k}$ is indeed invariant in the level of effort as expected for constant returns to scale in a symmetric equilibrium. We further want to know whether

$$\frac{h(\hat{x}_i^1, m_i)}{\hat{x}_i^{1*} \cdot m_i} \geq \frac{h(\hat{x}_i^1, m_i + 1)}{\hat{x}_i^1 \cdot (m_i + 1)}$$

holds. Suppose that $\beta = \frac{m_i+1}{m_i}$. Then, the RHS of the above inequality can be written as:

$$\frac{g(\hat{x}_i^1, m_i + 1)}{\hat{x}_i^1 \cdot (m_i + 1)} = \frac{h(\hat{x}_i^1, \beta \cdot m_i)}{\hat{x}_i^1 \cdot \beta \cdot m_i}. \quad (\text{B.1})$$

By constant returns to scale, (B.1) can be written as:

$$\frac{h(\hat{x}_i^1, \beta \cdot m_i)}{\hat{x}_i^1 \cdot \beta \cdot m_i} = \frac{h(\frac{\hat{x}_i^1}{\beta}, \beta \cdot m_i)}{\hat{x}_i^1 \cdot m_i},$$

which by $q(\cdot)$ being negatively (positively) group size biased is strictly smaller (larger) than $\frac{h(\hat{x}_i^1, m_i)}{\hat{x}_i^1 \cdot m_i}$. This implies that $\partial q(\hat{x}_i)/\partial x_i^k$ is indeed decreasing (increasing) in m_i for a symmetric equilibrium. \square

We now turn to the proof for negative group-size bias. The FOCs are given by:

$$\frac{\partial q(x_i^*)}{\partial x_i^k} \frac{Q_{/i}^*}{Q^{*2}} v_i - 1 = 0 \quad \forall i, k. \quad (\text{B.2})$$

We want to establish that a joint increase in m_i and p_i^* in equilibrium leads to a contradiction. Examining the FOC of group member 1 of group i :

$$\frac{\partial q(x_i^*)}{\partial x_i^1} = \frac{Q^*}{(1 - p_i^*) v_i} \quad \forall i, k, \quad (\text{B.3})$$

we know that Q^* must decrease if m_i and p_i^* increase, since by Lemma B.1. the LHS is decreasing in m_i and invariant in x_i^{1*} for every given symmetric equilibrium. The RHS is increasing in p_i^* . For the FOC to hold, Q^* must then decrease. Note that an increase in p_i^* implies a decrease in p_j^* for at least one group j . Looking at the FOC of group member 1 of this group j , we can derive the contradiction:

$$\frac{\partial q(x_j^*)}{\partial x_j^1} = \frac{Q^*}{(1 - p_j^*) v_j}. \quad (\text{B.4})$$

As we know, the LHS is constant for given group size m_j . The RHS however is both increasing in p_j^* and Q^* . Since both are decreasing, the FOC can no longer be fulfilled, which yields the contradiction.

The proof for positive group-size bias is similar. We again start with FOCs of an arbitrary member of an arbitrary group:

$$\frac{\partial q(x_i^*)}{\partial x_i^k} \frac{Q_{/i}^*}{Q^{*2}} v_i - 1 = 0 \quad \forall i, k. \quad (\text{B.5})$$

We want to establish that an increase in m_i and a decrease in p_i^* in equilibrium leads to a contradiction. Examining the FOC of group member 1 of group i :

$$\frac{\partial q(x_i^*)}{\partial x_i^1} = \frac{Q^*}{(1 - p_i^*) v_i} \quad \forall i, k, \quad (\text{B.6})$$

we know that Q^* must increase if m_i increases and p_i^* decreases, since by Lemma B.1. the LHS is decreasing in m_i and invariant in x_i^{1*} for every given symmetric equilibrium. The RHS is increasing in p_i^* . For the FOC to hold, Q^* must then increase. Note that a decrease in p_i^* implies an increase in p_j^* for at least one group j . Looking at the FOC of group member 1 of this group j , we can, as before, derive the contradiction:

$$\frac{\partial q(x_j^*)}{\partial x_j^1} = \frac{Q^*}{(1 - p_j^*)v_j}. \quad (\text{B.7})$$

As we know, the LHS is constant for given group size m_j . The RHS however is both increasing in p_j^* and Q^* . Since both are increasing, the FOC can no longer be fulfilled, which yields the contradiction. \square

Appendix C: Proof of Proposition 3

Proof. Let ρ be the degree of homogeneity. Euler's Theorem for homogenous functions implies that $q_{m_i}(x_i) = \frac{1}{\rho} \cdot \sum_k x_i^k \cdot \frac{\partial q_{m_i}(x_i)}{\partial x_i^k}$, which boils down to

$$q_{m_i}(x_i) = \frac{m_i \cdot x_1^k}{\rho} \cdot \frac{\partial q_{m_i}(x_i)}{\partial x_i^k} \quad (\text{C.1})$$

for a symmetric equilibrium, which can (again for a symmetric equilibrium) alternatively be expressed as

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i^k} = \frac{\rho \cdot q_{m_i}(x_i)}{m_i \cdot x_1^k}. \quad (\text{C.2})$$

(Weakly) negative group-size biasedness implies $q_{m_i}(x_i) \geq q_{\xi \cdot m_i}(x_i/\xi)$. Using (C.2) this inequality can be expressed as

$$\begin{aligned} q_{m_i}(x_i) = \frac{m_i \cdot x_1^k}{\rho} \cdot \frac{\partial q_{m_i}(x_i)}{\partial x_i^k} &\geq \frac{m_i \cdot x_1^k}{\rho} \cdot \frac{\partial q_{\xi \cdot m_i}(x_i/\xi)}{\partial x_i^k} \\ &= \frac{\xi \cdot m_i}{\rho} \cdot \frac{x_1^k}{\xi} \cdot \frac{\partial q_{\xi \cdot m_i}(x_i/\xi)}{\partial x_i^k} = q_{\xi \cdot m_i}(x_i/\xi). \end{aligned}$$

This inequality reduces to

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i^k} \geq \frac{\partial q_{\xi \cdot m_i}(x_i/\xi)}{\partial x_i^k}. \quad (\text{C.3})$$

Assume that $\xi = (m_i + 1)/m_i$. The first-order condition of a representative member of group i is equal to

$$\frac{\partial q(x_i^*)}{\partial x_i^k} = \frac{Q^*}{v_i(1 - \frac{Q_i^*}{Q^*})} = \frac{Q^*}{v_i(1 - p_i^*)} \quad \forall k \quad (\text{C.4})$$

in equilibrium. We can now prove by contradiction that m_i and p_i^* cannot rise at the same time. Suppose this would be the case. For the behavior of Q^* , we can now distinguish three cases, Q^* increases, remains constant, or decreases.

Suppose Q^* increases. The increase in p_i^* also implies that Q_i^* has to increase. For fixed $X_i = \sum x_i^k$, the definition of group-size biasedness implies that $q(x_i)$ goes down. In order to be consistent with the increase in Q_i^* it follows that $X_i = \sum x_i^k$ has to go up.

The above assumptions imply that the RHS of (C.4) increases. For this to be an equilibrium, the LHS of (C.4) has to increase as well. For fixed $X_i = \sum x_i^k$, however, (C.3) implies that $\partial q(x_i)/\partial x_i^k$ goes down. (Weakly) decreasing returns to scale imply that $\partial q(x_i)/\partial x_i^k$ is (weakly) decreasing along the symmetric array through the origin. Hence, $X_i = \sum x_i^k$ has to go down to reestablish the equality, a contradiction.

Second, suppose Q^* remains unchanged. The contradiction follows along the same lines as before: An increase from m_i to $m_i + 1$, (C.3) implies that $\partial q(x_i)/\partial x_i^k$ is reduced for constant $X_i = \sum x_i^k$. If p_i^* goes up, the LHS of (C.4) goes up. To reestablish the equality it follows that $X_i = \sum x_i^k$ has to go down, which c.p. reduces Q_i^* , and for Q^* being constant, has to increase Q_j^* for some j . This is, however, inconsistent with the conjecture that p_i^* increases.

Third, consider Q^* decreasing. Since p_i^* is increasing, there must exist a group j where p_j^* is decreasing. Take the FOC of this group j : Since Q^* is decreasing and p_j^* is decreasing, the RHS is decreasing, which implies that the LHS has to decrease. In addition, Q_j^* has to go down as well. For given m_j , the LHS can only decrease if $X_j = \sum x_j^k$ increases, which is inconsistent with the requirement that Q_j^* has to go down.

□

Appendix D: Proof of Corollary 1

Proof. From group size unbiasedness we know that (A.4) holds. Due to constant returns to scale, $g'(x_i^1 m_i)$ is a constant. The LHS of the first order condition (A.5) is thus constant. Suppose now p_i rises (falls) with a change in m_i . Then for the first order condition to still hold, Q^* needs to fall (rise). Also, the winning probability of at least one other group p_j needs to fall (rise). A fall (rise) in both Q^* and p_j is however incompatible with the first order condition of group j . Therefore, both p_i and Q^* need to remain constant. \square

Appendix E: Proof of Proposition 4

Proof. We assume throughout that we are in a symmetric, interior equilibrium. By homogeneity of degree r_i , we have from Euler's theorem

$$r_i \cdot q_{m_i}(x_i) = m_i \cdot x_i \cdot \frac{\partial q_{m_i}(x_i)}{\partial x_i}, \quad (\text{E.1})$$

and further

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i} = \frac{r_i \cdot q_{m_i}(x_i)}{m_i \cdot x_i} = \frac{r_i \cdot q_{m_i}(1)}{m_i \cdot (x_i)^{1-r_i}}. \quad (\text{E.2})$$

By the definition of group size biasedness we have that a function is positively group size biased, if $q_{m_i}(x_i) < q_{m_i\xi}(x_i/\xi)$. In the homogenous case, this is equivalent to $q_{m_i}(1) < q_{m_i\xi}(1)\xi^{-r_i}$. Using the above equation, we can reformulate this as:

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i} < \frac{\partial q_{m_i\xi}(x_i)}{\partial x_i} \xi^{-r_i}. \quad (\text{E.3})$$

By these conditions, a natural measure for group size bias emerges that does not depend on x_i :

$$b_i(\xi) = \frac{q_{m_i\xi}(1)}{q_{m_i}(1)\xi^{r_i}} = \frac{\frac{\partial q_{m_i\xi}(1)}{\partial x_i}}{\frac{\partial q_{m_i}(1)}{\partial x_i} \xi^{r_i}}. \quad (\text{E.4})$$

Clearly, if $b_i(\xi) > 1$, q is positively group size biased.¹² For homogenous functions, $q_{m_i\xi}$ can be uniquely determined by $b_i(\xi)$, q_{m_i} , and r_i . We can derive two helpful

¹²Note that this measure is a local measure which depends on ξ . A function that is positively group size biased for some increase in members may be negatively group size biased for others.

expressions for the derivatives of q with respect to x_i :

$$\frac{\partial q_{m_i}(x_i)}{\partial x_i} = \frac{r_i q_{m_i}(1)}{m_i x_i^{1-r_i}} = \frac{\partial q_{m_i \xi}(x_i)}{\partial x_i} \cdot \frac{\xi^{1-r_i}}{b_i(\xi)} \quad (\text{E.5})$$

$$= \frac{\partial q_{m_i}(1)}{\partial x_i} \cdot \frac{1}{x_i^{1-r_i}}. \quad (\text{E.6})$$

We now compare two classes of impact functions, q and \hat{q} for which $b_i(\xi) > \hat{b}_i(\xi)$ for some¹³ ξ and $r_i = \hat{r}_i$.

Suppose that the first-order condition holds for the impact function q_{m_i} at x_i^* . After adding $m_i \xi - m_i$ group members and employing the impact function $q_{m_i \xi}$, let the equilibrium contribution be $x_{i,\xi}^*$. Similarly, denote by \hat{x}_i^* , $\hat{x}_{i,\xi}^*$ the equilibrium efforts when using impact functions \hat{q}_{m_i} and $\hat{q}_{m_i \xi}$, respectively.

We can now employ the first-order condition to obtain that if the group size paradox holds, then $\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} > \frac{\partial q_{m_i \xi}(x_{i,\xi}^*)}{\partial x_i}$. The first-order condition of an interior solution becomes after rearranging terms:

$$\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} = \frac{Q^*}{v_i^k(1-p_i^*)}. \quad (\text{E.7})$$

By (E.6) we can reformulate the LHS:

$$\frac{\partial q_{m_i}(1)}{\partial x_i} \cdot (x_i^*)^{r_i-1} = \frac{Q^*}{v_i^k(1-p_i^*)}. \quad (\text{E.8})$$

Suppose the group-size paradox holds. Then we can see that for an increase in m_i , the LHS of the equation has to decrease. This follows since an increase in the LHS would imply that on the RHS Q^* must rise, because by assumption p_i^* decreases. Since p_i^* decreases and probabilities must sum to one, there must exist another group j , for which p_j increases. But then the RHS of the first order condition of group j must increase and since Q^* increases, q_j^* and thus x_j^* must increase as well. However, the LHS is decreasing in x_j , implying that there does not exist x_j such that the first-order condition is satisfied anymore. Concluding, if the group size paradox holds and m_i increases, then

$$\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} > \frac{\partial q_{m_i \xi}(x_{i,\xi}^*)}{\partial x_i}, \quad (\text{E.9})$$

¹³Alternatively we could assume that this condition holds for all ξ , from which we could derive the result that if the group size paradox holds for all ξ under q , then it must hold for all ξ under \hat{q} as well.

i.e. the LHS of the first-order condition of group i has to be lower in the new equilibrium. Similarly, it can be derived that if the winning probability does not change for a change in m_i , then $\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} = \frac{\partial q_{m_i\xi}(x_{i,\xi}^*)}{\partial x_i}$, and lastly, if the winning probability increases, $\frac{\partial q_{m_i}(x_i^*)}{\partial x_i} < \frac{\partial q_{m_i\xi}(x_{i,\xi}^*)}{\partial x_i}$.

We will now assume that the group-size paradox holds for q , but not for \hat{q} and show that this yields a contradiction. Combining the above properties of the partial derivatives of $q_{m_i}, q_{m_i\xi}$ with equation (E.9) and remembering that the derivative of a function that is homogenous of degree r is homogenous of degree $r - 1$, we obtain:

$$\frac{\partial q_{m_i}(x_i^*)}{\partial x_i^*} = \frac{\partial q_{m_i\xi}(1)}{\partial x_i} \cdot \frac{\xi^{1-r_i}}{b_i(\xi)(x_i^*)^{1-r_i}} > \frac{\partial q_{m_i\xi}(1)}{\partial x_i} \cdot \frac{1}{(x_{i,\xi}^*)^{1-r_i}} = \frac{\partial q_{m_i\xi}(x_{i,\xi}^*)}{\partial x_i} \quad (\text{E.10})$$

which implies:

$$b_i(\xi) < \left(\frac{\xi x_{i,\xi}^*}{x_i^*} \right)^{1-r_i} \quad (\text{E.11})$$

Similarly, with the inequality reversed (since we assume that the group size paradox does not hold for \hat{q}):

$$\hat{b}_i(\xi) > \left(\frac{\xi \hat{x}_{i,\xi}^*}{\hat{x}_i^*} \right)^{1-r_i} \quad (\text{E.12})$$

Combining these two inequalities via the assumption that q has a greater group size bias than \hat{q} :

$$\left(\frac{\xi \hat{x}_{i,\xi}^*}{\hat{x}_i^*} \right)^{1-r_i} < \hat{b}_i(\xi) < b_i(\xi) < \left(\frac{\xi x_{i,\xi}^*}{x_i^*} \right)^{1-r_i}. \quad (\text{E.13})$$

Exponentiating by $r_i/(1-r_i)$ and using homogeneity of degree r_i and (E.4), we have:

$$\frac{\hat{q}_{m_i\xi}(\hat{x}_{i,\xi}^*)}{\hat{q}_{m_i}(\hat{x}_i^*)\hat{b}_i(\xi)} < \frac{q_{m_i\xi}(x_{i,\xi}^*)}{q_{m_i}(x_i^*)b_i(\xi)}. \quad (\text{E.14})$$

Finally, we know that if the group-size paradox holds, $q_{m_i}(x_i^*) > q_{m_i\xi}(x_{i,\xi}^*)$ must hold since otherwise $x_{i,\xi}^*$ could not be a best response¹⁴. Together with $b_i(\xi) > \hat{b}_i(\xi)$, we conclude that $\hat{q}_{m_i\xi}(\hat{x}_{i,\xi}^*) < \hat{q}_{m_i}(\hat{x}_i^*)$. This contradicts the assumption that the group-size paradox does not hold for \hat{q} , since then a lower \hat{q} would have to yield a higher winning probability, in which case \hat{x}_i^* could not be a best response.

The reverse argument, stating that if we increase group size bias, the group size paradox cannot hold if it did not held before can be derived by reversing inequalities

¹⁴To see this, note that the winning probability is higher for a group impact $q_{m_i}(x_i^*)$. Since group impact is increasing in x_i , it would be profitable for some group member to choose $x_i^k < x_{i,\xi}^*$

in E.13 above. The proof for a change in $r_i > \hat{r}_i$ for $b_i(\xi) = \hat{b}_i(\xi)$ now follows the same lines. Assuming the group size paradox holds under q but not under \hat{q} , we have:

$$\left(\frac{\xi \hat{x}_{i,\xi}^*}{\hat{x}_i^*}\right)^{1-\hat{r}_i} < \hat{b}_i(\xi) = b_i(\xi) < \left(\frac{\xi x_{i,\xi}^*}{x_i^*}\right)^{1-r_i} \quad (\text{E.15})$$

This inequality, similarly to (E.13), implies (note that the term $\frac{r_i}{1-r_i}/\frac{\hat{r}_i}{1-\hat{r}_i}$ is strictly greater than 1 and since the RHS is strictly greater than 1 we can exponentiate it with the term):

$$\frac{\hat{q}_{m_i\xi}(\hat{x}_{i,\xi}^*)}{\hat{q}_{m_i}(\hat{x}_i^*)} < \frac{q_{m_i\xi}(x_{i,\xi}^*)}{q_{m_i}(x_i^*)}. \quad (\text{E.16})$$

From here, the same argument as above applies and therefore if the group size paradox holds for q with returns to scale r_i , then it must hold as well for \hat{q} with returns to scale $\hat{r}_i < r_i$. The reverse argument again follows from reversing the inequalities in (E.15) and assuming $\hat{r}_i > r_i$.

□

Appendix F: Proof of Lemma 1

Proof. We first check that the interior solution is a local maximum. The FOC of the maximization problem (2) can be written as

$$\frac{Q_{/i}Q_i}{Y_iQ^2} = \frac{(y_i^k)^{\frac{1}{\gamma}}}{y_i^k v_i^k}. \quad (\text{F.1})$$

The second-order condition is satisfied if

$$\frac{v_i^k Q_{/i}Q_i}{\gamma Q^2 Y_i^2} \left(\frac{1 - 2\frac{Q_i}{Q}}{\gamma} - 1 \right) - \frac{\frac{1}{\gamma} - 1}{\gamma} (y_i^k)^{\frac{1}{\gamma}-2} < 0. \quad (\text{F.2})$$

Solving the FOC for v_i^k and inserting the expression into the second-order condition we obtain, upon rearranging:

$$\frac{1 - \frac{1}{\gamma}}{\gamma} \left(1 - \frac{y_i^k}{Y_i} \right) - 2\frac{1}{\gamma^2} \frac{Q_i y_i^k}{Q Y_i} < 0, \quad (\text{F.3})$$

which holds for all $\gamma \in (-\infty, 1)$. Therefore, all solutions of the FOC are local maxima taking the other players' strategies as given. The best responses are either given by

the solution to the FOC, or by a corner solution. From equation (3) it is clear that the only possible corner solutions are non-participation with $x_i^k = 0$. We thus need to verify that whenever the best response of one member of the group is given by the solution to the FOC, it is not possible for any member of the group to have the best response $x_i^k = 0$. First, we will show that whenever there exists a solution of the FOC for one individual of a group, it exists for all individuals: From the FOCs of two representative group members l, k we obtain the within-group equilibrium condition:

$$\forall l, k < m_i : \frac{(y_i^k)^{\frac{1}{\gamma}-1}}{v_i^k} = \frac{(y_i^l)^{\frac{1}{\gamma}-1}}{v_i^l} \quad (\text{F.4})$$

for all members k, l of group i . Both, the LHS and RHS of (F.4) are strictly increasing in y_i^k, y_i^l if $\gamma \in (0, 1)$. For $\gamma \in (-\infty, 0)$ both LHS and RHS of (F.4) are strictly decreasing in y_i^k, y_i^l . Thus, for each y_i^k there exists a y_i^l such that the within-group equilibrium condition holds. Since for all group members the LHS of (F.1) is equal, there exists a positive solution to the FOC for either all group members or none.

Second, we need to show that $x_i^k = 0$ is not a best response if it is a best response for another individual l in the group to play $x_i^l > 0$. We do so by contradiction: Obviously, for a corner solution with $x_i^k = 0$ and $x_i^l > 0$ the following condition needs to hold:

$$\frac{\partial \pi_i^k}{\partial x_i^k} = \frac{Q_{/i} Q_i}{Q^2 Y_i} (x_i^k)^{\gamma-1} v_i^k - 1 \Big|_{x_i^k = 0, x_i^l > 0} \leq 0. \quad (\text{F.5})$$

From the fact that there is an individual l in the group, which participates with strictly positive effort, we know that

$$\frac{\partial \pi_i^l}{\partial x_i^l} = \frac{Q_{/i} Q_i}{Q^2 Y_i} (x_i^l)^{\gamma-1} v_i^l - 1 \Big|_{x_i^k = 0, x_i^l > 0} = 0. \quad (\text{F.6})$$

Inserting (F.6) into (F.5) yields:

$$\frac{(x_i^l)^{1-\gamma}}{v_i^l} - \frac{(x_i^k)^{1-\gamma}}{v_i^k} \Big|_{x_i^k = 0, x_i^l > 0} \leq 0 \quad (\text{F.7})$$

from which we obtain by inserting $x_i^k = 0$:

$$(x_i^l)^{1-\gamma} \Big|_{x_i^l > 0} \leq 0, \quad (\text{F.8})$$

which is a contradiction for all $\gamma < 1$. Thus there does not exist an equilibrium in which for one player in the group a corner solution at zero effort investments is obtained while for another an interior solution holds. \square

Appendix G: Proof of Lemma 2

Proof. If there exists a solution to the FOC, it is characterized by the following equation, obtained by solving (F.4) for y_i^l and summing over all l ,

$$Y_i = y_i^k \sum_l \left(\frac{v_i^l}{v_i^k} \right)^{\frac{\gamma}{1-\gamma}}. \quad (\text{G.1})$$

We can now solve equation (F.1) for Y_i explicitly:

$$Y_i = \left(\sqrt{Q_{/i} V_i(\gamma)} - Q_{/i} m_i^{\frac{1-\gamma}{\gamma}} \right)^\gamma. \quad (\text{G.2})$$

Thus, the condition for a strictly interior solution is $(1/m_i \sum_l v_i^l)^{\frac{\gamma}{1-\gamma}} > Q_{/i}$. Note that this condition is the same for all members of a group. In all other cases, we get $y_i^k = 0$ for $\gamma \in (0, 1)$ and $y_i^k = \infty$ for $\gamma \in (-\infty, 0)$ as was to be expected since both cases correspond to $x_i^k = 0$. In these cases we have $\forall l : y_i^k = y_i^l$ by equation (F.4) and by the definition of Q_i , we have: $Q_i = Y_i^{\frac{1}{\gamma}} / m_i^{(1-\gamma)/\gamma} = 0$. We can write a group best-response function as

$$\hat{Q}_i(\gamma, Q_{/i}) = \max \left(0, \sqrt{Q_{/i} V_i(\gamma)} - Q_{/i} \right). \quad (\text{G.3})$$

establishing part a), since by Lemma 1 either for all group members we obtain an interior solution or for none. Since the best-response function is continuous in $\gamma \neq 0$ and in the strategies of the other groups $Q_{/i}$, if a unique Nash equilibrium exists, the equilibrium strategies must also be continuous in γ . This establishes part c) of Lemma 2. What remains to be shown is which groups participate in equilibrium. Suppose a group ζ participates in equilibrium with strictly positive effort, while a group $\zeta + 1$ does not participate. Let $Q_i^*(\gamma)$ be Q_i in equilibrium (we ignore here that these are best responses and should thus be functions of $Q_{/i}^*$) and let the other variables introduced above be defined correspondingly in equilibrium. Then by the

above condition in equilibrium we have for any given γ :

$$\begin{aligned} V_\zeta(\gamma) &> Q_{/\zeta}^*(\gamma) \\ V_{\zeta+1}(\gamma) &\leq Q_{/\zeta+1}^*(\gamma) \end{aligned} \quad (\text{G.4})$$

Since by assumption $Q_{\zeta+1}^*(\gamma) = 0$, we have $Q_{/\zeta+1}^*(\gamma) = Q^*(\gamma)$. Solving (4) for $Q_{/i}$ tells us that in an equilibrium where group ζ participates, the following needs to be true:

$$Q_{/\zeta}^*(\gamma) = \frac{Q^*(\gamma)^2}{V_\zeta(\gamma)}. \quad (\text{G.5})$$

We now insert (G.5) into the first equation of (G.4) and the condition $\hat{Q}_{/\zeta+1} = \hat{Q}$ into the second equation. Thus the condition (G.4) becomes

$$\begin{aligned} V_p(\gamma) &> Q^*(\gamma) \\ V_{\zeta+1}(\gamma) &\leq Q^*(\gamma) \end{aligned} \quad (\text{G.6})$$

in equilibrium. It follows that $V_\zeta(\gamma) > V_{\zeta+1}(\gamma)$. We can thus order the groups such that $V_i(\gamma) \geq V_{i+1}(\gamma)$ and define $n^*(\gamma)$ as the group with the highest index number that still participates with strictly positive effort. By (G.6), all groups $i \leq n^*(\gamma)$ participate. This establishes part b) of Lemma 2. \square

Appendix H: Proof of Proposition 6

Proof. The winning probability of each group is given by:

$$\frac{Q_i^*(\gamma)}{Q^*(\gamma)} = \left(1 - \frac{n^*(\gamma) - 1}{\sum_{j=1}^{n^*(\gamma)} V_j(\gamma)^{-1}} \frac{1}{V_i(\gamma)} \right), \quad (\text{H.1})$$

where $V_i(\gamma) = \left(\frac{1}{m_i} \cdot \sum_l (v_i^l)^{\frac{\gamma}{1-\gamma}} \right)^{\frac{1-\gamma}{\gamma}}$. Since the winning probability increases in V_i , the winning probability is increased by an additional member x of the group if the following holds:

$$\left(\frac{\sum (v_i^l)^{\frac{\gamma}{1-\gamma}}}{m_i} \right)^{\frac{1-\gamma}{\gamma}} \leq \left(\frac{\sum (v_i^l)^{\frac{\gamma}{1-\gamma}} + (v_i^x)^{\frac{\gamma}{1-\gamma}}}{m_i + 1} \right)^{\frac{1-\gamma}{\gamma}}. \quad (\text{H.2})$$

This condition yields after solving:

$$v_i^x \geq V_i. \quad (\text{H.3})$$

Notice, if we substitute $\theta = \frac{\gamma}{1-\gamma}$, $V_i(\theta)$ is a power mean with the exponent ranging over $\theta \in (-1, \infty)$ where the greatest lower bound is given by $\gamma \rightarrow -\infty$ and the least upper bound given by $\gamma \rightarrow 1$. It follows from the power-mean inequality that the power mean is weakly increasing in its exponent θ and strictly increasing in θ if there are two distinct $v_i^k \neq v_i^l$ (Bullen (2003), chapter 3). Since θ is an increasing function of γ , V_i is strictly increasing in γ . This in turn implies that the minimum valuation a new group member must have in order to raise the winning probability of this group is also increasing in γ . \square

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